Cambridge Assessment International Education

# Cambridge O Level

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| CANDIDATE NAME | | | | | | | | | |
| CENTRE NUMBER | | | | | | CANDIDATE NUMBER | | | | |

*3119764393*

**ADDITIONAL MATHEMATICS** **4037/23**

Paper 2 **October/November 2023**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.


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***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series* $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series* $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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**1** The functions f and g are defined as follows, for all real values of $x$.

$$\mathrm{f}(x) = 2\sin x + 3\cos x$$

$$\mathrm{g}(x) = \mathrm{e}^{3x} - 1$$

**(a)** Find fg(0). [2]

**(b)** Find gg($x$). [1]

**(c)** Solve the equation $\mathrm{g}^{-1}(x) = \frac{1}{3}\ln 5$. [3]

**2** Find the values of $k$ for which the curve $y = x^2 + kx + (4k - 15)$ is completely above the $x$-axis. [4]

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**3** **(a)** Solve the following simultaneous equations.

$$3\log_2 x + 2\log_2 y = 24$$

$$5\log_2 x - 3\log_2 y = 2$$ [5]

**(b)** Solve the equation $\dfrac{2^{t+4}}{2^{1-2t}} = 512$. [4]

**4** Find the exact value of $\int_3^5 \frac{(x-1)^2}{x^3}\,\mathrm{d}x$. [6]

**5** The curved surface area of a cylinder with radius $r$ and height $h$ is $2\pi rh$.

A closed cylinder has radius $r$ cm and volume $1000\,\text{cm}^3$.

**(a)** Show that the total surface area of the cylinder is $2\pi r^2 + \frac{2000}{r}\,\text{cm}^2$. [3]

**(b)** Find the value of $r$ which makes this area a minimum. You should show that your value of $r$ gives a minimum for this area. [5]

**6** A particle travels in a straight line. Its displacement, $s$ metres, from the origin, at time $t$ seconds, where $t > 2$, is given by $s = \ln(4t^2 - 5) - t$.

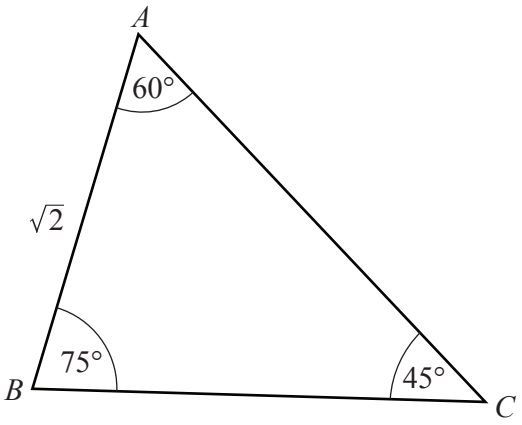
**(a)** Find expressions for the velocity, $v\,\text{ms}^{-1}$, and acceleration, $a\,\text{ms}^{-2}$, of the particle. [4]

**(b)** Find the time when the particle is at rest. [3]

**(c)** Find the acceleration at this time. [2]

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**7 DO NOT USE A CALCULATOR IN THIS QUESTION.**




You may use the following trigonometrical ratios.

$\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 60° = \frac{1}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 60° = \sqrt{3}$, $\tan 45° = 1$

**(a)** Given that the area of triangle $ABC$ is $\frac{3+\sqrt{3}}{4}$, show that $\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$. [5]

**(b)** Hence find the exact length of $AC$. [2]

**8** **(a)** Show that $\dfrac{\sin x}{\tan x - 1} - \dfrac{\cos x}{\tan x + 1} = \dfrac{\cos x}{\sin^2 x - \cos^2 x}$. [5]

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**(b)** Hence solve the equation $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = 1$ for $0° < x < 360°$. [5]

**9** A curve has equation $y = xe^{2x}$.

**(a)** Find $\frac{dy}{dx}$. [2]

**(b)** Find the equation of the normal to the curve at $x = 1$. [4]

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**(c)** Use your answer to **part (a)** to find the exact value of $\int_0^2 2xe^{2x}\,dx$. [5]

**10** **(a)** In an arithmetic progression the 5th term is 11. The 7th term is three times the 2nd term. Find the 1st term and the common difference. [4]

**(b)** A different arithmetic progression (AP) and a geometric progression (GP) have the following properties.

- The 1st terms of the AP and GP are both 3.
- The 2nd term of the AP is the same as the 3rd term of the GP.
- The 6th term of the AP is the same as the 5th term of the GP.
- The common ratio of the GP is greater than 1.

Find the common difference of the AP and the common ratio of the GP. [6]

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