Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

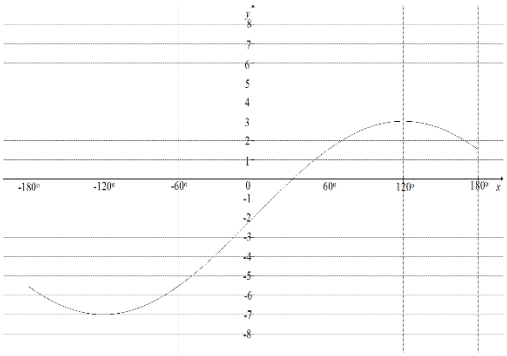
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

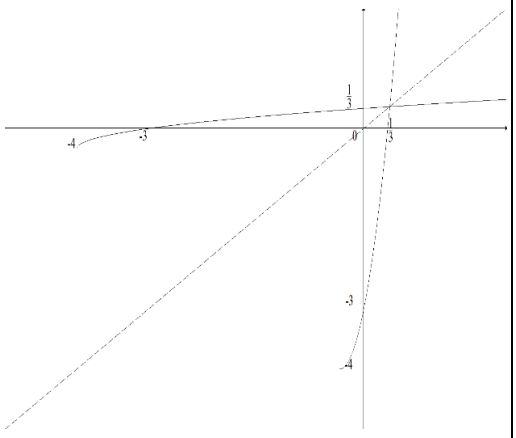
awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1(a)	$\left(\text{f}\left(x\right) \text{ or } y=\right)\ -3(3x+1)(x-1)(2x-5)$	**3**	**B1** for $k\left(x+\frac{1}{3}\right)(x-1)\left(x-\frac{5}{2}\right)$ and no other work that would gain marks. **B2** for $m(3x+1)(x-1)(2x-5)$ and no other work that would gain marks.
1(b)	$-\frac{1}{3}<x<1$	**B1**	Must be in terms of x
	$x>\frac{5}{2}$	**B1**	Must be in terms of x
2(a)	5	**B1**	
2(b)	480^{o}	**B1**	
2(c)		**3**	**To obtain any marks the graph must be a curve with one min in the third quadrant and one max in the first quadrant.** **B1** for the shape, starting in the 3rd quadrant and ending in the 1st quadrant. Must cross the x-axis only once, between 0^{o} and 60^{o}. Must extend for the complete domain starting with $-6<y<-5$ and ending with $1<y<2$ **B1** for passing through $(0,\ -2)$ **B1** for passing though $(120^{\text{o}},\ 3)$ and $(-120^{\text{o}},\ -7)$ soi
3	$\ln(y+2)=mx^2+c$ soi	**B1**	
	Either of: $9.37=2.25m+c$ $3.92=4.75m+c$	**M1**	For at least one correct equation involving m and c
	$m=-2.18,\ -\frac{109}{50}$ oe $c=14.3,\ 14.28,\ 14.275,\ \frac{571}{40}$	**2**	**Dep M1** for attempt to solve for at least one unknown. **A1** for both.
	$y=\text{e}^{\left(14.3-2.18x^2\right)}-2$ oe	**A1**	**FT** on the first **M1** for *their* m and c

Question	Answer	Marks	Guidance
3	**Alternative**		
	$\ln(y+2) = mx^2 + c$ soi	**(B1)**	
	Gradient = –2.18, $-\frac{109}{50}$ oe	**(B1)**	
	$9.37 = 2.25m + c$ $3.92 = 4.75m + c$	**(M1)**	Use of a correct equation with *their* gradient and *c*
	$c = 14.3,\ 14.28,\ 14.275,\ \frac{571}{40}$	**(A1)**	
	$y = e^{(14.3 - 2.18x^2)} - 2$ oe	**(A1)**	**FT** on *their m* and *c*
4(a)	$n = 16$	**B1**	
	$+\frac{n(n-1)}{2!}\left((-)\frac{x}{2}\right)^2$, ${}^nC_2\left((-)\frac{x}{2}\right)^2$ oe $\frac{n(n-1)}{8} = p,\ p = 30$	**2**	**M1** for attempt at third term allow unsimplified in terms of *n* or *their n*, but not just as part of an expansion unless used to find *p* **A1** for *p*.
	$+\frac{n(n-1)(n-2)}{3!}\left((-)\frac{x}{2}\right)^3$, ${}^nC_3\left((-)\frac{x}{2}\right)^3$ $\frac{n(n-1)(n-2)}{48} = q,\quad q = -70$	**2**	**M1** for attempt at fourth term, allow unsimplified in terms of *n* or *their n*, but not just as part of an expansion unless used to find *q* **A1** for *q*.
4(b)	${}^6C_4\left(\frac{2}{x^2}\right)^2\left(\frac{x}{3}\right)^4$	**B1**	For identifying the correct term and attempting to evaluate.
	$\frac{20}{27}$	**B1**	
5	$\cos\left(2\theta + \frac{\pi}{6}\right) = (\pm)\frac{\sqrt{3}}{2}$ oe or $\tan\left(2\theta + \frac{\pi}{6}\right) = (\pm)\frac{1}{\sqrt{3}}$ oe	**B1**	
	$\theta = -\frac{\pi}{6},\ 0,\ \frac{\pi}{3}$ oe	**4**	**M1** for a correct order of operations, may be implied by one correct solution. **A1** for 1 correct solution. **A1** for a 2nd correct solution **A1** for a 3rd correct solution with no extra solutions in the range. All solutions must be from correct working.

Question	Answer	Marks	Guidance
6(a)	$c=12$	**B1**	
	$\text{p}\left(\frac{1}{3}\right)$: $\frac{a}{27}+\frac{b}{9}+\frac{c}{3}-5=0$ soi	**M1**	Allow one arithmetic or sign error, may substitute in their c.
	$\text{p}(2)$: $8a+4b+2c-5=95$	**M1**	Allow one arithmetic or sign error, may substitute in their c.
	$a+3b=27$ or $\frac{a}{27}+\frac{b}{9}=1$ oe	**A1**	Allow multiples but c needs to have been eliminated and terms with powers evaluated.
	$2a+b=19$ oe	**A1**	Allow multiples but c needs to have been eliminated
	$a=6,\ b=7$	**2**	**M1**, **dep** on at least one previous **M1**, for attempt to solve *their* equations in a and b only, to find a or b. **A1** for both a and b.
6(b)	$(3x-1)(2x^2+3x+5)$ cao	**2**	**M1** for attempt at 2 terms in *their* quadratic factor. **A1** for both factors.
	For $2x^2+3x+5=0$, discriminant is less than zero, so no solutions. [Only solution is $x=\frac{1}{3}$.]	**B1**	Allow other valid arguments, but must be using a correct quadratic factor and an attempt to evaluate the discriminant
7(a)(i)	136 080	**B1**	
7(a)(ii)	(End in 0) 15 120 or ${}^{9}\text{P}_{5}$ or $9\times8\times7\times6\times5$	**B1**	
	(End in 5) 13 440 or $8\times{}^{8}\text{P}_{4}$ or $8\times8\times7\times6\times5$	**B1**	
	Total: 28 560	**B1**	
	Alternative 1		
	(Does not start with 5:) 26 880 or $16\times{}^{8}\text{P}_{4}$	**(B1)**	
	(Starts with 5:) 1680 or ${}^{8}\text{P}_{4}$	**(B1)**	
	Total: 28 560	**(B1)**	

Question	Answer	Marks	Guidance
7(a)(ii)	**Alternative 2**		
	(Number not divisible by 5:) 107 520 or $8\times8\times{}^{8}\mathrm{P}_{4}$	**(B1)**	
	136 080 – 107 520	**(B1)**	**FT** on *their* 136 080
	Total: 28 560	**(B1)**	
7(b)(i)	346 104	**B1**	
7(b)(ii)	18	**B1**	Do not isw subsequent work
7(b)(iii)	the number of committees with no dentists: 11 440	**M1**	Allow attempts at 7 options, but must have all of them: 8, 448, 6720, 39 200, 101 920, 122 304 and 64 064
	334 664	**A1**	May come from ${}^{24}\mathrm{C}_{7}-{}^{16}\mathrm{C}_{7}$
8(a)(i)	$a=-\frac{1}{3}$ or $x\geqslant-\frac{1}{3}$	**B1**	Allow -0.333 or better Allow a correct recurring decimal
8(a)(ii)	$\mathrm{f}\geqslant-4$	**B1**	
8(a)(iii)		**4**	**B1** for $y=\mathrm{f}(x)$, must have a correct shape (right hand side from the vertex of a quadratic curve), must be a 1:1 function, intersecting each of the *x* and *y* axes once, in quadrants 1, 3 and 4. **B1**, **dependent** on previous **B** for passing through $(0,-3)$ and $\left(\frac{1}{3},\ 0\right)$. **B1 dependent** on first **B1** for $y=\mathrm{f}^{-1}(x)$, being a correct reflection of *their* $y=\mathrm{f}(x)$, intersecting each of the *x* and *y* axes and $y=\mathrm{f}(x)$ once. **B1 dependent** on previous **B** for passing through $(-3,\ 0)$ and $\left(0,\ \frac{1}{3}\right)$.
8(b)	$3\left(\ln\left(2x^{2}+5\right)\right)-2(=4)$	**M1**	For correct order
	$x=\sqrt{\frac{\mathrm{e}^{2}-5}{2}}$ or exact equivalent	**2**	**M1 dep** for a correct attempt to deal with logarithms and obtain *x* = … Allow one arithmetic or sign slip.

Question	Answer	Marks	Guidance
9	$12\left(x^{\frac{2}{3}}\right)^2 - 11\left(x^{\frac{2}{3}}\right) - 5 = 0$	**B1**	For recognition of a 3-term quadratic equation in terms of $x^{\frac{2}{3}}$ or a suitable substitution
	$\left(3x^{\frac{2}{3}} + 1\right)\left(4x^{\frac{2}{3}} - 5\right) = 0$ $x^{\frac{2}{3}} = -\frac{1}{3},\ x^{\frac{2}{3}} = \frac{5}{4}$	**2**	**M1** for attempt to solve a 3-term quadratic equation in the form $12u^2 \pm 11u \pm 5 = 0$ to obtain at least one solution in the form $x^{\frac{2}{3}} = ...$ or 'u' = … **A1** for at least one correct solution.
	$x = 1.4$ only	**A1**	
10(a)	$27 = 12\theta$ $\theta = \frac{9}{4}$ oe	**B1**	$\angle AOB = \theta$
	Either $\tan(\pi - \theta) = \frac{CB}{12}$ soi Or $\frac{CB}{\sin(\pi - \theta)} = \frac{12}{\sin\left(\theta - \frac{\pi}{2}\right)}$	**M1**	Allow with *their* θ.
	Perimeter = $24 + 27 + 2(14.86..)$	**M1**	Allow with *their CB*.
	Perimeter = awrt 80.7	**A1**	From correct working only
10(b)	$\left(\frac{1}{2} \times 12^2 \times \textit{ their } \theta\right) + (12 \times \textit{ their } CB)$ 340 oe or 341 oe	**3**	**M1** for each area **A1** for awrt 340 or 341
11(a)	$\overrightarrow{AX} = \frac{\mathbf{b}}{2} - \mathbf{a}$	**B1**	Allow unsimplified
	$\overrightarrow{OZ} = \mathbf{a} + \lambda\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$	**2**	**M1** for $\overrightarrow{OZ} = \mathbf{a} + \lambda \times their\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$ Allow unsimplified Mark final answer
11(b)	$\overrightarrow{OY} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ oe	**B1**	Allow unsimplified
	$\overrightarrow{OZ} = \frac{\mu}{2}(\mathbf{a} + \mathbf{b})$	**B1**	**FT** on *their* $\overrightarrow{OY}$, allow unsimplified Mark final answer

Question	Answer	Marks	Guidance
11(c)	$\mathbf{a}+\lambda\left(\frac{\mathbf{b}}{2}-\mathbf{a}\right)=\frac{\mu}{2}(\mathbf{a}+\mathbf{b})$	**M1**	For equating *their* final answer for (a) and *their* final answer for (b) and attempt to equate like vectors at least once to obtain a scalar equation
	$\lambda=\mu=\frac{2}{3}$	**2**	**M1 dep** for solving *their* simultaneous equations to obtain at least one unknown. Each equation must be in terms of λ and μ **A1** for both.
11(d)	$\overrightarrow{OZ}=\frac{1}{3}(\mathbf{a}+\mathbf{b})$ oe	**B1**	Must be from correct work
12(a)	Cannot have the square root of a negative number. oe	**B1**	Must be a correct statement related to the question. Allow a numerical argument.
12(b)	$\frac{(x-3)\left(\frac{5}{2}\times(5x-2)^{-\frac{1}{2}}\right)-(5x-2)^{\frac{1}{2}}}{(x-3)^2}$ or $(x-3)^{-1}\frac{5}{2}(5x-2)^{-\frac{1}{2}}$ $+\left(-(x-3)^{-2}\right)(5x-2)^{\frac{1}{2}}$	**3**	**B1** for $\frac{5}{2}\times(5x-2)^{-\frac{1}{2}}$ seen **M1** for an attempt to differentiate a quotient or correct product **A1** for all other terms correct.
	$\frac{(5x-2)^{-\frac{1}{2}}}{2(x-3)^2}(5(x-3)-2(5x-2))$ oe	**M1**	**Dep** for an attempt to simplify to the given form, allow a sign error and an arithmetic error (e.g. Omission of a factor of 2 in the linear term).
	$\frac{-(5x+11)}{2(x-3)^2\sqrt{5x-2}}$ cao	**A1**	