Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1	$3y-(-5y-4)y=6$ oe or $3\left(\frac{-4-x}{5}\right)-x\left(\frac{-4-x}{5}\right)=6$ or $x+5\left(\frac{6}{3-x}\right)=-4$ oe	**M1**	
	$5y^2+7y-6=0$ or $x^2+x-42=0$	**A1**	
	Factorises *their* 3-term quadratic expression or solves *their* 3-term quadratic equation, e.g. $(5y-3)(y+2)$ [= 0] or $(x-6)(x+7)$ [= 0]	**M1**	
	$x = 6,\quad y = -2$ $x = -7,\ y = 0.6$	**A2**	**A1** for either $x = 6,\quad x = -7$ or $y = -2,\ y = 0.6$ or for an x, y pair from a correct factorisation or correct solving of a correct equation. The method of solution **must** be seen in this case.
2	$e^{2x-3-(5-x)}=\frac{7}{4}$ oe or $e^{5-x-(2x-3)}=\frac{4}{7}$ oe	**M1**	
	$e^{3x-8}=\frac{7}{4}$ oe, soi or $e^{8-3x}=\frac{4}{7}$ oe, soi	**A1**	
	$3x-8=\ln\frac{7}{4}$ oe or $8-3x=\ln\frac{4}{7}$ oe	**M1**	**FT** expression of similar structure which has at most one sign or arithmetic slip
	$x=\frac{\ln 1.75+8}{3}$ oe or $x=\frac{8-\ln\frac{4}{7}}{3}$ oe or 2.85[32…] rot to 3 or more sf	**A1**	

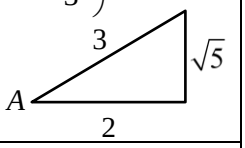
Question	Answer	Marks	Guidance
2	**Alternative method**		
	$\ln 4 + \ln e^{2x-3} = \ln 7 + \ln e^{5-x}$ oe, soi	**(M1)**	
	$\ln 4 + 2x - 3 = \ln 7 + 5 - x$ oe	**(A1)**	
	$2x + x = \ln 7 + 5 - \ln 4 + 3$ or $3x - 8 = \ln \frac{7}{4}$ oe or $8 - 3x = \ln \frac{4}{7}$ oe	**(M1)**	**FT** expression of similar structure which has at most one sign or arithmetic slip
	$x = \frac{\ln 7 - \ln 4 + 8}{3}$ oe or 2.85[32…] rot to 3 or more sf	**(A1)**	
3	$\left[m_{\text{tangent}} =\right] -ax^{-2} + 3$ oe	**B1**	
	[When $x = 1$, m_{normal} =] $\frac{-1}{3-a}$ oe or gradient of tangent = 4 soi	**B1**	**FT** $\frac{-1}{their \left.\frac{dy}{dx}\right\|_{x=1}}$ if appropriate
	$\frac{-1}{their(3-a)} = -\frac{1}{4}$ oe or *their* $(3 - a) = 4$ oe	**M1**	**FT** $\frac{-1}{their \left.\frac{dy}{dx}\right\|_{x=1}}$ or *their* $\left.\frac{dy}{dx}\right\|_{x=1}$ and *their* evaluation of $\frac{-1}{-\frac{1}{4}}$
	$a = -1$ nfww	**A1**	
	[When $x = 1$] *their* $0 = -\frac{1}{4}[1] + b$ oe	**M1**	**FT** $y =$ **(***their a***)** + 1 providing at least 2 of the first 3 marks awarded
	$b = \frac{1}{4}$ nfww	**A1**	

Question	Answer	Marks	Guidance
4	$\log_3\left(\frac{11x-8}{x^2}\right)=1$ or $\log_3(11x-8)=\log_3(3x^2)$ soi OR $\log_x\left(\frac{11x-8}{3}\right)=2$ or $\log_x(11x-8)=\log_x(3x^2)$ soi	**M2**	**M1** for correct use change of base in a correct equation so that all logs have consistent base: $\log_x 3=\frac{\log_3 3}{\log_3 x}$ or $\log_x 3=\frac{1}{\log_3 x}$ oe, soi **OR** $\log_3(11x-8)=\frac{\log_x(11x-8)}{\log_x 3}$ oe, soi
	$3x^2-11x+8\,[=0]$ oe, nfww	**A1**	
	$(3x-8)(x-1)$ [= 0]	**M1**	**FT** *their* 3-term quadratic **dep** on at least **M1** previously awarded
	$x=\frac{8}{3}$ or 2.67 or 2.666[6…] rot to 3 or more dp as only solution	**A1**	
5	$6x^3-5x^2-13x+12 \quad [=0]$	**B1**	
	Uses the correct factor $x-1$ to find a quadratic factor with at least 2 terms correct	**M1**	
	$6x^2+x-12$	**A1**	
	Factorises or solves *their* 3-term quadratic: $(2x+3)(3x-4)$ [= 0] or $[x=]\frac{-1\pm\sqrt{1-4(6)(-12)}}{2(6)}$ oe	**M1**	**dep** on previous **M1**
	$x = 1, -1.5, \frac{4}{3}$ nfww	**A1**	**dep** on all previous marks awarded

Question	Answer	Marks	Guidance
6(a)	510	**3**	**M2** for a fully correct method e.g. [starts with 5, 7, 9 and ends in 3 or two of 5, 7, 9] $3 \times 6 \times 5 \times 3$ or 270 **and** [starts with 6, 8 and ends in 3,5,7, 9] $2 \times 6 \times 5 \times 4$ or 240 **OR** [ends with 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 1$ or 150 **and** [ends with 5, 7, 9 and starts with 6, 8 or two of 5, 7, 9] $4 \times 6 \times 5 \times 3$ or 360 or **M1** for a partially correct method equivalent to one of the above two steps
6(b)	540	**3**	**M2** for a fully correct method e.g. [starts with 5, 7 and ends with 2, 3 and 5 or 7] $2 \times 6 \times 5 \times 3 = 180$ **and** [starts with 6, 8, 9 and ends with 2, 3, 5, 7] $3 \times 6 \times 5 \times 4 = 360$ OR [ends with 2, 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 2$ or 300 **and** [ends with 5, 7 and starts with 6, 8, 9 and 5 or 7] $2 \times 6 \times 5 \times 4$ or 240 or **M1** for a partially correct method equivalent to one of the above two steps

Question	Answer	Marks	Guidance
7(a)	$\frac{\sin^2 x+(1-\cos x)^2}{(1-\cos x)\sin x}$ or $\frac{\sin^2 x}{(1-\cos x)\sin x}+\frac{(1-\cos x)^2}{(1-\cos x)\sin x}$	**M1**	
	$\frac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$	**A1**	OR $\frac{1-\cos^2 x+(1-\cos x)^2}{(1-\cos x)\sin x}$
	$\frac{1+1-2\cos x}{(1-\cos x)\sin x}$ or $\frac{1-\cos^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$	**A1**	OR $\frac{(1-\cos x)(1+\cos x)+(1-\cos x)^2}{(1-\cos x)\sin x}$
	Fully correct justification of given answer: $\frac{2(1-\cos x)}{(1-\cos x)\sin x}=2\operatorname{cosec} x$ or $\frac{2-2\cos x}{(1-\cos x)\sin x}=\frac{2}{\sin x}=2\operatorname{cosec} x$ or equivalent	**A1**	All steps correct and final step justified OR $\frac{1+\cos x+1-\cos x}{\sin x}=2\operatorname{cosec} x$
	Alternative		
	$\frac{\sin x(1+\cos x)}{(1-\cos x)(1+\cos x)}+\frac{(1-\cos x)\sin x}{\sin x\sin x}$ or $\frac{\sin x(1+\cos x)}{1-\cos^2 x}+\frac{(1-\cos x)\sin x}{\sin^2 x}$	**(M1)**	
	$\frac{\sin x+\sin x\cos x}{\sin^2 x}+\frac{\sin x-\cos x\sin x}{\sin^2 x}$	**(A1)**	
	$\frac{2\sin x}{\sin^2 x}$	**(A1)**	
	Fully correct justification of given answer: $\frac{2}{\sin x}=2\operatorname{cosec} x$	**(A1)**	All steps correct and final step justified

Question	Answer	Marks	Guidance
7(b)	$3\sin^2 x - \sin x - 2 \quad [=0]$ soi	**B1**	
	$(3\sin x + 2)(\sin x - 1)$ [= 0] oe	**M1**	
	$\sin x = -\frac{2}{3}$, $\sin x = 1$	**A1**	
	90 221.8 or 221.81[03…] rot to 2 or more dp 318.2 or 318.18[96…] rot to 2 or more dp	**A1**	and no extras in range If **B1 M1 A0 A0** allow **SC1** for 221.8 or 221.81[03…] rot to 2 or more dp **and** 318.2 or 318.18[96…] rot to 2 or more dp and no extras in range
8(a)	$2\pi rh + 2\pi r^2 + \pi r^2$ [= 300] oe	**M1**	
	$h = \frac{300 - 3\pi r^2}{2\pi r}$ oe, isw	**A1**	
8(b)	$V = \pi r^2\left(their\ \frac{300 - 3\pi r^2}{2\pi r}\right) + \frac{2}{3}\pi r^3$ oe	**M2**	**FT** *their h* providing in terms of *r* and derived from a dimensionally correct equation in **(a)**; **M1** for $V = \pi r^2\left(their\ \frac{300 - 3\pi r^2}{2\pi r}\right) + k\pi r^3$, $k \neq \frac{2}{3}$ oe
	Correct completion to given answer: $150r - \frac{5}{6}\pi r^3$ nfww	**A1**	
8(c)	Derivative of *V*: $150 - \frac{5}{2}\pi r^2$ oe, soi, isw	**B1**	
	$their\left(150 - \frac{5}{2}\pi r^2\right) = 0$ and solves as far as r = …	**M1**	**FT** $their\ \frac{\mathrm{d}V}{\mathrm{d}r}$ providing that at least one term is correct
	$r = \sqrt{\frac{300}{5\pi}}$ oe or 4.37 (cm)	**A1**	
	$150(their 4.37) - \frac{5}{6}\pi(their 4.37)^3$	**M1**	**FT** *their* 4.37
	437 or awrt 437 (cm^3) isw	**A1**	

Question	Answer	Marks	Guidance
9(a)	$\frac{1}{2}(\sqrt{5}-1)(\sqrt{5}+1)\sin A = \frac{2\sqrt{5}}{3}$	**M1**	OR $\frac{1}{2}\times(\sqrt{5}+1)\times \text{height} = \frac{2\sqrt{5}}{3}$ **and** $\sin A = (\textit{their}\ \text{height}) \div (\sqrt{5}-1)$
	Simplifies to $\frac{1}{2}\times(5-1)\times\sin A = \frac{2\sqrt{5}}{3}$ oe	**A1**	OR $\sin A = \frac{4\sqrt{5}}{3(\sqrt{5}+1)} \div (\sqrt{5}-1)$ or $\sin A = \frac{5-\sqrt{5}}{3} \div (\sqrt{5}-1)$
	$\frac{\sqrt{5}}{3}$ isw	**A1**	
9(b)	$\cos A = \frac{2}{3}$ or exact equivalent	**B2**	**B1** for $\cos A = \sqrt{1-\textit{their}\left(\frac{\sqrt{5}}{3}\right)^2}$ or for $\cos A = \cos\left(\sin^{-1}\frac{\sqrt{5}}{3}\right)$ or for a sketch 
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2$ $-2(\sqrt{5}-1)(\sqrt{5}+1)\times\cos A$ soi	**M1**	
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2$ $-2(\sqrt{5}-1)(\sqrt{5}+1)\times\frac{2}{3}$ soi	**A1**	
	$x = \sqrt{\frac{20}{3}}$ or $\frac{2}{3}\sqrt{15}$ or $2\sqrt{\frac{5}{3}}$ oe, isw	**A1**	

Question	Answer	Marks	Guidance
9(c)	$\frac{their\ x}{their \sin A} = \frac{\sqrt{5}+1}{\sin B}$ oe, soi or $\frac{1}{2}\times(\sqrt{5}-1)\times their\ x\times\sin B = \frac{2\sqrt{5}}{3}$ oe, soi	**M1**	**FT** *their x* and *their* sin*A*
	Correct expression $\frac{\sqrt{\frac{20}{3}}}{\frac{\sqrt{5}}{3}} = \frac{\sqrt{5}+1}{\sin B}$ oe or $\frac{1}{2}\times(\sqrt{5}-1)\times\sqrt{\frac{20}{3}}\times\sin B = \frac{2\sqrt{5}}{3}$ oe	**A1**	
	$\frac{\sqrt{15}+\sqrt{3}}{6}$ or $\frac{\sqrt{3}}{6}(\sqrt{5}+1)$ or $\frac{\sqrt{5}+1}{2\sqrt{3}}$ oe, isw	**A1**	
10(a)	$ar^2 = 4.5$ and $ar^5 = 15.1875$ soi	**B1**	
	Correctly eliminates one unknown using correct equations e.g $\left(\frac{4.5}{r^2}\right)r^5 = 15.1875$ or $\sqrt{\frac{4.5}{a}} = \sqrt[5]{\frac{15.1875}{a}}$ oe, soi	**M1**	
	$r = 1.5$, $a = 2$	**A2**	**A1** for either
10(b)	$S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ and $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe	**B2**	**M1 FT** *their a* and *r* for $S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ or $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe
	Correct plan: $S_{25} - S_{15}$ oe attempted	**M1**	**FT** *their a* and *r*
	99 253	**A1**	

Question	Answer	Marks	Guidance
10(b)	**Alternative 1**		
	first term = (*their*2)(*their*1.5)^{15} and an attempt at S_{10}	**(M1)**	**FT** *their a* and *r*
	Correct sum $S_{10} = \frac{875.7...(1-1.5^{10})}{1-1.5}$ oe	**(B2)**	**M1 FT** *their* first term and *r* for $S_{10} = \frac{\textit{their}875.7...(1-\textit{their}1.5^{10})}{1-\textit{their}1.5}$
	99 253	**(A1)**	
	Alternative 2		
	Correct sum: $2(1.5)^{15} + 2(1.5)^{16} + 2(1.5)^{17} + 2(1.5)^{18} + 2(1.5)^{19} + 2(1.5)^{20} + 2(1.5)^{21} + 2(1.5)^{22} + 2(1.5)^{23} + 2(1.5)^{24}$ oe or $2(1.5)^{15}\{1 + 1.5 + (1.5)^2 + (1.5)^3 + (1.5)^4 + (1.5)^5 + (1.5)^6 + (1.5)^7 + (1.5)^8 + (1.5)^9\}$	**(M3)**	**M2 FT** *their a* and *r* for sum starting with $2(1.5)^{15}$ **and** ending with $2(1.5)^{24}$ with at most one omission or error or **M1 FT** *their a* and *r* for sum starting with $2(1.5)^{15}$ **or** ending with $2(1.5)^{24}$ with at most two omissions or errors
	99 253	**(A1)**	
11(a)	(–2, 2)	**B2**	**B1** for one correct coordinate nfww **M1** for $\overrightarrow{AC} = \frac{1}{3}\begin{pmatrix} 9 \\ -12 \end{pmatrix}$ or $\overrightarrow{CA} = \frac{1}{3}\begin{pmatrix} -9 \\ 12 \end{pmatrix}$ for $x = -5 + 3$ or $x = 4 - 6$ or for $y = 6 - 4$ or $y = -6 + 8$ or for $x + 5 = \frac{4-x}{2}$ or $6 - y = \frac{y+6}{2}$ or for $2\left(\overrightarrow{OC} - \begin{pmatrix} -5 \\ 6 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \overrightarrow{OC}$ oe

Question	Answer	Marks	Guidance
11(b)	$m_{AB} = \frac{-6-6}{4-(-5)}$ oe or $-\frac{12}{9}$ or $-\frac{4}{3}$	**B1**	
	$m_{CD} = \frac{3}{4}$	**M1**	**FT** $\frac{-1}{\textit{their } m_{AB}}$
	$y - 2 = \frac{3}{4}(x+2)$ oe or $y = \frac{3}{4}x + c$ and $2 = \left(\frac{3}{4}\right)(-2)+c$ oe soi	**M1**	**FT** *their* (–2, 2) and $\frac{-1}{\textit{their } m_{AB}}$
	$y = \frac{3}{4}x + \frac{7}{2}$ or equivalent in form $y = mx + c$	**A1**	
11(c)	$(x-4)^2 + (y+6)^2 = 125$ oe, soi	**B1**	
	Uses *their* $y = \frac{3}{4}x + \frac{7}{2}$ to eliminate one unknown	**M1**	if correct implies **B1**
	Correct equation in one unknown $[BD^2 =](x-4)^2 + \left(\frac{3}{4}x + \frac{7}{2} + 6\right)^2 = 125$ oe	**A1**	
	Writes in solvable form: $25x^2 + 100x - 300 = 0$ oe	**A1**	
	Factorises or solves a correct 3-term quadratic	**A1**	
	(2, 5) and (–6, –1)	**A1**	If **B1, M0** award: **SC2** for identifying one correct point by inspection from the length equation and testing it in the correct equation of *CD* **and** **SC2** for identifying the second correct point by inspection from the length equation and testing it in the correct equation of *CD*

Question	Answer	Marks	Guidance
11(c)	**Alternative**		
	$[BC =]$ $\sqrt{(4-(their-2))^2+(-6-(their2))^2}$	**(B1)**	**FT** *their C*
	$CD = \sqrt{125 - their100}$	**(M1)**	**FT** *their* BC^2 providing $125 - their\ 100 > 0$
	$CD = 5$	**(A1)**	
	$\begin{pmatrix}4\\3\end{pmatrix}+\begin{pmatrix}-2\\2\end{pmatrix}$ or $\begin{pmatrix}-4\\-3\end{pmatrix}+\begin{pmatrix}-2\\2\end{pmatrix}$ OR finds $25x^2 + 100x - 300 = 0$ oe	**(A2)**	**A1** for $\overrightarrow{CD_1}=\begin{pmatrix}4\\3\end{pmatrix}$ or $\overrightarrow{CD_2}=\begin{pmatrix}-4\\-3\end{pmatrix}$ soi OR **A1** for finds $(x+2)^2+\left(\frac{3}{4}x+\frac{7}{2}-2\right)^2=25$
	(2, 5) and (–6, –1)	**(A1)**	