Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **10** printed pages.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

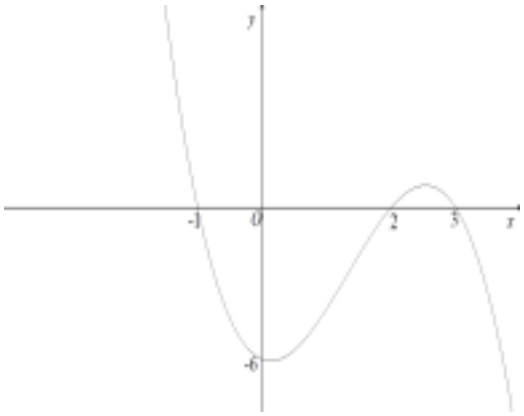
B Mark for a correct result or statement independent of Method marks.

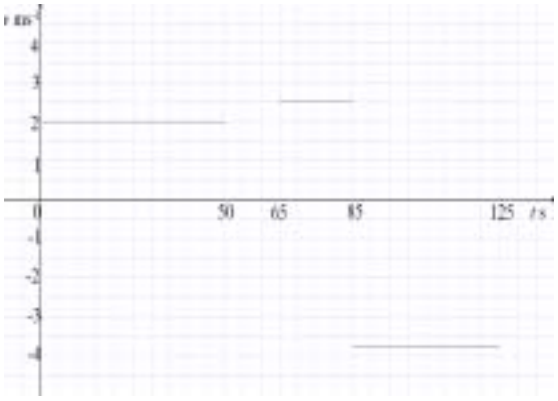
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1(a)		**3**	**B1** for a well-drawn cubic graph in correct orientation Both arms extending beyond x-axis Maximum above x-axis **B1** for x-intercepts **B1** for y-intercept
1(b)	$x < -1$	**B1**	Dep on a cubic curve in the correct orientation and -1 correct on x-axis
	$2 < x < 3$ or $3 > x > 2$	**B1**	Dep on a cubic curve in the correct orientation and 2 and 3 correct on x-axis
2(a)	$\frac{dy}{dx} = \frac{(x^2+1)2e^{2x-3} - 2xe^{2x-3}}{(x^2+1)^2}$ oe or $\frac{dy}{dx} = \frac{2e^{2x-3}}{(x^2+1)} - \frac{2xe^{2x-3}}{(x^2+1)^2}$ oe	**3**	**B1** for $\frac{d}{dx}(e^{2x-3}) = 2e^{2x-3}$ seen in a quotient rule or product rule expression **M1** for correct method for differentiating a quotient or equivalent product **A1 FT** from *their* $2e^{2x-3}$
2(b)	When $x = 2$, $\frac{dy}{dx} = \frac{6e}{25}$	**M1**	For evaluation of *their* $\frac{dy}{dx}$ when $x = 2$
	$\frac{6e}{25} \times \frac{dx}{dt} = 2$ oe	**M1**	For correct substitution of *their* evaluated $\frac{dy}{dx}$ and $\frac{dy}{dt} = 2$ in a correct rates of change equation
	$\frac{dx}{dt} = \frac{25}{3e}$, $\frac{50}{6e}$	**A1**	
3(a)(i)	$x > \frac{1}{2}$	**B1**	Must be using x

Question	Answer	Marks	Guidance
3(a)(ii)	$x = 4\ln(2y-1)$ $e^{\frac{x}{4}} = 2y-1$ $y = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$	**M1**	For full method for inverse using correct order of operations
	$f^{-1}(x) = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$ or $f^{-1}(x) = \frac{1}{2}\left(1+\sqrt[4]{e^x}\right)$	**A1**	Must be using correct notation
	$x \in \mathbb{R}$	**B1**	
3(b)	$\sqrt{2x-3}+5=7$	**M1**	For correct order
	$x = \frac{2^2+3}{2}$	**M1**	Dep on previous M mark, for obtaining x by simplifying and solving using correct order of operations, including squaring
	$x = \frac{7}{2}$ or 3.5	**A1**	
4(a)(i)		**3**	**B1** For $v = 2$ for $0 \leqslant t \leqslant 50$ **B1** For $v = 2.5$ for $65 \leqslant t \leqslant 85$ **B1** For $v = 3.75$ for $85 \leqslant t \leqslant 125$ and $v = 0$ for $50 \leqslant t \leqslant 65$
4(a)(ii)	300	**B1**	
4(b)	$\frac{dx}{dt} = -18\sin\left(3t+\frac{\pi}{3}\right)$	**M1**	$\pm 18\sin\left(3t+\frac{\pi}{3}\right)$
	$\frac{d^2x}{dt^2} = -54\cos\left(3t+\frac{\pi}{3}\right)$	**M1**	$\pm 54\cos\left(3t+\frac{\pi}{3}\right)$
	−27 nfww	**A1**	

Question	Answer	Marks	Guidance
5	$(1+x)\left(1+n\left(-\frac{x}{2}\right)+\frac{n(n-1)}{2}\left(\frac{x^2}{4}\right)...\right)$	**2**	**B1** For $\binom{n}{1}\left(-\frac{x}{2}\right)$ **B1** For $\binom{n}{2}\left(\frac{x^2}{4}\right)$
	$\frac{1}{4}\binom{n}{2}x^2-\frac{1}{2}\binom{n}{1}x^2=\frac{25}{4}x^2$	**M1**	Correctly using two terms in n to obtain an x^2 term and equating to $\frac{25}{4}x^2$ Dep on one B1
	$\frac{n(n-1)}{8}-\frac{n}{2}=\frac{25}{4}$ oe	**A1**	
	$n=10$ only	**A1**	
6(a)	$\lg y=\lg A+bx^2$	**B1**	Stated or may be implied by later work
	If using $\lg y=\lg A+bx^2$ as a starting point $5.25=\lg A+3.63b$ and $6.88=\lg A+4.83b$ or $5.25=\lg A+1.358(3.63)$ or $6.88=\lg A+1.358(4.83)$ OR If finding the equation of the straight line and then finding $\lg A$ and b by inspection $\lg y-6.88=1.358(x^2-4.83)$ or $\lg y-5.25=1.358(x^2-3.63)$ or $\lg y=1.358x^2+0.31..$(or 0.32..)	**M1**	For correctly finding required equation(s)
	$b=1.36$, $\frac{163}{120}$ or $1\frac{43}{120}$	**B1**	Must be b = and from correct working
	A in range 2.05 to 2.09	**A1**	
6(b)	$\lg y=0.3132+(4\times1.36)$ $y=2.09\times10^{4\times1.36}$	**M1**	For $\lg y=(their\ \lg A)+4(their\ b)$ or $y=(their\ A)\left(10^{4(their\ b)}\right)$
	Allow 553 000 to 576 000	**A1**	
6 (c)	$4=2.09\left(10^{1.36x^2}\right)$ or $\lg 4=0.3132+1.36x^2$	**M1**	$4=(their\ A)\left(10^{their\ bx^2}\right)$ or $\lg 4=(their\ \lg A)+(their\ b)x^2$
	awrt 0.46	**A1**	

Question	Answer	Marks	Guidance
7(a)	$-4a+b+5=0$ oe	**B1**	Allow multiples of equation
	$a+b-25=0$ oe	**B1**	Allow multiples of equation
	$a=6,\ b=19$	**2**	**M1** for solving *their* 2 equations and obtaining two solutions **A1** for both $a=6,\ b=19$
	$(x+4)(6x^2-5x+1)$ $A=6,\ B=-5,\ C=1$	**2**	**M1** for attempt to obtain quadratic factor by inspection or by algebraic long division **A1** $(6x^2-5x+1)$ or $A=6,\ B=-5,\ C=1$
	Alternative $a+b-25=0$ oe	**(B1)**	Allow multiples of equation
	Comparing coefficients $C=1$ and $A=a$	**(B1)**	
	$4A+B=b$	**(B1)**	
	Leading to $5A+B=25$	**(M1)**	For use of *their* $a+b-25=0$ to obtain an equation in A and B
	$4B+1=-19$	**(B1)**	
	$(x+4)(6x^2-5x+1)$ $A=6,\ B=-5,\ C=1$	**(A1)**	
7(b)	$(x+4)(3x-1)(2x-1)$	**B1**	Must follow from a correct solution to **(a)**
7(c)	–19	**B1**	
8(a)	$\angle AOB=1.45$ (radians)	**B1**	
8(b)	Sector area $=\frac{1}{2}(r^2)(1.45)$	**B1**	For correct sector area. Allow unsimplified
	Area of triangle $=\frac{1}{2}\times 0.5r\times r\times\sin(\pi-$ *their* $1.45)$	**B1**	For correct area of triangle Allow unsimplified
	Total area = $0.973r^2$	**B1**	

Question	Answer	Marks	Guidance
8(c)	$(AC^2) = r^2 + 0.25r^2 - (2 \times r \times 0.5r \cos(\pi - 1.45))$	**M1**	For correct substitution in cosine rule using $(\pi -$ *their* $1.45)$
	$AC = 1.17r$	**A1**	
	Perimeter = $2.95r + 1.17r$	**B1**	**FT** on *their AC*
	$r = 2.91$	**A1**	
9(a)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$	**B1**	
	$\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}\overrightarrow{AB}$ or $\overrightarrow{OX} = \mathbf{b} + \frac{1}{4}\overrightarrow{BA}$ $\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OX} = \mathbf{b} + \frac{1}{4}(\mathbf{a} - \mathbf{b})$	**M1**	For correct use of ratio, using *their* $\overrightarrow{AB}$ or $\overrightarrow{BA}$
	$\overrightarrow{OX} = \frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}$	**A1**	
9(b)	$\overrightarrow{AC} = 2\mathbf{b} - \mathbf{a}$	**B1**	
9(c)	$\overrightarrow{AY} = -\mathbf{a} + h\left(\frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}\right)$	**B1**	**FT** on *their* $\overrightarrow{OX}$
9(d)	$-\mathbf{a} + h\left(\frac{\mathbf{a}}{4} + \frac{3}{4}\mathbf{b}\right) = m(2\mathbf{b} - \mathbf{a})$	**M1**	For equating appropriate vectors and attempt to equate like vectors
	$-1 + \frac{h}{4} = -m$	**A1**	**FT** from *their* $\overrightarrow{AY}$ and $\overrightarrow{AC}$
	$\frac{3h}{4} = 2m$	**A1**	**FT** from *their* $\overrightarrow{AY}$ and $\overrightarrow{AC}$
	$h = \frac{8}{5}, \ m = \frac{3}{5}$	**A1**	For both
10(a)	$\frac{3x + 10 + 2(x+1)}{(x+1)(3x+10)} = \frac{3x + 10 + 2x + 2}{(x+1)(3x+10)}$ $= \frac{5x + 12}{3x^2 + 13x + 10}$	**B1**	For expansion and simplification to obtain given answer

Question	Answer	Marks	Guidance
10(b)	$P\left(0,\ \frac{6}{5}\right)$ and $Q\left(-\frac{6}{5},\ 0\right)$ oe	**B1**	
	Area of triangle = $\frac{18}{25}$ or 0.72	**B1**	
	Area under curve = $\int_0^2 \frac{1}{x+1}+\frac{2}{3x+10}\ \mathrm{d}x$	**M1**	For use of part **(a)** and attempt to integrate to obtain at least one ln term.
	$=\left[\ln(x+1)+\frac{2}{3}\ln(3x+10)\right]_0^2$	**2**	**B1** For $\ln(x+1)$ **B1** For $\frac{2}{3}\ln(3x+10)$
	$=\ln 3+\frac{2}{3}\ln 16-\frac{2}{3}\ln 10$	**M1**	For correct use of limits. Dep on previous M mark.
	$=\frac{2}{3}\ln 3\sqrt{3}+\frac{2}{3}\ln 16-\frac{2}{3}\ln 10$	**M1**	For use of $\ln 3=\frac{2}{3}\ln 3\sqrt{3}$
	$=\frac{2}{3}\ln 3\sqrt{3}+\frac{2}{3}\ln\left(\frac{16}{10}\right)=\frac{2}{3}\ln\left(\frac{48\sqrt{3}}{10}\right)$	**M1**	For use of multiplication and division rule
	Total area = $\frac{18}{25}+\frac{2}{3}\ln\left(\frac{24\sqrt{3}}{5}\right)$ oe	**A1**	For correct answer in the required form dep on the three preceding M marks Must not be obtained using a calculator
11(a)	$2\cos x=3\frac{\sin x}{\cos x}\ \Rightarrow 2\cos^2 x=3\sin x$	**M1**	For use of $\tan x=\frac{\sin x}{\cos x}$ and multiplying by $\cos x$
	$2\left(1-\sin^2 x\right)=3\sin x$	**M1**	For use of correct identity
	$2\sin^2 x+3\sin x-2=0$	**A1**	For correct rearrangement to obtain the given answer
	Alternative $2\sin^2 x+3\sin x-2$ $=2\left(1-\cos^2 x\right)+3\sin x-2$	**(M1)**	For use of correct identity
	$=-2\cos x\cos x+3\sin x$ $=-3\tan x\cos x+3\sin x$	**(M1)**	For use of $2\cos x=3\tan x$
	$-3\sin x+3\sin x=0$	**(A1)**	For use of $\tan x\cos x=\sin x$ and answer 0

Question	Answer	Marks	Guidance
11(b)	$\sin\left(2\alpha+\frac{\pi}{4}\right)=\frac{1}{2}$ only	**B1**	For solution of quadratic from **(a)** to obtain $\sin\left(2\alpha+\frac{\pi}{4}\right)=\frac{1}{2}$ only
	$2\alpha+\frac{\pi}{4}=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $2\alpha=\frac{7\pi}{12}, \frac{23\pi}{12}$	**M1**	For correct order of operations in attempt to solve $\sin\left(2\alpha+\frac{\pi}{4}\right)=\frac{1}{2}$, may be implied by one correct solution
	$\alpha=\frac{7\pi}{24}$	**A1**	
	$\alpha=\frac{23\pi}{24}$	**A1**	