Cambridge
O Level

**Cambridge Assessment International Education**
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

**ADDITIONAL MATHEMATICS** **4037/23**

Paper 2 **October/November 2019**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

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Cambridge Assessment International Education

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

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**1** Solve $|3x+2| = x+4$. [3]

**2** **(i)** Show that $\dfrac{\text{cosec}x - \cot x}{1-\cos x} = \text{cosec}x$. [3]

**(ii)** Hence solve $\dfrac{\text{cosec}x - \cot x}{1-\cos x} = 2$ for $0° < x < 180°$. [2]

**3** The first four terms in the expansion of $(1+ax)^5(2+bx)$ are $2+32x+210x^2+cx^3$, where $a$, $b$ and $c$ are integers. Show that $3a^2-16a+21=0$ and hence find the values of $a$, $b$ and $c$. [8]

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**4** **(i)** Given that $y = 2x^2 - 4x - 7$, write $y$ in the form $a(x-b)^2 + c$, where $a$, $b$ and $c$ are constants. [3]

**(ii)** Hence write down the minimum value of $y$ and the value of $x$ at which it occurs. [2]

**(iii)** Using your answer to **part (i)**, solve the equation $2p - 4\sqrt{p} - 7 = 0$, giving your answer correct to 2 decimal places. [3]

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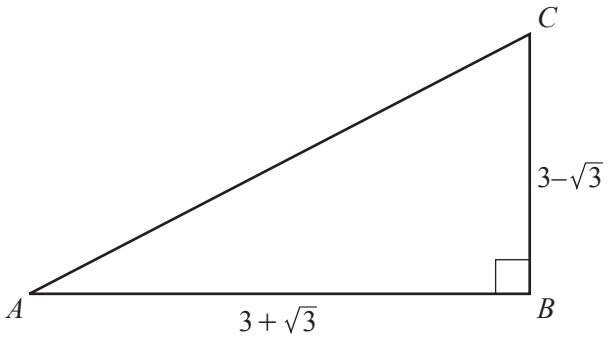
**[Turn over**

**5** **(a)** Solve $3\cot^2\left(y-\frac{\pi}{4}\right)=1$ for $0<y<\pi$ radians. [4]

**(b)** Solve $7\cot z+\tan z=7\operatorname{cosec} z$ for $0°\leqslant z\leqslant 360°$. [6]

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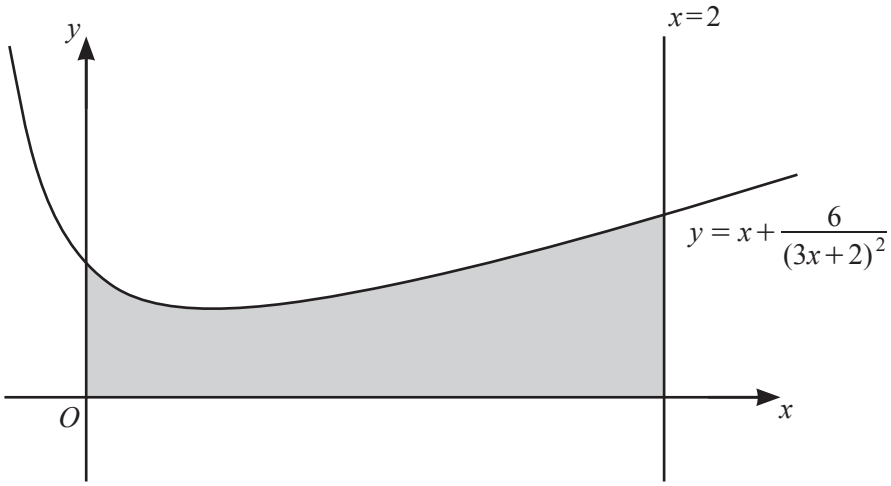
**6 Do not use a calculator in this question.**




**(i)** Find tan $ACB$ in the form $r + s\sqrt{3}$, where $r$ and $s$ are integers. [3]

**(ii)** Find $AC$ in the form $t\sqrt{u}$, where $t$ and $u$ are integers and $t \neq 1$. [3]

**7**




The diagram shows part of the curve $y = x + \frac{6}{(3x+2)^2}$ and the line $x = 2$.

**(i)** Find, correct to 2 decimal places, the coordinates of the stationary point. [6]

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**(ii)** Find the area of the shaded region, showing all your working. [4]

**8** The roots of the equation $x^3 + ax^2 + bx + 24 = 0$ are 2, 3 and $p$, where $p$ is an integer.

**(i)** Find the value of $p$. [1]

**(ii)** Show that $a = -1$ and find the value of $b$. [4]

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Given that a curve has equation $y = x^3 - x^2 + bx + 24$ find, using your value of $b$,

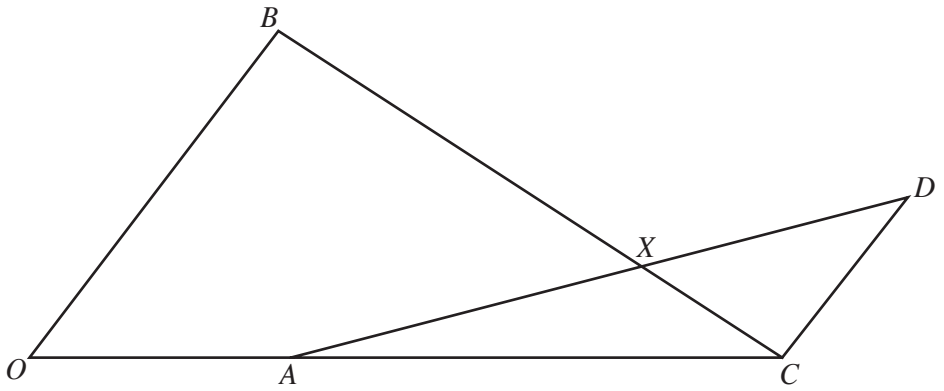
**(iii)** $\frac{dy}{dx}$, [1]

**(iv)** the integer value of $x$ for which the gradient of the curve is 2 and the corresponding value of $y$. [3]

The coordinates of the point $P$ on the curve are given by the values of $x$ and $y$ found in **part (iv)**.

**(v)** Find the equation of the tangent to the curve at $P$. [1]

**9**




The diagram shows points $O$, $A$, $B$, $C$, $D$ and $X$. The position vectors of $A$, $B$, and $C$ relative to $O$ are $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = 2\mathbf{b}$ and $\overrightarrow{OC} = 3\mathbf{a}$. The vector $\overrightarrow{CD} = \mathbf{b}$.

**(i)** Given that $\overrightarrow{AX} = \lambda \overrightarrow{AD}$, find $\overrightarrow{OX}$ in terms of $\lambda$, **a** and **b**. [2]

**(ii)** Given that $\overrightarrow{BX} = \mu \overrightarrow{BC}$, find $\overrightarrow{OX}$ in terms of $\mu$, **a** and **b**. [2]

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**(iii)** Hence find the value of $\lambda$ and of $\mu$. [4]

**(iv)** Find the ratio $\frac{AX}{XD}$. [1]

**10** The functions f and g are defined by

$$\mathrm{f}(x) = \ln(3x+2) \quad \text{for } x > -\frac{2}{3},$$
$$\mathrm{g}(x) = \mathrm{e}^{2x} - 4 \quad \text{for } x \in \mathbb{R}.$$

**(i)** Solve $\mathrm{gf}(x) = 5$. [5]

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**(ii)** Find $f^{-1}(x)$. [2]

**(iii)** Solve $f^{-1}(x) = g(x)$. [4]

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