Cambridge
O Level

**Cambridge Assessment International Education**
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*9243226810*

**ADDITIONAL MATHEMATICS** **4037/22**

Paper 2 **October/November 2019**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.


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***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{r} a^{n-r}\, b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

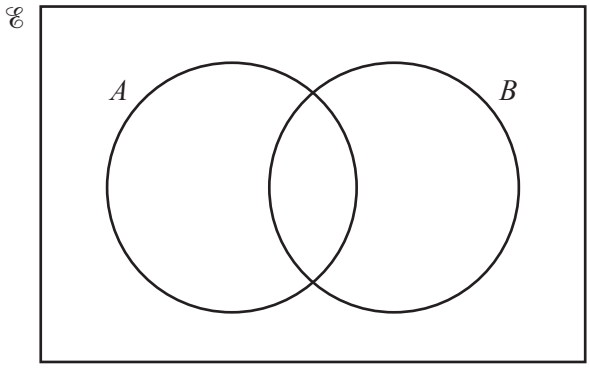
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

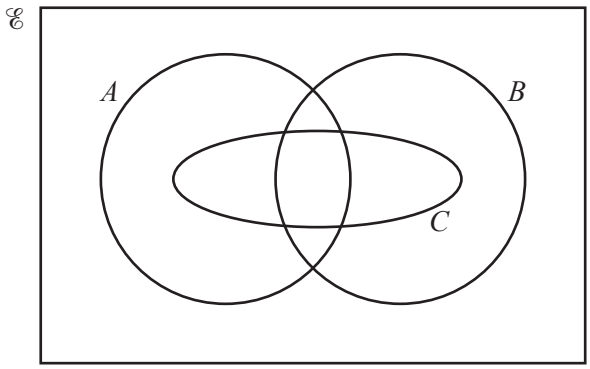
$$\Delta = \frac{1}{2} bc \sin A$$

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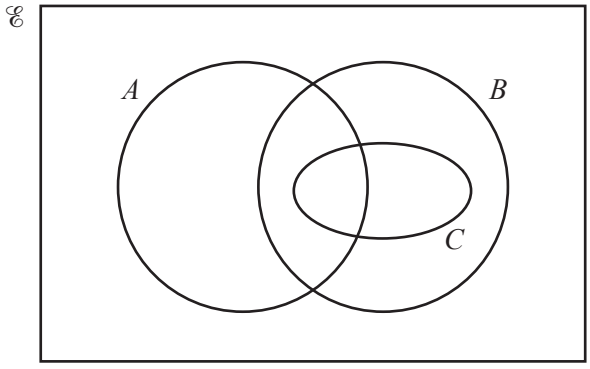
**1** On each of the Venn diagrams below, shade the region indicated.




$(A' \cap B) \cup (A \cap B')$




$(A \cap B) \cup C$




$A \cap B \cap C'$

[3]

**2** Given that $y = 2\sin 3x + \cos 3x$, show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 3y = k\sin 3x$, where $k$ is a constant to be determined. [5]

**3** A 5-digit code is formed using the following characters.

| | | | | | | |
|---|---|---|---|---|---|---|
| Letters | a | e | i | o | u | |
| Numbers | 1 | 2 | 3 | 4 | 5 | 6 |
| Symbols | @ | * | # | | | |

No character can be repeated in a code. Find the number of possible codes if

**(i)** there are no restrictions, [2]

**(ii)** the code starts with a symbol followed by two letters and then two numbers, [2]

**(iii)** the first two characters are numbers, and no other numbers appear in the code. [2]

**4** Find the values of $k$ for which the line $y = kx + 3$ does not meet the curve $y = x^2 + 5x + 12$. [5]

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**5** At the point where $x = 1$ on the curve $y = \dfrac{k}{(x+1)^2}$, the normal has a gradient of $\frac{1}{3}$.

**(i)** Find the value of the constant $k$. [4]

**(ii)** Using your value of $k$, find the equation of the tangent to the curve at $x = 2$. [3]

**6** **(i)** Show that $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=\frac{2}{\sin x}$. [5]

**(ii)** Hence solve the equation $\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

**7** **(a)** The cubic equation $x^3 + ax^2 + bx - 40 = 0$ has three positive integer roots. Two of the roots are 2 and 4. Find the other root and the value of each of the integers $a$ and $b$. [4]

**(b)** **Do not use a calculator in this question.**

Solve the equation $x^3 - 5x^2 - 46x - 40 = 0$ given that it has three integer roots, only one of which is positive. [4]

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**8** **(i)** A particle $A$ travels with a speed of $6.5\,\text{ms}^{-1}$ in the direction $-5\mathbf{i}-12\mathbf{j}$. Find the velocity, $\boldsymbol{v}_A$, of $A$. [2]

**(ii)** A particle $B$ travels with velocity $\boldsymbol{v}_B = 12\mathbf{i}-9\mathbf{j}$. Find the speed, in $\text{ms}^{-1}$, of $B$. [2]

Particle $A$ starts moving from the point with position vector $20\mathbf{i}-7\mathbf{j}$. At the same time particle $B$ starts moving from the point with position vector $-67\mathbf{i}+11\mathbf{j}$.

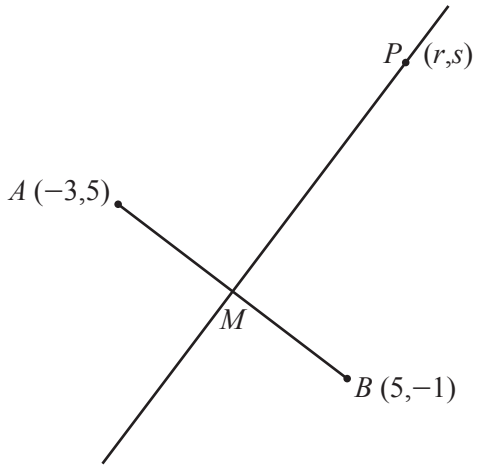
**(iii)** Find $\boldsymbol{r}_A$, the position vector of $A$ after $t$ seconds, and $\boldsymbol{r}_B$, the position vector of $B$ after $t$ seconds. [2]

**(iv)** Find the time when the particles collide and the position vector of the point of collision. [3]

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**[Turn over**

**9**




The diagram shows the points $A$ (–3, 5) and $B$ (5, –1). The mid-point of $AB$ is $M$ and the line $PM$ is perpendicular to $AB$. The point $P$ has coordinates ($r$, $s$).

**(i)** Find the equation of the line $PM$ in the form $y = mx + c$, where $m$ and $c$ are exact constants. [5]

**(ii)** Hence find an expression for $s$ in terms of $r$. [1]

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**(iii)** Given that the length of $PM$ is 10 units, find the value of $r$ and of $s$. [5]

**10** **(i)** Given that $y = \frac{\ln x}{x^2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

**(ii)** Find the coordinates of the stationary point on the curve $y = \frac{\ln x}{x^2}$. [3]

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**(iii)** Using your answer to **part (i)**, find $\int \frac{\ln x}{x^3} \mathrm{d}x$ . [3]

**(iv)** Hence evaluate $\int_1^2 \frac{\ln x}{x^3} \mathrm{d}x$ . [2]

**Question 11 is printed on the next page.**

**11 Do not use a calculator in this question.**

Solve the quadratic equation $(\sqrt{5}-3)x^2+3x+(\sqrt{5}+3)=0$, giving your answers in the form $a+b\sqrt{5}$, where $a$ and $b$ are constants. [6]

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