Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

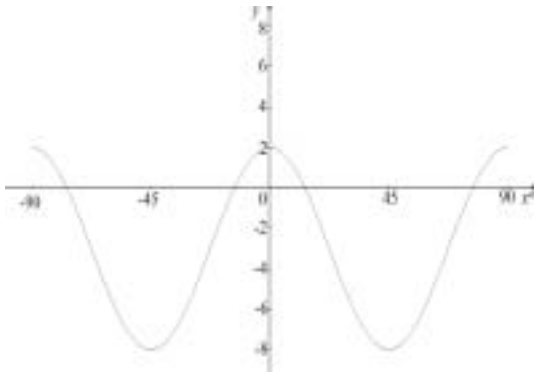
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

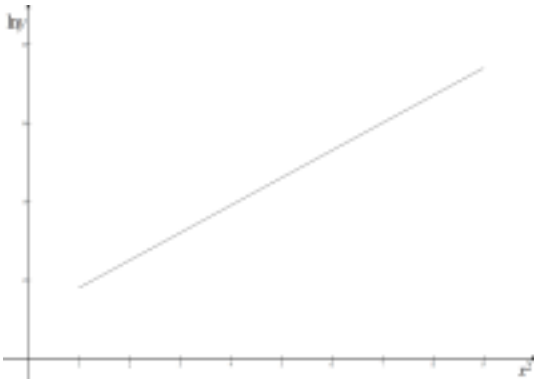
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1(i)		**M1**	for a Venn diagram showing at least 4 correct 'parts' in terms of x
		A1	for all 7 'parts' correct in terms of x on a Venn diagram or in working. May be implied by a correct equation.
	$80+24+x+23-x+3+x=145$ $50+28+x+28-x+24+x=145$ $75+28+x+24-x+3+x=145$ $50+80+75-(23+28+24)+x=145$ or equivalents	**M1**	for forming an equation in x using sum of 'parts' = 145 or $50+80+75-(23+28+24)+x=145$ Equations must be seen
	$x=15$	**A1**	from correct working only
1(ii)	43	**B1ft**	for *their* x plus 28
2(i)		**B4**	**B1** for a maximum at $(0, 2)$ **B1** for minimums at $y=-8$ and no other minimums **B1** for starting at $(-90^{o}, 2)$ and finishing at $(90^{o}, 2)$ **B1** for a fully correct curve with correct shape, particularly at end points, that has earned all three previous B marks.
2(ii)	5	**B1**	
2(iii)	90^{o}	**B1**	
3(i)	$\frac{dy}{dx}=kx\left(3x^2-1\right)^{-\frac{4}{3}}$	**M1**	
	$\frac{dy}{dx}=-\frac{1}{3}\times 6x\left(3x^2-1\right)^{-\frac{4}{3}}$	**A1**	
3(ii)	When $x=\sqrt{3}$, $\frac{dy}{dx}=-\frac{\sqrt{3}}{8}$ $-\frac{\sqrt{3}}{8}p$ or $-0.217p$	**B1**	**FT** on *their* $\frac{dy}{dx}$ of the form $kx\left(3x^2-1\right)^{-\frac{4}{3}}$

Question	Answer	Marks	Guidance
3(iii)	When $x=\sqrt{3}, \quad y=\frac{1}{2}$	**B1**	for $y=\frac{1}{2}$
	Normal: $y-\frac{1}{2}=\frac{8}{\sqrt{3}}\left(x-\sqrt{3}\right)$	**M1**	Dep on M1in part(i). An equation of the normal using *their* normal gradient, $\sqrt{3}$ and *their y*
		A1	allow unsimplified
4(i)	$-\frac{1}{13}\begin{pmatrix}-1 & -2\\ -4 & 5\end{pmatrix}$ oe	**B2**	**B1** for $-\frac{1}{13}$ **B1** for $\begin{pmatrix}-1 & -2\\ -4 & 5\end{pmatrix}$
4(ii)	$\frac{1}{13}\begin{pmatrix}1 & 2\\ 4 & -5\end{pmatrix}\begin{pmatrix}12\\ 7\end{pmatrix}$	**M1**	for pre-multiplication by *their* inverse from (i)
	$=\frac{1}{13}\begin{pmatrix}26\\ 13\end{pmatrix}$	**M1**	for correct method for matrix multiplication
	$=\begin{pmatrix}2\\ 1\end{pmatrix}$	**A1**	
	$x=1.11$	**B1**	
	$y=\frac{\pi}{4}$ or 0.785	**B1**	
5(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(x^2+3\right)\right)=\frac{2x}{\left(x^2+3\right)}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2+3\right)\frac{2x}{\left(x^2+3\right)}+2x\ln\left(x^2+3\right)$	**M1**	for product rule
		A1	**FT** *their* $\frac{2x}{\left(x^2+3\right)}$
5(ii)	$\left(x^2+3\right)\ln\left(x^2+3\right)=\int 2x+2x\ln\left(x^2+3\right)\,\mathrm{d}x$	**M1**	for using *their* result from (i) for $2x+kx\ln\left(x^2+3\right)$
	$\int x\ln\left(x^2+3\right)\,\mathrm{d}x$ $=\frac{1}{2}\left(x^2+3\right)\ln\left(x^2+3\right)-\frac{x^2}{2}(+c)$	**A1**	

Question	Answer	Marks	Guidance
6(i)	$\ln y = \ln A + x^2 \ln b$ or $\lg y = \lg A + x^2 \lg b$	**B1**	May be implied by a table of values for x^2 and $\ln y$ or $\lg y$ or axes labelled $\ln y$ or $\lg y$ and x^2
		**M1**	for attempt to plot either $\ln y$ or $\lg y$ against x^2 using an evenly spaced scale on each axis.
		A2	**A2** All points on a correct line (for $1 \leqslant x^2 \leqslant 9$) with axes correctly labelled **A1** One point not on the correct line or a correct line with axes not correctly labelled. **A0** Two or more points not on the correct line or one point not on the line and axes incorrect
6(ii)	Gradient = $\ln b$ or $\lg b$ $\ln b \approx 0.7$ or $\lg b \approx 0.3$ leading to	**M1**	for a complete method using the gradient of *their* straight-line graph of lg*y* or ln*y* against x^2 to obtain *b*
	$b = 2$ (allow 1.6 – 2.4)	**A1**	from correct working
	Intercept = $\ln A$ or $\lg A$ $\ln A \approx 1.1$ $\lg A \approx 0.5$ leading to	**M1**	for a complete method using intercept of *their* straight-line graph of lg*y* or ln*y* against x^2 to find *A*
	$A = 3$ (allow 2.5 – 3.6)	**A1**	from correct working
6(iii)	$100 = 3\left(2^{x^2}\right)$ or $\ln 100 = \textit{their}1.1 + \textit{their}0.7x^2$ or $\lg 100 = \textit{their}0.5 + \textit{their}0.3x^2$ or reading from lg*y* = 2 to obtain x^2 or from ln*y* = 4.6 to obtain x^2	**M1**	for a valid method to find x^2 Substitution methods should be using values of *A* and *b* in range
	leading to $x = 2.25$ (allow 2.0 – 2.7)	**A1**	for an answer in range from correct working
7(a)(i)	15 120	**B1**	
7(a)(ii)	1680	**B1**	
7(a)(iii) Method 1	Total = 2310	**B3**	B1 1st digit is 7 or 9 1680 or 210×8 B1 1st digit is 8 630 or 210×3
7(a)(iii) Method 2	Total = 2310	**B3**	B1 for 5th digit is 2,4 or 6 1890 or 210×9 B1 for 5th digit is 8 420 or 210×2

Question	Answer	Marks	Guidance
7(b)(i)	3003	**B1**	
7(b)(ii)	28	**B1**	
7(b)(iii)	Total 1419	**B3**	B1 Including husband and wife 495 B1 Excluding husband and wife 924
8(a)(i)	$\log_a a + 2\log_a y + \log_a x$	**M1**	for $\log_a a + \log_a x + \log_a y^2$ **and** $\log_a y^2 = 2\log_a y$
	$1 + 2q + p$	**A1**	
8(a)(ii)	$3\log_a x - \log_a y - \log_a a$	**M1**	for $\log_a x^3 - (\log_a a + \log_a y)$ **and** $\log_a x^3 = 3\log_a x$
	$3p - q - 1$ or $3p - (q+1)$	**A1**	
8(a)(iii)	$\frac{1}{p} + \frac{1}{q}$	**B1**	
8(b)	$m - 3m^2 + 4 = 0$	**M1**	for obtaining a quadratic in m or 3^x
	$m = \frac{4}{3}$, (-1) $x = \frac{\lg\frac{4}{3}}{\lg 3}$, $x = \frac{\ln\frac{4}{3}}{\ln 3}$ or $\lg_3\frac{4}{3}$	**M1**	Dep for attempt to solve quadratic and deal with 3^x correctly
	$x = 0.262$ only	**A1**	
9(i)	$100 = 2r + 2r\theta + 3r\theta$	**M1**	for addition of $2r$ and two arc lengths with at least one correct arc length
	$\theta = \frac{100 - 2r}{5r}$ or $\frac{20}{r} - \frac{2}{5}$ oe	**A1**	
9(ii)	$\frac{1}{2}9r^2\theta - \frac{1}{2}4r^2\theta$	**M1**	for subtraction of two sector areas with at least one sector area correct.
	$\frac{5r^2}{2}\left(\frac{100 - 2r}{5r}\right)$	**A1**	Must expand and simplify to obtain given answer $50r - r^2$
9(iii)	$\frac{\mathrm{d}A}{\mathrm{d}r} = 50 - 2r$ $0 = 50 - 2r$ leading to $r = 25$	**M1**	for differentiation and equating to zero and obtaining r or for using completing the square $-(25 - r)^2 + 25^2$
	Max when $A = 625$	**A1**	

Question	Answer	Marks	Guidance
9(iv)	When $r = 10$, $\frac{\mathrm{d}A}{\mathrm{d}r} = 30$	**B1**	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{30}$	**M1**	for $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{their\,30}$ where *their* 30 has been obtained from an evaluation of $\frac{\mathrm{d}A}{\mathrm{d}r}$ at $r = 10$
	$\frac{\mathrm{d}r}{\mathrm{d}t} = 0.1$ or $\frac{1}{10}$	**A1**	
9(v)	$\frac{\mathrm{d}\theta}{\mathrm{d}r} = -\frac{20}{r^2}$ oe	**B1**	
	$\frac{\mathrm{d}\theta}{\mathrm{d}r} = -\frac{1}{5}$ oe $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{10} \times -\frac{1}{5}$ oe	**M1**	for $their\frac{\mathrm{d}r}{\mathrm{d}t} \times their\frac{\mathrm{d}\theta}{\mathrm{d}r}$ with both evaluated at $r = 10$
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{1}{50}$ or -0.02	**A1**	
10(a)(i)	$\pm\frac{20 - -20}{5}$	**M1**	for finding the gradient of the relevant part
	8	**A1**	
10(a)(ii)	7.5	**B1**	
10(a)(iii)	$\frac{1}{2}(5 + 7.5)20 + \left(\frac{1}{2} \times 2.5 \times 20\right)$ or $20 \times 5 + \left(\frac{1}{2} \times 2.5 \times 20\right) + \left(\frac{1}{2} \times 2.5 \times 20\right)$ oe	**M1**	for a correct expression for total area using *their* 7.5
	150	**A1**	
10(b)(i)	$x = 3\mathrm{e}^{2t} + t + c$	**M1**	for $k\mathrm{e}^{2t} + t$ Condone omission of c
	$0 = 3\mathrm{e}^{0} + 0 + c$ When $t = 0$, $x = 0$ so $c = -3$	**M1**	Dep for substitution to find c
	$x = 3\mathrm{e}^{2t} + t - 3$	**A1**	

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Question	Answer	Marks	Guidance
10(b)(ii)	$\frac{\mathrm{d}v}{\mathrm{d}t} = 12\mathrm{e}^{2t}$ so $12\mathrm{e}^{2t} = 24$	**M1**	for $k\mathrm{e}^{2t}$ equated to 24
	$2t = \ln 2$	**M1**	Dep for correct order of operations to obtain $2t$
	$t = \frac{1}{2}\ln 2$, $\ln\sqrt{2}$ or 0.347	**A1**	