Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

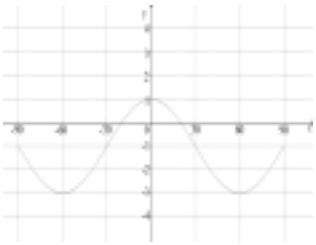
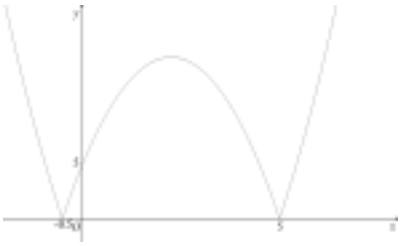
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1(i)		**B3**	**B1** for y intercept $(0,1)$, must have a graph **B1** for starting and finishing at $(\pm 90,-1)$ **B1** for all correct, must be attempt at a curve passing through $(\pm 30,-1)$ and $(\pm 60,-3)$
1(ii)	2	**B1**	
1(iii)	120^{o} or $\frac{2\pi}{3}$	**B1**	
2	$\lg y^2 = mx + c$	**B1**	May be implied by subsequent work
	Gradient $= -4 \quad (= m)$	**B1**	
	$c = 32$	**B1**	
	$y = 10^{\text{their } \frac{c}{2} + \text{their } \frac{mx}{2}}$	**M1**	Dep on first B1 Use of $\lg y^2 = 2\lg y$ and $10^{\text{their } \frac{c}{2} + \text{their } \frac{mx}{2}}$ Or use of $y^2 = 10^{(\text{their } c + \text{their } mx)}$ and $10^{\text{their } \frac{c}{2} + \text{their } \frac{mx}{2}}$
	$y = 10^{16-2x}$	**A1**	
3	$\left(1 - \frac{x}{7}\right)^{14} = 1 - 2x + \frac{13}{7}x^2$	**B2**	All terms correct or **B1** for 2 correct terms
	$(1-2x)^4 = 1 - 8x + 24x^2 \ldots$	**B2**	First three terms correct or **B1** for one incorrect term
	Product $= 1 - 10x + \frac{293}{7}x^2$	**M1**	For attempt to multiply out to obtain $(1) - 10x + mx^2$, $m \neq 16$
	$a = -10,\ b = \frac{293}{7}$	**A1**	For both, need to identify a and b
4(i)		**B4**	**B1** for shape, with max in first quadrant **B1** for $(-0.5,0)$ and $(5,0)$ **B1** for $(0,5)$ **B1** all correct, with cusps and correct curvature for $x < 0.5$ and $x > 5$

Question	Answer	Marks	Guidance
4(ii)	$k = 0$	**B1**	Not from incorrect work
	Stationary point when $y = \pm\frac{121}{8}$ or ± 15.125	**M1**	For attempt to find *y*-coordinate of stationary point, must be a complete method i.e. Use of calculus Use of discriminant, Use of completing the square Use of symmetry Allow if seen in part (i), but must be used in (ii)
	$k > \frac{121}{8}$	**A1**	cao
5a(i)	fg	**B1**	
5a(ii)	g^{-1}	**B1**	
5a(iii)	f^{-1}	**B1**	
5a(iv)	g^2	**B1**	
5(b)(i)	Undefined at $x = 0$ oe	**B1**	
5(b)(ii)	$4 = a + b$ $h'(x) = \frac{p}{x^3}$ and attempt at $h'(1)$	**M1**	For attempt at $h(1)$ and differentiation to obtain $h'(1)$, must have the form $h'(x) = \frac{p}{x^3}$ oe
	$b = -8$ $a = 12$	**A1**	For both
6(a)	$p^{\frac{7}{2}}q^{\frac{5}{3}}r^{-\frac{7}{3}}$	**B3**	**B1** for each term or for each of $a = \frac{7}{2}$, $b = \frac{5}{3}$, $c = -\frac{7}{3}$

Question	Answer	Marks	Guidance
6(b)	**Either** $\log_7 x + \frac{2}{\log_7 x} = 3$	**M1**	For change of base.
	$(\log_7 x)^2 - 3\log_7 x + 2 = 0$ $\log_7 x = 1, \quad \log_7 x = 2$	**M1**	Dep for forming a 3 term quadratic equation in $\log_7 x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 7 logarithm correctly
		A1	For both
	Or $\frac{1}{\log_x 7} + 2\log_x 7 = 3$	**M1**	For change of base
	$2(\log_x 7)^2 - 3\log_x 7 + 1 = 0$ $\log_x 7 = 1, \quad \log_x 7 = 0.5$	**M1**	Dep for forming a 3 term quadratic equation in $\log_x 7$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base x logarithm correctly
		A1	For both
	Or $\frac{\lg x}{\lg 7} + 2\frac{\lg 7}{\lg x} = 3$ or $\lg 1000$	**M1**	For change of base
	$(\lg x)^2 - 3\lg 7(\lg x) + 2(\lg 7)^2 = 0$ $\lg x = 2\lg 7 \qquad \lg x = \lg 7$	**M1**	Dep for forming a 3 term quadratic equation in $\lg x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 10 logarithm correctly
		A1	For both, must be exact
7(i)	$\frac{dy}{dx} = \left(e^{x^2} + 1\right) + 2xe^{x^2}(x + 5)$	**B1**	For $2xe^{x^2}$
		M1	For attempt at differentiating a product or expanding brackets and differentiating a product
		A1	For all other terms, apart from $2xe^{x^2}$, correct

Question	Answer	Marks	Guidance
7(ii)	When $x = 0.5$, $\frac{dy}{dx} = 9.35$	**M1**	For attempt to find *their* $\frac{dy}{dx}$ when $x = 0.5$ and multiplication by p
	Approximate change = $9.35p$	**A1**	
7(iii)	$\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt}$ $9.346 \times \frac{dx}{dt} = 2$	**M1**	For use of correct rates of change equation using *their* $\frac{dy}{dx}$ when $x = 0.5$ and $\frac{dy}{dt} = 2$
	$\frac{dx}{dt} = 0.214$	**A1**	FT on $\frac{2}{\textit{their } 9.346}$ Must be correct to at least 3 sf
8(a)(i)	**Either** $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \\ 0 & 1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$	**B2**	For correct matrices in correct order or **B1** if one correct matrix and a slip in one element of the other matrix
	Or $\begin{pmatrix} 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 0 & 3 \\ 1 & 3 & 1 & 1 & 0 \\ 1 & 0 & 2 & 3 & 1 \end{pmatrix}$ or $\begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 & 0 & 3 \\ 1 & 3 & 1 & 1 & 0 \end{pmatrix}$	**B2**	For correct matrices in correct order or **B1** if one correct matrix and a slip in one element of the other matrix
8(a)(ii)	$\begin{pmatrix} 10 \\ 10 \\ 6 \\ 2 \\ 12 \end{pmatrix}$ or $\begin{pmatrix} 10 & 10 & 6 & 2 & 12 \end{pmatrix}$ Team E	**M1**	For matrix multiplication of *their* (i), with at least 2 elements correct, **must be in correct form**, may be unsimplified
		A1	All correct and identifying team E
8(b)(i)	$\frac{1}{6} \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$	**B2**	**B1** for $\frac{1}{6}$ and **B1** for $\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$

Question	Answer	Marks	Guidance
8(b)(ii)	$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$	**M1**	For pre-multiplication by *their* inverse from **(i)**
	$\mathbf{C} = \frac{1}{6}\begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 1 & -2 \end{pmatrix}$	**M1**	Dep for matrix multiplication, using *their* inverse from (i), at least 2 elements correct
	$= \frac{1}{6}\begin{pmatrix} 21 & -2 \\ -9 & -2 \end{pmatrix}$ oe	**A1**	
9(i)	$\pi r^2 h = 1200\pi$	**B1**	
	$h = \frac{1200}{r^2}$ or $\pi rh = \frac{1200\pi}{r}$ and substitution into *their S*	**B1**	Must have attempt to use in an equation for S
	$S = 2\pi r^2 + \left(2\pi r \times \frac{1200}{r^2}\right)$ leading to given answer	**B1**	
9(ii)	$\frac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \frac{2400\pi}{r^2}$	**M1**	Must obtain the form $Ar + \frac{B}{r^2}$
	When $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$, $r = \sqrt[3]{600}$, 8.43	**M1**	Dep for equating to zero and attempt to solve to obtain r = …
		A1	For correct r
	$S_{\min} = 1340$ or 1341	**A1**	
	Either $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 4\pi + \frac{4800\pi}{r^3}$ $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0$ so minimum	**B1**	For a correct method to reach a correct conclusion If r is not calculated, then must state that $r > 0$
	Or Consideration of gradient e.g. r | < 8.43 | 8.43 | > 8.43 $\frac{\mathrm{d}S}{\mathrm{d}r}$ | – | 0 | + Minimum point	**B1**	Must be making a correct and convincing argument with sufficient detail

Question	Answer	Marks	Guidance
10(i)	**Either** $18^2 = 10^2 + 10^2 - 200\cos AOB$	**M1**	Attempt to use cosine rule
	$\cos AOB = -0.62$	**A1**	Allow unsimplified
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	**A1**	Must justify 2 dp
10(i)	**Or** $\sin\frac{AOB}{2} = \frac{9}{10}$ or $\tan\frac{AOB}{2} = \frac{9}{\sqrt{19}}$ or $\cos\frac{AOB}{2} = \frac{\sqrt{19}}{10}$	**M1**	Attempt at trig using a right angled triangle
	$\frac{AOB}{2} =$ awrt 1.12	**A1**	
	$AOB = 2.2395$ or greater accuracy, so 2.24 (to 2 dp) or $AOB = 2.239...$ so 2.24 (to 2 dp) $AOB = 2.240$ so 2.24 (to 2 dp)	**A1**	Must justify 2 dp
10(ii)	$AOC = 2\pi - 2(2.2395)$ or $\frac{AOC}{2}$ or $ABC = \pi - (2.2395)$ oe	**M1**	For attempt to find angle AOC or ABC $AOC = 2\pi - 2(their\ AOB)$ $ABC = \pi - (their\ AOB)$ oe
	$AOC = 1.804$ or 1.803	**A1**	Condone 1.8 or 1.80
	Arc length = 18.04 or 18.03	**M1**	For attempt at arc length using $10 \times their\ AOC$
	$AC = 20\sin\frac{AOC}{2}$ or $36\sin\frac{ABC}{2}$ or $\sqrt{10^2 + 10^2 - 200\cos AOC}$ or $\sqrt{18^2 + 18^2 - 648\cos ABC}$ $= 15.69$ or 15.7	**M1**	For attempt at AC using *their AOC*, or ABC but $AOC \neq 2.24$ or $\frac{2\pi}{3}$
	Perimeter = 33.7	**A1**	Allow awrt 33.7

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Question	Answer	Marks	Guidance
10(iii)	Area of sector $= 50 \times 1.804$ $= 90.2$ or 90.15	**M1**	For attempt at sector area $\frac{1}{2} \times 10^2 \times$ *their AOC* *AOC* must be in radians
	Area of triangle $= 50 \sin 1.804 = 48.6$ or 48.66	**M1**	For attempt at area of triangle $\frac{1}{2} \times 10^2 \times \sin$ *their AOC* *AOC* must be in radians
	Shaded area $= 41.6$ or 41.5	**A1**	Lack of accuracy is penalised here
11	$\frac{dy}{dx} = 2(3x-1)^{\frac{1}{3}} + c$	**M1**	For $\left(\frac{dy}{dx} =\right) \quad a(3x-1)^{\frac{1}{3}}$, condone omission of $+ c$
		A1	All correct, condone omission of c
	$6 = 4 + c$	**M1**	Dep for attempt to find c
	$\left(\frac{dy}{dx} =\right) \quad 2(3x-1)^{\frac{1}{3}} + 2$	**A1**	All correct, may be implied by $c = 2$
	$y = \frac{1}{2}(3x-1)^{\frac{4}{3}} + 2x + d$	**M1**	For attempt to integrate *their* $\frac{dy}{dx}$ to obtain the form $y = b(3x-1)^{\frac{4}{3}} \quad (+mx + d)$
		A1	All correct, condone omission of d
	$11 = 14 + d$	**M1**	Dep for attempt to find d, a second arbitrary constant, having used an arbitrary constant for $\frac{dy}{dx}$
	$y = \frac{1}{2}(3x-1)^{\frac{4}{3}} + 2x - 3$	**A1**	

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