Cambridge
O Level

**Cambridge International Examinations**
Cambridge Ordinary Level

**ADDITIONAL MATHEMATICS** **4037/12**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

**Published**

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**Abbreviations**

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| **1 (a) (i)** | 10 | **B1** | |
| **(ii)** | 22 | **B1** | |
| **(iii)** | 4 | **B1** | |
| **(b) (i)** | $Q \subset R$ | **B1** | |
| **(ii)** | $P \cap Q = \varnothing$ , or { } | **B1** | |
| **2** | $a = 1,\ b = -3,\ c = -1$ | **B3** | **B1** for each |
| **3** | $3y^2 + 5y - 2 = 0$ | **B1**, **B1** | **B1** for $5y$ or $5\log_3 x$ , **B1** for $-2$ |
| | $y = \frac{1}{3},\ y = -2$ | **M1** | for correct attempt at the solution of *their* quadratic equation |
| | $x = 3^{\frac{1}{3}},\ x = 3^{-2}$ | **M1** | for dealing with one base 3 logarithm correctly |
| | $x = 1.44,\ x = \frac{1}{9}$ | **A1**, **A1** | **A1** for each |
| **4 (i)** | $32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$ | **B3** | **B1** for each term, powers of $x$ must be simplified |
| **(ii)** | Coefficients needed: | | |
| | $\left(3 \times \textit{their} - \frac{80}{3}\right) + \left(1 \times \textit{their}\ 32\right)$ | **M1** | for dealing with 2 terms |
| | $= -48$ | **A1** | Allow **A1** for $-48x^7$ |

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|---|---|---|---|
| **5 (i)** | $\frac{dy}{dx} = \frac{3}{2(3x+2)}$ | **B1** | for correct derivative of log function |
| | When $x = -\frac{1}{3}$, $y = 0$, $\frac{dy}{dx} = \frac{3}{2}$ | **B1** | for $y = 0$ |
| | Equation of normal: $y = -\frac{2}{3}\left(x + \frac{1}{3}\right)$ | **M1**<br>**A1** | **M1** for attempt at a gradient of a perpendicular from differentiation and the equation of the normal |
| **(ii)** | $Q\left(0, -\frac{2}{9}\right)$ or $(0, 0.22)$ or better | **B1 ft** | Follow through on *their c* from part (i) |
| | $R\left(0, \frac{1}{2}\ln 2\right)$ or $(0, 0.35)$ or better | **B1** | |
| | Area of $PQR$ $= \frac{1}{2}\left(\frac{1}{2}\ln 2 + \frac{2}{9}\right) \times \frac{1}{3}$ | | |
| | $= 0.0948$ | **B1** | Allow 0.095 |
| **6 (a)** | **YX**, **XZ** | **B2** | **B2** for both with no extras<br>**B1** for 1 correct with or without extras<br>**B1** for both correct with extras<br>**B0** for anything else |
| **(b) (i)** | $\frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$ | **B1**, **B1** | **B1** for $\frac{1}{18}$, **B1** for $\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$ |
| **(ii)** | $\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$ | | |
| | $= \frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} -4 & 2 \\ 10 & 4 \end{pmatrix}$ | **M1** | for pre-multiplication |
| | $= \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$ | **A1**, **A1** | **A1** for any correct pair of elements, but must be from correct matrices |

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| **7 (i)** | $(0, \sqrt{3})$ or $(0, 1.73)$ or better | **B1** | |
| **(ii)** | $\left(\frac{\pi}{6}, 2\right)$ or $(0.524, 2)$ or better | **B1**, **B1** | **B1** for each |
| **(iii)** | $\cos\left(x - \frac{\pi}{6}\right) = 0$ | **M1** | for correct attempt to solve trigonometric equation |
| | $x = \frac{2\pi}{3}$ oe or 2.09 or better | **A1** | |
| **(iv)** | $2\sin\left(x - \frac{\pi}{6}\right)$ ( +c) | **B1** | |
| **(v)** | Area $= \left[2\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\frac{2\pi}{3}}$<br>$= 2 + 1$<br>$= 3$ | **M1**<br><br>**A1** | for correct use of **their** limits, in radians, into $k\sin\left(x - \frac{\pi}{6}\right)$. |
| **8 (i)** | $47 - 24 = 12\theta$<br>$\theta = \frac{23}{12}$ , so $\theta = 1.917$ or better<br>$\theta = 1.92$ to 2dp | <br>**M1**<br>**A1** | <br>for complete correct method to get $\theta =$<br>must have evidence of working to more than 2 dp, allow if 1.916 seen (truncated) |
| **(ii)** | $\sin\frac{\theta}{2} = \frac{CD/2}{12}$<br>*CD* = awrt 19.6 or 19.7 | **M1**<br>**A1** | for a complete method, may use cosine rule to get *CD* |
| **(iii)** | Area of sector = awrt 138<br>Area of triangle *AOB* = awrt 67 or 68<br>Area of segment = awrt 70 or 71<br><br>*AD* × *AB* + segment area = 425<br>leading to *AD* = awrt 18.1 or 18.0<br><br>**Alternative method:**<br>Area of sector = awrt 138<br>Difference in length between *BC* (or *AD*) and *OM* where *M* is the midpoint of *CD* = 6.88, allow awrt 6.9<br>Remaining area consists of two trapezia each of width 9.85 and each of area 143.4<br>$\frac{1}{2}(2BC - 6.88) \times 9.85 = 143.4$ oe<br><br>leading to *AD* = awrt 18.1 or 18.0 | **B1**<br>**M1**<br>**M1**<br><br>**M1**<br>**A1**<br><br><br>**B1**<br>**M1**<br><br>**M1**<br><br><br>**M1**<br>**A1** | for sector area, allow unsimplified<br>for a correct attempt at area<br>for segment area (*their* sector area – *their* triangle area)<br>for complete method to find *AD*<br>Allow **A1** for 18<br><br><br>for sector area<br>for attempt to find difference between parallel sides<br><br>for area of one trapezium<br>$\frac{1}{2}(2BC - their\ 6.88) \times their\ 9.85$ oe<br><br>for attempt to find either *BC* or *AD* |

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| **9 (i)** | $\mathrm{p}\left(\frac{3}{2}\right)$: $\frac{27a}{8}-\left(4\times\frac{9}{4}\right)+\frac{3b}{2}+18\ \ (=0)$ | **M1** | for attempt at $\mathrm{p}\left(\frac{3}{2}\right)$ |
| | $\mathrm{p}'\left(\frac{3}{2}\right)=\left(3a\times\frac{9}{4}\right)-\left(8\times\frac{3}{2}\right)+b\ \ (=0)$ | **M1** | for differentiation and attempt at $\mathrm{p}'\left(\frac{3}{2}\right)$ |
| | leading to $9a+4b+24=0$ oe<br>and $27a+4b-48=0$ oe | **M1** | for solution of simultaneous equations, to get either $a$ or $b$ |
| | leading to $a=4,\ b=-15$ | **A1** | for both |
| **(ii)** | $(x+2)(2x-3)^2$ oe | **M1, A1** | **M1** for attempt at long division or factorisation |
| **(iii)** | $(x+2)(2x-3)^2=x+2$<br>$x+2=0,\ x=-2$ | **B1** | Must be using $(x+2)$ correctly using part (ii) to get $x=-2$ |
| | $(2x-3)^2=1$<br>leading to $x=1,\ x=2$ | **M1**<br>**A1** | for solution of the quadratic equation |
| **10 (a) (i)** | $20U+\frac{1}{2}\left(U+\frac{U}{2}\right)10=165$ | **M1** | for realising that area under the graph is needed and attempt to find an area |
| | | **DM1** | for equating their area to 165 and attempt to solve |
| | leading to $U=6$ | **A1** | |
| **(ii)** | Gradient of line: $-0.3$ | **M1, A1** | **M1** for use of the gradient, must be negative |
| **(b) (i)** | 27 | **B1** | |
| **(ii)** | $t^2=8\ln 4$<br>$t=3.33$ or better | **M1**<br>**A1** | for a correct attempt to solve $\mathrm{e}^{\frac{t^2}{8}}=4$ |
| **(iii)** | acceleration $=3\frac{2t}{8}\mathrm{e}^{\frac{t^2}{8}}\left(\mathrm{e}^{\frac{t^2}{8}}-4\right)^2$ | **M1, A1** | **M1** for a correct attempt to differentiate using the chain rule |
| | When $t=1,\ a=6.98$ | **M1, A1** | **M1**for use of $t=1$ in their acceleration |

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| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| **11 (i)** | $\ln y = \ln A + x \ln b$ | **B1** | may be implied, if equation not seen specifically, by correct values for $A$ and $b$ |
| | Gradient: $\ln b = -\dfrac{0.12}{8}$ , $= -0.015$ | **M1** | for use of gradient to obtain ln $b$ |
| | $b = 0.985$ | **A1** | Allow **A1** for $e^{-0.015}$ |
| | Intercept: $\ln A = 0.26$ | **DM1** | for use of one of the given points correctly |
| | $A = 1.30$ | **A1** | Allow **A1** for $e^{0.26}$ or 1.3 |
| | **Alternative 1** | | |
| | $\ln y = \ln A + x \ln b$ | **B1** | |
| | $0.2 = 4 \ln b + \ln A$ | **M1** | for one correct equation |
| | $0.08 = 12 \ln b + \ln A$ | **DM1** | for attempt to obtain either ln$A$ or ln$b$ from simultaneous equations |
| | $A = 1.30$ and $b = 0.985$ | **A1**, **A1** | Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3 |
| | **Alternative 2** | | |
| | $1.22 = Ab^4$ | **B1** | |
| | $1.08 = Ab^{12}$ | **B1** | |
| | | **M1** | for correct attempt to obtain $b$ or $A$, must already have **B2** |
| | $A = 1.30$ and $b = 0.985$ | **A1**, **A1** | Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3 |
| **(ii)** | When $x = 6$, $\ln y = 0.17$ | **M1** | for $\ln y = \textit{their} \ln A + 6\ \textit{their} \ln b$ or $y = \textit{their}\ A \times (\textit{their}\ b)^6$ |
| | $y = 1.19$ | **A1** | allow awrt 1.18 to 1.20 |
| **(iii)** | When $y = 1.1$, $\ln y = 0.095$ | **M1** | for $\ln 1.1 = \textit{their} \ln A + x\ \textit{their} \ln b$ or $1.1 = \textit{their} A \times (\textit{their} b)^x$ |
| | $x = 11$ | **A1** | allow 10.5 to 11.5 |

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