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Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/23 Paper 2, maximum raw mark 80

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1	**(i)**	$f(2)=0 \rightarrow 3(2)^3+8(2)^2-33(2)+p=0$ correct working to $p = 10$ AG method for quadratic factor $f(x)=(x-2)(3x^2+14x-5)$	**M1** **A1** **M1** **A1**	
	(ii)	$f(x)=(x-2)(3x-1)(x+5)$ $f(x)=0 \rightarrow x=2,\ -5,\ \frac{1}{3}$	**M1** **A1**	factorise or solve quadratic factor = 0
2	**(i)**	${}^{12}C_4=495$	**B1**	
	(ii)	${}^7C_2\times{}^5C_2=21\times10$ $=210$	**M1** **A1**	
	(iii)	not K and B = ${}^6C_2\times{}^4C_1=15\times4=60$ K and not B = ${}^6C_1\times{}^4C_2=6\times6=36$ 60 + 36 96 OR K and B = ${}^6C_1\times{}^4C_1=6\times4=24$ not K and not B = ${}^6C_2\times{}^4C_2=15\times6=90$ 210 – 90 – 24 96	**B1** **B1** **M1** **A1** **B1** **B1** **M1** **A1**	
3	**(i)**	C is (1, 6) D is $(1,6)+(12,9)$ = (13, 15)	**B1** **M1** **A1ft**	
	(ii)	gradient of $CD=\frac{15-6}{13-1}\left(=\frac{3}{4}\right)$ gradient of $AB=\frac{10-2}{-2-4}\left(=\frac{8}{-6}=\frac{-4}{3}\right)$ $\frac{3}{4}\times\frac{-4}{3}=-1$ lines are perpendicular	**B1ft** **B1** **B1**	 correct completion www
	(iii)	area $=\frac{1}{2}\times AB\times CD=\frac{1}{2}\times10\times15$ =75 or array method	**M1** **A1**	good attempt at two relevant lengths for $\frac{1}{2}$ base × height method

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4	**(i)**	$2000 = 1000e^{a+b} \rightarrow a+b = \ln 2$	**B1**	
	(ii)	$3297 = 1000e^{2a-b} \rightarrow 2a+b$ $= \ln 3.297$ oe	**M1** **A1**	substitution of 2, 3297 and rearrange
	(iii)	Solve for one value $a = 0.5$ and $b = 0.193$ or 0.19	**M1** **A1**	
	(iv)	$n = 10$ $P = 1000e^{5.193}$ $= \$180\,000$.	**M1** **A1**	
5	**(i)**	$\overrightarrow{OX} = \mu(a+b)$	**B1**	
	(ii)	$\overrightarrow{RP} = b-3a$ or $\overrightarrow{RX} = \lambda(b-3a)$ oe $\overrightarrow{OX} = 3a+\lambda(b-3a)$	**B1** **B1**	
	(iii)	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate both coefficients $\mu = 3-3\lambda$ $\mu = \lambda$ $\mu = \lambda = 0.75$ $\frac{RX}{XP} = 3$ or 3:1	**M1** **A1** **A1ft**	$\frac{\lambda}{1-\lambda}$
6	**(i)**	$m = 4$ equation of line is $\frac{\ln y - 39}{3^x - 9} = \frac{39-19}{9-4}$ $\ln y = 4(3^x)+3$	**B1** **M1** **A1ft**	forms equation of line ft only on their gradient
	(ii)	$x = 0.5 \rightarrow \ln y = 4\sqrt{3}+3 = 9.928$ $y = 20\,500$	**M1** **A1**	correct expression for ln$y$
	(iii)	Substitutes y and rearrange for 3^x Solve $3^x = 1.150$ $x = 0.127$	**M1** **M1** **A1**	

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7	**(i)**	$x = \frac{2}{y} + 1 \quad \rightarrow \quad y = \frac{2}{x-1}$	**M1**	any valid method
		$f^{-1}(x) = \frac{2}{x-1}$	**A1**	
	(ii)	$gf(x) = \left(\frac{2}{x} + 1\right)^2 + 2$	**B2/1/0**	–1 each error
	(iii)	$fg(x) = \frac{2}{x^2+2} + 1$	**B2/1/0**	–1 each error
	(iv)	$ff(x) = \frac{2}{\frac{2}{x}+1} + 1 = \frac{2x}{x+2} + 1$	**M1**	correct starting expression
		$= \frac{3x+2}{x+2}$	**A1**	correct algebra to given answer
		$\frac{3x+2}{x+2} = x \quad \rightarrow \quad x^2 - x - 2 = 0$	**M1**	form and solve 3 term quadratic
		$(x-2)(x+1) = 0$ $x = 2$ only	**A1**	
8	**(i)**	$v = C + K\sin 2t \qquad C \neq 0$	**M1**	
		$v = 5 + 6\sin 2t$	**A1**	
		$a = 12\cos 2t$	**A1ft**	
	(ii)	$a = 0 \quad \rightarrow \quad \cos 2t = 0$ and solve	**M1**	set $a = 0$ and solve for t
		$t = \frac{\pi}{4}$ or 0.785 or 0.79	**A1**	
		$v = 5 + 6\sin\frac{\pi}{2} = 11$	**A1ft**	ft only on K
	(iii)	$v = 2 \quad \rightarrow \quad \sin 2t = -\frac{1}{2}$ and solve	**M1**	set $v = 2$ and solve for t
		$t = \frac{7\pi}{12}$ or $1.83 - 1.84$	**A1**	
		$a = 12\cos\frac{7\pi}{6} = -6\sqrt{3}$ or -10.4	**A1**	

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9	**(i)**	$\frac{dy}{dx} = 4 - \frac{1}{(x-2)^2}$	**B1**	
		$\frac{dy}{dx} = 0 \rightarrow (x-2)^2 = \frac{1}{4}$ $(4x^2 - 16x + 15 = 0)$	**M1**	solve 3 term quadratic from $\frac{dy}{dx} = 0$
		$x = 2.5$ or 1.5	**A1**	x values or 1 pair
		$y = 12$ or 4	**A1**	y values or 1 pair
		$\frac{d^2y}{dx^2} = 2(x-2)^{-3}$	**M1**	use $\frac{d^2y}{dx^2}$ with solution from $\frac{dy}{dx} = 0$
		$x = 2.5 \rightarrow \frac{d^2y}{dx^2} > 0 \rightarrow$ minimum $x = 1.5 \rightarrow \frac{d^2y}{dx^2} < 0 \rightarrow$ maximum	**A1**	both identified www
	(ii)	$x = 3 \rightarrow \frac{dy}{dx} = 3$	**B1**	
		Use $m_1 m_2 = -1$ for gradient normal from gradient tangent	**M1**	must use numerical values
		Eqn of normal : $\frac{y-13}{x-3} = -\frac{1}{3}$	**A1ft**	
		Intersection of norm and curve		
		$14 - \frac{x}{3} = 4x + \frac{1}{x-2}$	**M1**	equation and attempt to simplify
		$13x^2 - 68x + 87 = 0$	**DM1**	attempt to solve 3 term quadratic
		$x = \frac{29}{13}$ or 2.23	**A1**	
10	**(i)**	$\text{LHS} = \frac{1+\cos x + 1 - \cos x}{(1-\cos x)(1+\cos x)}$	**B1**	correct fraction
		$= \frac{2}{1-\cos^2 x}$	**B1**	correct evaluation
		$= \frac{2}{\sin^2 x} = \text{RHS}$	**B1**	use of $1 - \cos^2 x = \sin^2 x$ and completion of fully correct proof
	(ii)	$2\text{cosec}^2 x = 8$	**M1**	identity used
		$\sin^2 x = \frac{1}{4}$	**A1**	
		$\sin x = \pm\frac{1}{2}$	**A1**	
		$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	**A1**	

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