UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2010**

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper (1 sheet)

1054186529

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

DC (CW/DJ) 34216
© UCLES 2010

UNIVERSITY of CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

© UCLES 2010

1 The equation of a curve is given by $y = 2x^2 + ax + 14$, where a is a constant.

Given that this equation can also be written as $y = 2(x - 3)^2 + b$, where b is a constant, find

(i) the value of a and of b, [2]

(ii) the minimum value of y. [1]

2 **(i)** Sketch, on the same set of axes, the graphs of $y = \cos x$ and $y = \sin 2x$ for $0° \leqslant x \leqslant 180°$. [2]

(ii) Hence write down the number of solutions of the equation $\sin 2x - \cos x = 0$ for $0° \leqslant x \leqslant 180°$. [1]

3 Show that $\dfrac{\cos x}{1 - \sin x} + \dfrac{\cos x}{1 + \sin x} = 2 \sec x$. [4]

4 Factorise completely the expression $2x^3 - 11x^2 - 20x - 7$. [5]

5 A curve has the equation $y = 2x \sin x + \dfrac{\pi}{3}$. The curve passes through the point $P\left(\dfrac{\pi}{2}, a\right)$.

(i) Find, in terms of π, the value of a. [1]

(ii) Using your value of a, find the equation of the normal to the curve at P. [5]

6 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $(2 - 5x)^6$, giving your answer in the form $a + bx + cx^2$, where a, b and c are integers. [3]

(ii) Find the coefficient of x in the expansion of $(2 - 5x)^6\left(1 + \dfrac{x}{2}\right)^{10}$. [3]

7 **(a)** Sets A and B are such that

$A = \{x : \sin x = 0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$,

$B = \{x : \cos (x - 30°) = -0.5 \text{ for } 0° \leqslant x \leqslant 360°\}$.

Find the elements of

(i) A, [2]

(ii) $A \cup B$. [2]

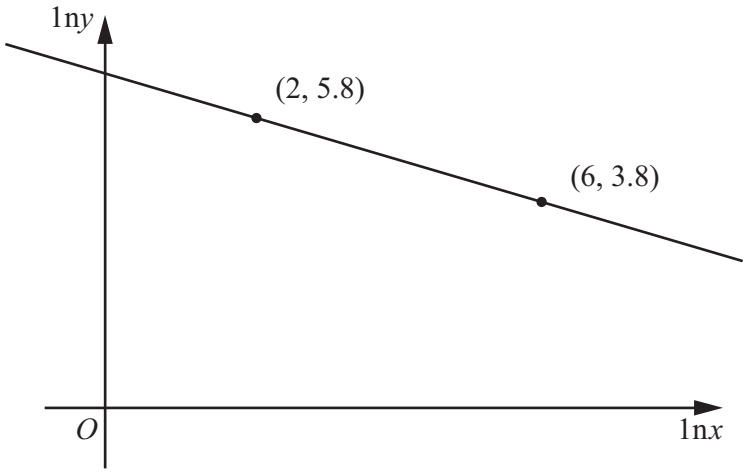
(b) Set C is such that

$C = \{x : \sec^2 3x = 1 \text{ for } 0° \leqslant x \leqslant 180°\}$.

Find $n(C)$. [3]

© UCLES 2010

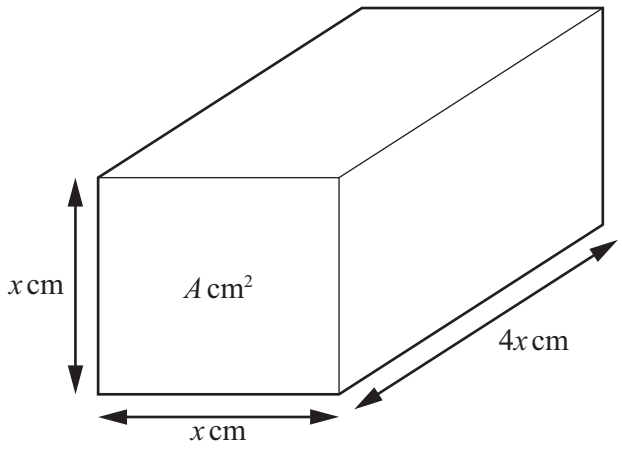
8 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points (2, 5.8) and (6, 3.8) is obtained.



(i) Find the value of $\ln y$ when $\ln x = 0$. [2]

(ii) Given that $y = Ax^b$, find the value of A and of b. [5]

9



The figure shows a rectangular metal block of length $4x$ cm, with a cross-section which is a square of side x cm and area A cm^2. The block is heated and the area of the cross-section increases at a constant rate of $0.003\,\text{cm}^2\text{s}^{-1}$. Find

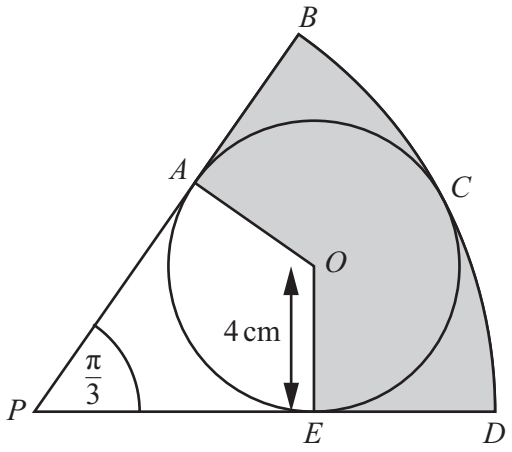
(i) $\dfrac{\mathrm{d}A}{\mathrm{d}x}$ in terms of x, [1]

(ii) the rate of increase of x when $x = 5$, [3]

(iii) the rate of increase of the volume of the block when $x = 5$. [4]

© UCLES 2010

10



The diagram shows a circle, centre O, radius 4 cm, enclosed within a sector $PBCDP$ of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.

(i) Show that $PA = 4\sqrt{3}$ cm and $PB = 12$ cm. [2]

Find, to 1 decimal place,

(ii) the area of the shaded region, [4]

(iii) the perimeter of the shaded region. [4]

11 **(i)** Find $\int \frac{1}{\sqrt{1+x}}\,\mathrm{d}x$. [2]

(ii) Given that $y = \frac{2x}{\sqrt{1+x}}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{\sqrt{1+x}} + \frac{Bx}{\left(\sqrt{1+x}\right)^3}$, where A and B are to be found. [4]

(iii) Hence find $\int \frac{x}{\left(\sqrt{1+x}\right)^3}\,\mathrm{d}x$ and evaluate $\int_0^3 \frac{x}{\left(\sqrt{1+x}\right)^3}\,\mathrm{d}x$. [4]

© UCLES 2010

12 Answer only **one** of the following two alternatives.

EITHER

A curve is such that $\frac{dy}{dx} = 4x^2 - 9$. The curve passes through the point (3, 1).

(i) Find the equation of the curve. [4]

The curve has stationary points at A and B.

(ii) Find the coordinates of A and of B. [3]

(iii) Find the equation of the perpendicular bisector of the line AB. [4]

OR

A curve has the equation $y = Ae^{2x} + Be^{-x}$ where $x \geqslant 0$. At the point where $x = 0$, $y = 50$ and $\frac{dy}{dx} = -20$.

(i) Show that $A = 10$ and find the value of B. [5]

(ii) Using the values of A and B found in part **(i)**, find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

© UCLES 2010

BLANK PAGE

© UCLES 2010

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© UCLES 2010