UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2010 question paper for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1	**(i)** $a = -12$, $b = -4$	B1, B1 [2]	B1 for each
	(ii) -4	√ B1 [1]	Follow through on their y value
2	**(i)** Graphs	B1 B1 [2]	B1 for one correct curve B1 for a second correct curve consistent with the first curve
	(ii) 3	√ B1 [1]	Follow through on number of clear points of intersection
3	$\dfrac{\cos x(1+\sin x)+\cos x(1-\sin x)}{1-\sin^2 x}$ $\dfrac{2\cos x}{\cos^2 x}$ $2\sec x$	M1 DM1 M1 A1 [4]	M1 for attempt to get in terms of a single fraction DM1 simplifying numerator M1 simplifying denominator
4	$x = -1$ or 7 or $-\dfrac{1}{2}$ seen Either $(x+1)(2x^2-13x-7)$ or $(x-7)(2x^2+3x+1)$ or $(2x+1)(x^2-6x-7)$ leading to $(x+1)(x-7)(2x+1)$	M1 DM1 A1 DM1, A1 [5]	M1 for attempt to find a root DM1 for attempt to obtain quadratic factor A1 correct quadratic factor DM1 attempt to factorise quadratic factor
5	**(i)** $a = \pi + \dfrac{\pi}{3}$, $a = \dfrac{4\pi}{3}$	B1 [1]	Must be in terms of π
	(ii) $\dfrac{dy}{dx} = 2x\cos x + 2\sin x$ at P, $\dfrac{dy}{dx} = 2$, $\Rightarrow$ grad of normal = $-\dfrac{1}{2}$ normal: $y - \dfrac{4\pi}{3} = -\dfrac{1}{2}\left(x - \dfrac{\pi}{2}\right)$ $\left(2y = \dfrac{19\pi}{6} - x\right)$	M1, A1 M1 M1, A1 [5]	M1 for attempt to differentiate a product M1 for $m_1m_2 = -1$, must have used differentiation M1 for attempt at a normal equation, must have used differentiation, allow unsimplified

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6	**(i)** $64 - 960x + 6000x^2$	B1, B1, B1 [3]	B1 for each correct term, allow 2^6
	(ii) 1 × (their x term) + $\frac{10}{2}$ × (their 64) –960 + 320 = –640	M1 B1 A1 [3]	M1 for 2 terms B1 for $\frac{10}{2}$ or 5
7	**(a)** **(i)** $x = 30°, 150°$	B1, B1 [2]	B1 for each
	(ii) $x - 30° = 120°, 240°$ $x = 150°, 270°$ $A \cup B = \{30°, 150°, 270°\}$	B1 √ B1 [2]	B1 for $x = 150°, 270°$ only Follow through on their A and B
	(b) $\cos 3x = \pm 1$ or $\tan 3x = 0$ $3x = 0°, 180°, 360°, 540°$ $x = 0°, 60°, 120°, 180°$ n(C) = 4	M1 A1 √ B1 [3]	M1 for dealing with sec and $3x$ A1 for all solutions correct Follow through on their number of solutions
8	**(i)** and **(ii)** Gradient = – 0.5 Use of ratios or $\ln y = -0.5 \ln x + c$ $\ln y = 6.8$ $\ln y = b \ln x + \ln A$ $A = e^{(\text{their } 6.8)}$ $A = 898$, $b = -0.5$	M1 M1 A1 B1 M1 A1, A1 [7]	M1 for attempt at gradient M1 for attempt at y intercept A1 for $\ln y = 6.8$ B1 for $\ln y = b \ln x + \ln A$ M1 for use of e A1 for A and A1 for b
9	**(i)** $A = x^2$, $\Rightarrow \frac{dA}{dx} = 2x$	B1 [1]	
	(ii) When $x = 5$, $\frac{dA}{dx} = 10$ $\frac{dx}{dt} = \frac{0.003}{10}$ = 0.0003	√ B1 M1 A1 [3]	Follow through on their $\frac{dA}{dx}$ M1 for 0.003 ÷ their 10
	(iii) $V = 4x^3$, $\frac{dV}{dx} = 12x^2$ $\frac{dV}{dt} = 12x^2 \times 0.0003$ = 0.09	B1, B1 M1 A1 [4]	B1 for each

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10	**(i)** $\tan\frac{\pi}{6}=\frac{4}{PA}$, $PA=4\sqrt{3}$ $PB=\frac{4}{\sin\frac{\pi}{6}}+4$, $PB=12$ allow equivalent methods	B1 B1 [2]	B1 for PA (answer given) B1 for PB (answer given)
	(ii) Sector area = $\frac{1}{2}12^2\times\frac{\pi}{3}$ Area of kite = $2\times\frac{1}{2}\times4\sqrt{3}\times4$ Shaded area = 47.7	√ B1 M1, A1 A1 [4]	√ B1 sector area, ft on their PB M1 for attempt to find area of kite or appropriate triangle
	(iii) $P=\left(12\times\frac{\pi}{3}\right)+2\left(12-4\sqrt{3}\right)+2(4)$ = 30.7	B1, B1, B1 B1 [4]	B1 for each of the 3 terms B1 for final answer
11	**(i)** $2(1+x)^{\frac{1}{2}}(+c)$	M1, A1 [2]	M1 for $(1+x)^{\frac{1}{2}}$, A1 for 2
	(ii) $\frac{dy}{dx}=\frac{2\sqrt{1+x}-2x\frac{1}{2}(1+x)^{-\frac{1}{2}}}{1+x}$ $=\frac{2}{\left(\sqrt{1+x}\right)}-\frac{x}{\left(\sqrt{1+x}\right)^3}$	M1 A2, 1, 0 A1 [4]	M1 attempt at differentiation –1 each error A1 all correct
	(iii) $\int\frac{x}{\left(\sqrt{1+x}\right)^3}\,dx=\int\frac{2}{\left(\sqrt{1+x}\right)}\,dx-\frac{2x}{\sqrt{1+x}}$ $=4\sqrt{1+x}-\frac{2x}{\sqrt{1+x}}(+c)$ $\int_0^3\frac{x}{\left(\sqrt{1+x}\right)^3}\,dx=(8-3)-(4),=1$	M1 A1 M1, A1 [4]	M1 for idea of using **(ii)** 'in reverse' A1 all correct M1 for attempt evaluation

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12 EITHER		
(i) $y = \frac{4x^3}{3} - 9x(+c)$	M1, A1	M1 for attempt to integrate
when $x = 3$, $y = 1$, so $c = -8$	M1, A1 [4]	M1 for attempt to find c
(ii) $4x^2 - 9 = 0$, leads to $x = \pm 1.5$	M1	M1 for attempt to solve $\frac{dy}{dx} = 0$
Points (1.5, –17), (–1.5, 1)	A1, A1 [3]	A1 for each pair
(iii) Midpoint AB: (0, –8)	M1	M1 for attempt to find midpoint
Gradient of AB = –6, perp grad = $\frac{1}{6}$	M1	M1 for attempt to find grad of perp
Equation: $x - 6y = 48$	M1, A1 [4]	M1 must be working with perp
12 OR		
(i) $50 = A + B$	B1	
$\frac{dy}{dx} = 2Ae^{2x} - Be^{-x}$	M1	M1 for attempt to differentiate
	A1	A1 all correct
$-20 = 2A - B$	DM1	DM1 for attempt to solve equations.
leads to $A = 10$ and $B = 40$	A1 [5]	
(ii) $\frac{dy}{dx} = 20e^{2x} - 40e^{-x}$, $20e^{2x} = 40e^{-x}$	M1	M1 for equating to zero and attempt at solution
$e^{3x} = 2$	M1	M1 for dealing with exponentials
$x = \frac{1}{3}\ln 2$ or 0.231	M1	M1 for attempt to obtain x
$y = 47.6$	A1 [4]	A1 for both
(iii) $\frac{d^2y}{dx^2} = 40e^{2x} + 40e^{-x}$	M1	M1 for attempt at second derivative or other valid method
Always +ve, so min	A1 [2]	A1 for a correct conclusion

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