Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **10** printed pages.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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Question	Answer	Marks	Guidance
1	$a = 5$	**B1**	
	$b = 4$	**B1**	
	$c = -3$	**B1**	
2	$\tan^2\theta = \frac{1}{y+2}$ soi **or** $x = 1 + \tan^2\theta$ soi	**B1**	Must be in terms of $\tan^2\theta$
	Use of $\tan^2\theta + 1 = \sec^2\theta$ $\frac{1}{y+2} + 1 = x$ oe	**M1**	For a valid attempt to eliminate θ
	$y = \frac{1}{x-1} - 2$ or $y = \frac{3-2x}{x-1}$ oe	**2**	**Dep M1** for attempt to rearrange to obtain in the required form **A1** for a correct form
	Alternative $x = \frac{1}{\cos^2\theta}$ **and** $y + 2 = \frac{\cos^2\theta}{\sin^2\theta}$ soi	**(B1)**	
	$y + 2 = \frac{\frac{1}{x}}{1 - \frac{1}{x}}$ oe	**(M1)**	For a valid attempt to eliminate θ, making use of $\sin^2\theta + \cos^2\theta = 1$
	$y = \frac{1}{x-1} - 2$ or $y = \frac{3-2x}{x-1}$ oe	**(2)**	**Dep M1** for attempt to rearrange to obtain in the required form **A1** for a correct form
3(a)	Gradient = 4 soi	**B1**	
	Intercept = -3 soi	**B1**	
	$\lg(2y+1) = 4x^2 - 3$ oe	**M1**	For $\lg(2y+1) = \textit{their } m\left(x^2\right) + \textit{their } c$
	$y = \frac{1}{2}\left(10^{4x^2-3} - 1\right)$ or $y = \frac{\frac{10^{4x^2}}{1000} - 1}{2}$	**A1**	
3(b)	$y = 0$	**B1**	Must have at least 3 marks from part **(a)**

Question	Answer	Marks	Guidance
3(c)	$2=\frac{1}{2}\left(10^{4x^2-3}-1\right)$ oe and attempt to obtain x = …	**M1**	**Dep** on **M** mark in part **(a)** for use of $y=2$ in *their* $y=\frac{1}{2}\left(10^{4x^2-3}-1\right)$, $y=\frac{\frac{10^{4x^2}}{1000}-1}{2}$ or $\lg(2y+1)=4x^2-3$ and attempt to obtain x = …
	$x=(\pm)\,0.962$ or better	**A1**	
4(a)	$\frac{1}{17}\begin{pmatrix}-15\\8\end{pmatrix}$ oe	**2**	**B1** for 17 seen
4(b)	$2a+4b-12=4b-4a$ $-5+3=4a+8b$	**M1**	For equating like vectors to obtain at least one equation
	$a=2,\ b=-\frac{5}{4}$ oe	**2**	**Dep M** mark for solving *their* 2 equations to obtain both a and b
5	$\left(1+\frac{x}{6}\right)^{12}=1+2x+\frac{11x^2}{6}$	**2**	**B2** for 3 correct terms **B1** for 2 correct terms
	$(2-3x)^3=8-36x+54x^2...$	**2**	**B2** for 3 correct terms **B1** for 2 correct terms
	Term in x: $-36x+16x$	**M1**	A correct method using *their* terms or coefficients, must be considering 2 terms
	$p=-20$ soi	**A1**	
	Term in x^2: $\frac{88}{6}x^2+54x^2-72x^2$	**M1**	A correct method using *their* terms or coefficients, must be considering 3 terms
	$q=-\frac{10}{3}$ soi	**A1**	
6(a)	$\mathrm{p}\left(\frac{1}{2}\right)$: $a+4b+15=0$ oe	**B1**	For $\mathrm{p}\left(\frac{1}{2}\right)$ equated to zero
	$\mathrm{p}(2)$: $4a+b=60$ oe	**B1**	For $\mathrm{p}(2)$ equated to 120
	$a=17,\ b=-8$	**2**	**Dep M1** on both previous **B** marks, for solving *their* equations to obtain a and b **A1** for both

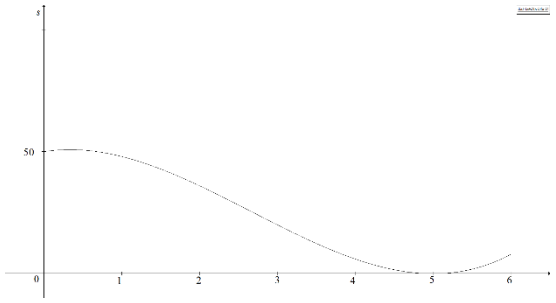
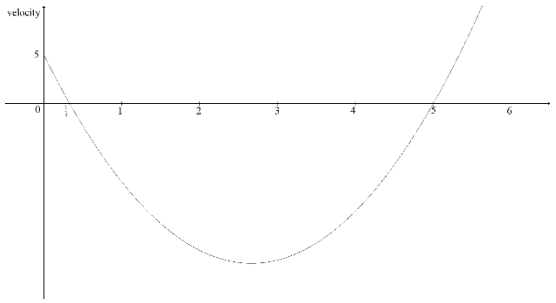
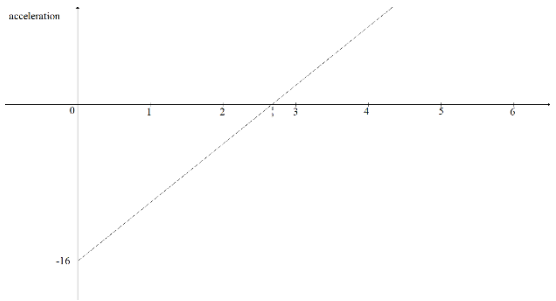
Question	Answer	Marks	Guidance
6(b)	-8	**B1**	**FT** on *their* **integer** b
6(c)	$\mathrm{p}'(x) = 18x^2 + 34x + 6$ $\mathrm{p}''(x) = 36x + 34$	**M1**	For attempt to differentiate *their* p(x), may be implied by correct **FT** answer
	$\mathrm{p}''(0) = 34$	**A1**	**FT** on $2\times$ *their* **integer** a
7(a)	$\dfrac{2(x-1)^2 - (x-1)(2x+3) + (2x+3)}{(x-1)^2(2x+3)}$	**M1**	Attempt at a fraction, allow with an extra $(x-1)$ term in each term of the numerator and the denominator
	$= \dfrac{8-3x}{(x-1)^2(2x+3)}$	**A1**	**AG** – must see sufficient detail to justify the given result, if an extra $(x-1)$ term involved, it must be dealt with correctly
7(b)	$\left[\ln(2x+3) - \ln(x-1) - \dfrac{1}{(x-1)}\right]_2^a$	**3**	**B1** for each correct term
	$\left(\ln(2a+3) - \ln(a-1) - \dfrac{1}{a-1}\right) - (\ln 7 - 1)$	**M1**	**Dep** on at least one ln term from integration, for applying limits correctly in *their* integral
	$\dfrac{a-2}{a-1} + \ln\left(\dfrac{2a+3}{7(a-1)}\right)$ oe	**2**	**A1** for $\dfrac{a-2}{a-1}$ or $1 - \dfrac{1}{a-1}$ **A1** for $\ln\left(\dfrac{2a+3}{7(a-1)}\right)$
	Alternative 1 final 2 marks $\dfrac{-1}{a-1} + \ln\left(\dfrac{\mathrm{e}(2a+3)}{7(a-1)}\right)$ oe	**(2)**	**A1** for $\dfrac{-1}{a-1}$ **A1** for $\ln\left(\dfrac{\mathrm{e}(2a+3)}{7(a-1)}\right)$
	Alternative 2 final 2 marks $\dfrac{a-2}{a-1} - \ln 7 + \ln\left(\dfrac{2a+3}{(a-1)}\right)$ oe	**(2)**	**A1** for $\dfrac{a-2}{a-1} - \ln 7$ or $1 - \dfrac{1}{a-1} - \ln 7$ Allow 1.946 or better for ln 7 **A1** for $\ln\left(\dfrac{2a+3}{(a-1)}\right)$

Question	Answer	Marks	Guidance
7(b)	**Alternative 3** final 2 marks $\frac{-1}{a-1} - \ln 7 + \ln\left(\frac{e(2a+3)}{(a-1)}\right)$ oe	**2**	**A1** for $\frac{-1}{a-1} - \ln 7$ Allow 1.946 or better for ln 7 **A1** for $\ln\left(\frac{e(2a+3)}{(a-1)}\right)$
8(a)	With the sisters: 70 or ${}^{8}C_{4}$ oe	**B1**	
	Without the sisters: 28 or ${}^{8}C_{6}$ oe	**B1**	
	Total: 98	**B1**	
8(b)(i)	60480	**B1**	
8(b)(ii)	The start of the password and the end of the password can each be chosen 6 ways	**B1**	6 or ${}^{3}P_{2}$ oe seen twice
	The remaining characters can be chosen in 20 ways	**B1**	20 or ${}^{5}P_{2}$ oe seen
	Total number of ways: 720	**B1**	

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Question	Answer	Marks	Guidance
9	When $x = 0,\ y = \ln 2$ soi	**B1**	May be implied in later work
	$\frac{dy}{dx} = \frac{(x+1)\frac{6x}{(3x^2+2)} - \ln(3x^2+2)}{(x+1)^2}$	**3**	**B1** for $\frac{6x}{(3x^2+2)}$ allow when seen **M1** for attempt at a quotient or product **A1** for all other terms apart from $\frac{6x}{(3x^2+2)}$ correct
	When $x = 0, \frac{dy}{dx} = -\ln 2$	**M1**	**Dep** on previous **M** mark for attempt to find the gradient using *their* $\frac{dy}{dx}$
	Equation of normal: $y - \ln 2 = \frac{1}{\ln 2}x$	**M1**	For attempt at a normal equation using *their y* (not 3 ln2) and $-\frac{1}{their(-\ln 2)}$, must be from an attempt at differentiation
	When $y = 0,\ x = -(\ln 2)^2$	**M1**	For attempt to find the value of x when $y = 0$ using *their* normal equation
	Gradient $BC = \frac{3\ln 2}{(\ln 2)^2}$	**M1**	**Dep** on both previous **M** marks
	$\frac{3}{\ln 2}$ or $3(\ln 2)^{-1}$	**A1**	Must have correct exact working throughout

Question	Answer	Marks	Guidance
10(a)(i)	xy^2 soi	**B1**	Simplification of the left-hand side of the first equation
	$1 = \lg 10$ soi	**B1**	Simplification of right-hand side of equation
	$x - 3\left(\frac{10}{x}\right) = 13$	**M1**	For substitution of y^2 into linear equation oe and attempt to simplify
	$x^2 - 13x - 30 = 0$	**A1**	**AG** – must see sufficient detail to justify the given answer
	Alternative $y^2 = \frac{(x-13)}{3}$	**(B1)**	
	$\lg x + \lg \frac{(x-13)}{3} = 1$	**(M1)**	For attempt at substitution in the log equation
	$\frac{x(x-13)}{3} = 10$ oe	**(B1)**	
	$x^2 - 13x - 30 = 0$	**(A1)**	**AG** – must see sufficient detail to justify the given answer
10(a)(ii)	$x = 15$ only	**B1**	
	$y = \sqrt{\frac{2}{3}}$ or $\frac{\sqrt{6}}{3}$ or exact equivalent only	**B1**	isw once exact value seen
10(b)	$\log_a x + \frac{3}{\log_a x}$ or $\frac{1}{\log_x a} + 3\log_x a$	**B1**	For an appropriate change of base
	$(\log_a x)^2 - 4\log_a x + 3 = 0$ or $3(\log_x a)^2 - 4\log_x a + 1 = 0$	**M1**	For an attempt to obtain a 3-term quadratic equation in terms of $\log_a x$ or $\log_x a$, equated to zero.
	$\log_a x = 3 \quad \log_a x = 1$ or $\log_x a = \frac{1}{3}, \quad \log_x a = 1$	**M1**	**Dep** on previous **M** mark for correct solution of *their* quadratic equation
	$x = a$	**A1**	Must be from completely correct work
	$x = a^3$	**A1**	Must be from completely correct work

Question	Answer	Marks	Guidance
11(a)	$\frac{ds}{dt} = 3t^2 - 16t + 5$ oe	**2**	**M1** for attempt at differentiation of a product or expansion and differentiation with at least two out of three terms of *their* expansion differentiated correctly **A1** all correct, allow factorised
	$t = \frac{1}{3},\ \ t = 5$	**2**	**Dep M1** for attempt to solve *their* $\frac{ds}{dt} = 0$, must be in quadratic form **A1** for both
11(b)		**2**	**B1** for a correct curve in the first quadrant only. There must be an indication of a max point in the correct position and a min point at $(5,\ 0)$ **B1** for $(0,\ 50)$ provided basic curve shape is correct
11(c)		**2**	**B1** for a quadratic curve in the first and fourth quadrants **B1** for $(0,\ 5)$, $\left(\frac{1}{3},\ 0\right)$ or $(0.333,\ 0)$ marked and passing through $(5,\ 0)$ on the *x*-axis
11(d)(i)	Acceleration = $6t - 16$	**B1**	
11(d)(ii)		**2**	**B1** for a straight line with a positive gradient in the first and fourth quadrants, meeting the vertical axis **Dep B1** for $\left(\frac{8}{3},\ 0\right)$ or $(2.67,\ 0)$ and $(0,-16)$