Cambridge Assessment International Education

# Cambridge O Level

**ADDITIONAL MATHEMATICS** **4037/11**

Paper 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 80

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Maths-Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

## MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

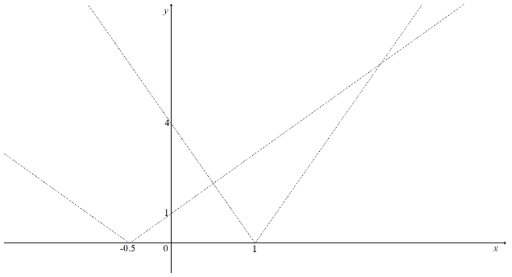
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

### Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $p^{-\frac{3}{2}}q^{\frac{8}{3}}r^{-2}$ | **3** | **B1** for $a=-\frac{3}{2}$<br>**B1** for $b=\frac{8}{3}$<br>**B1** for $c=-2$ |
| 2(a) | $\frac{ds}{dt}=-\frac{3}{2}(1+3t)^{-\frac{3}{2}}$ | **2** | **M1** for $a(1+3t)^{-\frac{3}{2}}$<br>**A1** all correct |
| | When t = 1, $\frac{ds}{dt}=-\frac{3}{16}$<br>Speed = $\frac{3}{16}$ | **A1** | |
| 2(b) | Acceleration = $\frac{27}{4}(1+3t)^{-\frac{5}{2}}$ | **B1** | Allow unsimplified |
| | $(1+3t)^{-\frac{5}{2}}$ is always positive (so acceleration can never be zero.) | **B1** | Any valid explanation. |
| 3(a) | $f(x) \in \mathbb{R}$ oe | **B1** | Must be using correct notation, allow $y \in$ |
| 3(b) | $5(\ln(3x+1))-7=13$ | **M1** | For correct order |
| | $x=\frac{e^4-1}{2}$ | **2** | **M1** for a correct attempt to solve to get $x=$, allow one sign error<br>**Dep** on previous M mark<br>**A1** all correct must be exact |
| 3(c) | $(f'(x)=)\ \frac{2}{2x+1}$ | **2** | **M1** for $\frac{a}{2x+1}$<br>**A1** all correct |
| | $(g^{-1}(x)=)\ \frac{x+7}{5}$ | **B1** | soi |
| | $2x^2+15x-3=0$ | **M1** | for equating and forming a 3-term quadratic equation = 0 |
| | $x = 0.195, -7.69$ | **M1** | For solution of *their* 3-term quadratic |
| | $x = 0.195$ | **A1** | For discounting negative root. |
| 4(a) | $[f(x)=]\pm 4(x+2)(x-1)(x-3)$ | **3** | **B1** for $\pm$<br>**B1** for 4<br>**B1** for $(x+2)(x-1)(x-3)$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b)(i) |  | **3** | **B1** for 2 V shapes which intersect twice in the first quadrant, with vertices on the $x$-axis, must be straight lines, not curves.<br><br>**B1** for –0.5 and 1 on the $x$-axis<br>**B1** for 1 and 4 on the $y$-axis |
| 4(b)(ii) | $2x + 1 = 4(x - 1)$ | **M1** | For attempt to solve to get $x =$ |
| | $x = 2.5$ | **A1** | |
| | $2x + 1 = -4(x - 1)$ oe | **M1** | For attempt to solve to get $x =$ |
| | $x = 0.5$ | **A1** | |
| | **Alternative**<br><br>$4x^2 + 4x + 1 = 16x^2 - 32x + 16$ | **(M1)** | For attempt to square each equation and equate |
| | $12x^2 - 36x + 15 = 0$ oe | **(M1)** | Dep on previous M mark for attempt to simplify to a 3-term quadratic equation, equated to zero and attempt to solve |
| | $x = 2.5 \quad x = 0.5$ | **(A2)** | **A1** for each |
| 5(a) | $\begin{pmatrix} -7.5 \\ 4 \end{pmatrix}$ or $-\frac{1}{2}\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ oe | **2** | **B1** for $\begin{pmatrix} 7.5 \\ -4 \end{pmatrix}$ oe<br>or<br>**B1** for $\begin{pmatrix} -15 \\ 8 \end{pmatrix}$ |
| 5(b) | $15a + 2a + 1 = 6b + 6a$<br>$5b + 2 = 2$ | **M1** | For equating like vectors in order to obtain at least one equation |
| | $a = 1, b = 2$ | **2** | **Dep M1** for attempt to solve both equations<br>**A1** for both |
| 6(a) | $k = 14$ | **B1** | |
| | $k = 6$ | **B1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b)(i) | $\dfrac{(1+\tan\theta)(1+\cos\theta)+(1-\tan\theta)(1-\cos\theta)}{1-\cos^2\theta}$ | **M1** | Allow $(1+\cos\theta)(1-\cos\theta)$ in the denominator |
| | Expansion of numerator and simplification of denominator | **M1** | **Dep** on previous M mark |
| | Use of $\tan\theta\cos\theta = \sin\theta$ | **B1** | soi |
| | $\dfrac{2(1+\sin\theta)}{\sin^2\theta}$ | **A1** | Sufficient simplification to justify obtaining the given answer |
| 6(b)(ii) | $2(1+\sin\theta) = 3\sin^2\theta$<br>$3\sin^2\theta - 2\sin\theta - 2 = 0$ | **M1** | For use of part **(a)** and attempt to simplify to a 3-term quadratic equation equated to zero. |
| | $\sin\theta = \dfrac{1-\sqrt{7}}{3}$ or –0.5485… | **M1** | **M1** for attempt to solve and obtain a value for $\theta$, may be implied by one correct solution |
| | 213.3° and 326.7° | **A2** | **A1** for one solution<br>If 0 scored **SC1** for awrt 213 and 327<br>Penalise excess solutions in the range |
| 7(a) | Common difference = $2\lg 3$ | **B1** | Must be exact |
| | $\dfrac{n}{2}(2\lg 3 + (n-1)2\lg 3) = 256\lg 81$<br>or $\dfrac{n}{2}(\lg 9 + (n-1)\lg 9) = 512\lg 9$ | **M1** | For use of the sum formula |
| | $\lg 81 = 4\lg 3$ soi<br>or $\lg 81 = 2\lg 9$ soi | **B1** | Allow when working with decimal |
| | $n^2 = 1024$ oe | **M1** | Dep on first M mark, for attempt to simplify the sum equation by dividing through by $\lg 3$ oe to obtain an equation in $n$ only |
| | $n = 32$ cao | **A1** | Must have exact working through out |
| 7(b) | $\ln 256 = 4\ln 4$, $\ln 16 = 2\ln 4$ oe | **M1** | For use of power rule to obtain the common ratio |
| | Common ratio = 0.5 | **A1** | |
| | $S_\infty = \dfrac{4\ln 4}{1-\textit{their } r}$ oe | **M1** | Allow $\ln 256$ for first term and *their r* provided it is positive and < 1 |
| | $16\ln 2$ | **A1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $x^2 + 2\sqrt{5}x - 20 = 3\sqrt{5}x + 10$<br>$x^2 - \sqrt{5}x - 30 = 0$ | **M1** | For equating $x$ terms and simplifying to a 3-term quadratic equation equated to zero. |
| | $x = \dfrac{\sqrt{5} \pm \sqrt{5 - (4 \times -30)}}{2}$ oe | **M1** | **Dep** on previous M mark for attempt to solve to obtain $x =$, sufficient detail must be shown |
| | $x = 3\sqrt{5} \quad x = -2\sqrt{5}$ | **A1** | For both |
| | $y = 55, y = -20$ | **A1** | For both |
| 8(b) | Use of $\text{cosec}^2\,\theta = 1 + \cot^2\,\theta$ | **B1** | May be implied by later work |
| | $\text{cosec}^2\theta = 1 + \dfrac{\left(2+\sqrt{3}\right)^2}{\left(\sqrt{3}-1\right)^2}$ | **M1** | For attempting to deal with tan correctly, forming a single fraction and simplifying, with sufficient detail – at least 4 terms in the numerator |
| | $\text{cosec}^2\theta = \dfrac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}}$ | **A1** | |
| | $\text{cosec}^2\theta = \dfrac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}} \times \dfrac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$ | **M1** | For attempt to rationalise *their* expression, with sufficient detail in the simplification of the numerator – at least 3 terms |
| | $14 + \dfrac{15\sqrt{3}}{2}$ | **A1** | |
| | **Alternative 1**<br>Use of $\text{cosec}^2\,\theta = 1 + \cot^2\,\theta$ | **(B1)** | May be implied by later work |
| | $\cot\theta = \dfrac{2+\sqrt{3}}{\sqrt{3}-1}$<br>$= \dfrac{3\sqrt{3}+5}{2}$ | **(2)** | **M1** for attempting to rationalise cot $\theta$ or tan with sufficient detail in the simplification of the numerator – at least 3 terms |
| | $\text{cosec}^2\theta = 1 + \left(\dfrac{3\sqrt{3}+5}{2}\right)^2$<br>$= 14 + \dfrac{15\sqrt{3}}{2}$ | **(2)** | **M1** for expressing as a single fraction and attempt to simplify to required form. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(b) | **Alternative 2**<br>Use of $\text{cosec}^2\,\theta = 1 + \cot^2\,\theta$<br>**and** $\cot^2\,\theta = \frac{1}{\tan^2\,\theta}$ | **(B1)** | May be implied by later work |
| | $\tan^2\,\theta = \frac{4-\sqrt{3}}{7+4\sqrt{3}}$<br>$= 52 - 30\sqrt{3}$ | **(2)** | **M1** for attempting to rationalise $\tan^2\,\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms |
| | $\text{cosec}^2\,\theta = 1 + \left(\frac{1}{52-30\sqrt{3}}\right)^2$<br>$= 14 + \frac{15\sqrt{3}}{2}$ | **(2)** | **M1** for attempting to rationalise $\cot^2\,\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms **and** expressing as a single fraction and attempt to simplify to required form. |
| | **Alternative 3**<br>Use of right-angled triangle<br>$\text{Hyp}^2 = 11 + 2\sqrt{3}$ | **(2)** | **M1** For attempt to calculate the square of hypotenuse |
| | $\text{cosec}^2\theta = \frac{11+2\sqrt{3}}{4-2\sqrt{3}}$ | **(B1)** | for correct use of $\text{cosec}^2\,\theta$ with *their* squared hypotenuse |
| | $\text{cosec}^2\theta = \frac{11+2\sqrt{3}}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$ | **(M1)** | For attempt to rationalise *their* expression, with sufficient detail in the simplification of the numerator – at least 3 terms |
| | $14 + \frac{15\sqrt{3}}{2}$ | **A1** | |
| 9(a) | $\frac{1}{2}r^2\theta = 10,\ \theta = \frac{20}{r^2}$ | **B1** | |
| | $[P =]\ 2r + r\theta$ | **M1** | For substituting *their* $\theta$ in $P$ |
| | $[P =]\ 2r + \frac{20}{r^2}$ | **A1** | |
| 9(b) | $\frac{\mathrm{d}P}{\mathrm{d}r} = 2 - \frac{20}{r^2}$ | **M1** | For attempt to differentiate *their* answer to part **(a)** to obtain the form of $\left[\frac{\mathrm{d}P}{\mathrm{d}r} =\right] 2 + \frac{a}{r^2}$ |
| | When $\frac{\mathrm{d}P}{\mathrm{d}r} = 0,\ r = \sqrt{10}$ | **2** | **Dep M1** for equating *their* $\frac{\mathrm{d}P}{\mathrm{d}r}$ to zero and attempt to solve<br>**A1** cao |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(c) | $\frac{d^2P}{dr^2} = \frac{40}{r^3}$<br>As $r$ is positive, $\frac{d^2P}{dr^2}$ is also positive<br>so minimum | **2** | **M1** for a complete method, allow valid alternatives, if differentiated, must be in the form of $\left[\frac{d^2P}{dr^2} = \right]\frac{k}{r^3}$<br>**A1** for a correct conclusion |
| 9(d) | $\theta = 2$ | **B1** | |
| 10 | $-1 = \tan\left(3p + \frac{\pi}{2}\right)$<br>$p = \frac{\pi}{12}$ | **2** | **M1** for a complete method to find the value of $p$ |
| | $\frac{dy}{dx} = 3\sec^2\left(3x + \frac{\pi}{2}\right)$ | **2** | **M1** for $a\sec^2\left(3x + \frac{\pi}{2}\right)$<br>**A1** all correct |
| | When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$ | **M1** | For attempt to find the gradient using *their p* from differentiation |
| | Equation of normal:<br>$y + 1 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$ | **M1** | For attempt at normal equation using *their p* and $-\frac{1}{\textit{their}\text{ value for }\frac{dy}{dx}}$ |
| | When $x = 0$, $y = \frac{\pi}{72} - 1$ | **M1** | For attempt to find $B$ using *their* normal equation (must be from differentiation) |
| | When $y = 0$, $x = \frac{\pi}{12} - 6$ | **M1** | For attempt to find $A$ using *their* normal equation (must be from differentiation) |
| | Mid-point $\left(\frac{\pi}{24} - 3,\ \frac{\pi}{144} - \frac{1}{2}\right)$ | **2** | **A1** for $x$ value (must be exact)<br>**A1** for $y$ value (must be exact) |

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