Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

© UCLES 2021

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

© UCLES 2021

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

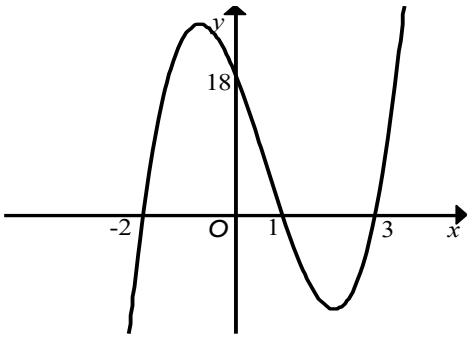
B Mark for a correct result or statement independent of Method marks.

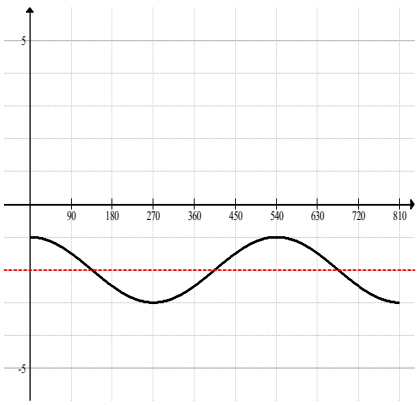
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

© UCLES 2021

Question	Answer	Marks	Partial Marks
1	$1+4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}$	**B2**	mark final answer for **B2** **B1** for any 3 correct simplified terms in a sum or all 5 simplified terms listed but not summed or for a correct, simplified expansion that is not their final answer or **M1** for a correct unsimplified expansion e.g. $1+4e^{2x}+\frac{4\times3}{2}(e^{2x})^2+\frac{4\times3\times2}{6}(e^{2x})^3+(e^{2x})^4$ If 0 scored, **SC1** for a complete, correct, simplified expansion as final answer found by multiplying out the brackets
2	Correct graph and intercepts  	**B3**	**B1** for correct shape; the ends must extend above and below the x-axis **B1** for correct roots indicated; must have attempted a cubic shape **B1** for correct y-intercept indicated; must have attempted a cubic shape
3	Uses b^2-4ac : $6^2-4(2k-1)(k+1)$	**M1**	
	$-8k^2-4k+40*0$ oe	**M1**	**dep** on first **M1** where * is = or any inequality sign condone one sign or arithmetic slip in simplification
	Factorises or solves *their* 3-term quadratic expression or equation for CVs e.g. $(5+2k)(8-4k)$ oe	**M1**	
	Finds correct CVs: -2.5 oe, 2	**A1**	
	$-2.5 \leqslant k \leqslant 2$	**A1**	mark final answer

Question	Answer	Marks	Partial Marks
4	$\frac{m}{27}-\frac{29}{9}+\frac{39}{3}+n=0$ oe	**B1**	
	$m - 29 + 39 + n = 6$ oe	**B1**	
	Eliminates one unknown correctly for a pair of linear equations in m and n and solves for one unknown	**M1**	
	$m = 6$, $n = -10$	**A2**	**A1** for either
	[p(2) =] 48 – 116 + 78 – 10 = 0 oe, nfww	**A1**	
5(a)	1	**B1**	
5(b)	$360 \div \frac{2}{3}$ oe	**M1**	
	540	**A1**	If 0 scored, **SC1** for 3π
5(c)	Correct sketch for domain $0° \leqslant x \leqslant 810°$ 	**B2**	**B1** for correct cosine shape from (0, –1) with amplitude 1 for $0° \leqslant x \leqslant 810°$ **B1** for attempt at correct cosine shape with period 540° for $0° \leqslant x \leqslant 810°$ If 0 scored, **SC1** for a fully correct graph for $0° \leqslant x \leqslant 540°$ **Maximum of 1 mark if not fully correct.**
6(a)	$\sqrt{(11-5)^2+(6--4)^2}$ oe	**M1**	
	11.7 or 11.66[19...] rot to 4 or more figs	**A1**	
6(b)(i)	[y =] 1	**B1**	

Question	Answer	Marks	Partial Marks
6(b)(ii)	$m_{AC} = \frac{6--4}{11-5}$ or $\frac{10}{6}$ nfww oe	**B1**	
	$m_{BD} = \frac{-1}{their\frac{10}{6}}$ oe	**M1**	
	$y - their\ 1 = -\frac{3}{5}(x-8)$ oe isw	**A1**	**FT** *their* 1 from (b)(i) and *their* perpendicular gradient
6(b)(iii)	$\begin{pmatrix}-5\\3\end{pmatrix}$ and $\begin{pmatrix}5\\-3\end{pmatrix}$	**B2**	**B1** for either If 0 scored, **SC1** for $-5\mathbf{i} + 3\mathbf{j}$ and $5\mathbf{i} - 3\mathbf{j}$
7(a)	[Arc length + 2 × tangent length] $18 \times \frac{7\pi}{9} + 2 \times 18 \times \tan\frac{7\pi}{18}$ oe	**M2**	**M1** for [Arc length] $18 \times \frac{7\pi}{9}$ oe or [Tangent length] $18 \times \tan\frac{7\pi}{18}$ oe or [Tangent length] $\frac{18}{\tan\frac{\pi}{9}}$ oe or [Tangent length] $\frac{18}{\sin\frac{\pi}{9}} \times \sin\frac{7\pi}{18}$ oe
	143 or 142.9 or awrt 142.9 (cm)	**A1**	
7(b)	[Area of kite – area of sector] $18 \times their\left(18 \times \tan\frac{7\pi}{18}\right) - \frac{1}{2} \times 18^2 \times \frac{7\pi}{9}$ oe	**M2**	**FT** *their BC* or *CD* from (a) providing it is not 18 **M1** for [area of sector] $\frac{1}{2} \times 18^2 \times \frac{7\pi}{9}$ oe or [area of kite] $18 \times their\left(18 \times \tan\frac{7\pi}{18}\right)$ oe or [area of kite] $18 \times their(18 \times \tan 70)$ oe
	494 or 494.3 or awrt 494.3 (cm^2)	**A1**	

Question	Answer	Marks	Partial Marks
8(a)	Factorises or solves $3t^2 - 30t + 72 = 0$ or $t^2 - 10t + 24 = 0$	**M1**	
	$t = 4$, $t = 6$	**A1**	
	Integrates v to find F(t): $\frac{3t^3}{3} - \frac{30t^2}{2} + 72t$	**M2**	**M1** for any two terms correct
	Correct substitution for F(6) – F(4) or F(4) – F(6)	**M1**	**dep** on at least **M1** for integration **FT** *their* 4 and *their* 6 provided they are **both positive**
	4 (m)	**A1**	**dep** on all previous marks being awarded
8(b)	$[a =] 6t - 30$	**B1**	
	[When $t = 2$: $a = 6(2) - 30 =$] -18 (ms^{-2}) cao	**B1**	
9	Correctly eliminates x or y e.g. $4x^2 + 3x\left(-\frac{4}{x}\right) + \left(-\frac{4}{x}\right)^2 = 8$ oe or $4\left(-\frac{4}{y}\right)^2 + 3\left(-\frac{4}{y}\right)y + y^2 = 8$ oe	**M1**	
	Rearranges to a 3-term quadratic in x^2 or y^2 soi e.g. $4x^4 - 20x^2 + 16 = 0$ or $y^4 - 20y^2 + 64 = 0$	**A1**	
	Factorises or solves *their* 3-term quadratic in x^2 or y^2 soi : $(x^2 - 1)(x^2 - 4)$ or $(y^2 - 16)(y^2 - 4)$	**M1**	
	$x^2 = 1$, $x^2 = 4$ oe, nfww or $y^2 = 16$, $y^2 = 4$ oe, nfww	**A1**	
	$x = \pm 1$ $x = \pm 2$ $y = \mp 4$ $y = \mp 2$ oe, nfww	**A2**	**A1** for all 4 x values or all 4 y values
10(a)	$\frac{1}{3}e^{3x+3} + c$ or $\frac{1}{3}e^3 \times e^{3x} + c$ nfww	**B2**	**B1 for** ke^{3x+3} or $ke^3 \times e^{3x}$ where $k \neq \frac{1}{3}$ or 0

Question	Answer	Marks	Partial Marks
10(b)(i)	$\frac{\mathrm{d}(\sin 4x)}{\mathrm{d}x} = 4\cos 4x$ soi	**B1**	
	Applies correct form of product rule: $4x\cos 4x + [1]\sin 4x$ isw	**B1**	**FT** *their* $4\cos 4x$ if possible
10(b)(ii)	$\left[\int (4x\cos 4x)\,\mathrm{d}x = \right] x\sin 4x - \int \sin 4x\,\mathrm{d}x$	**M1**	**FT** use of *their* $mx\cos 4x + n\sin 4x$ where m and n are constants
	$x\sin 4x + \frac{1}{4}\cos 4x[+c]$ soi	**A1**	
	$\frac{\pi}{3}\sin\left(4\times\frac{\pi}{3}\right)+\frac{1}{4}\cos\left(4\times\frac{\pi}{3}\right)-$ $\left[\frac{\pi}{4}\sin\left(4\times\frac{\pi}{4}\right)+\frac{1}{4}\cos\left(4\times\frac{\pi}{4}\right)\right]$	**A1**	
	Correct completion to **given answer** $\frac{1}{8}-\frac{\pi\sqrt{3}}{6}$	**A1**	
11(a)	$500 = \frac{4}{6}\pi x^3 + \pi x^2 y$ oe	**M1**	
	$y = \frac{1}{\pi x^2}\left(500 - \frac{4}{6}\pi x^3\right)$ oe, isw	**A1**	if first **M0**, **SC1** for $y = \frac{1}{\pi x^2}\left(500 - \frac{4}{3}\pi x^3\right)$ oe seen
	$S = 2\pi x^2 + \pi x^2 + 2\pi x\left(\frac{500}{\pi x^2} - \frac{2}{3}x\right)$	**M1**	**dep** on first **M1**
	Correct completion to given answer: $S = \frac{5}{3}\pi x^2 + \frac{1000}{x}$	**A1**	
11(b)	Differentiates S: $\frac{10}{3}\pi x - \frac{1000}{x^2}$ oe	**B2**	**B1** for each term
	$\frac{10}{3}\pi x - \frac{1000}{x^2} = 0$ and attempt to solve	**M1**	**FT** *their* $\frac{\mathrm{d}S}{\mathrm{d}x}$ providing at least **B1** awarded
	$x = \sqrt[3]{\frac{300}{\pi}}$ isw or 4.57[07...] nfww	**A1**	
12(a)(i)	$A(0,1)$ and $B(1, 0)$	**B1**	

Question	Answer	Marks	Partial Marks
12(a)(ii)	$[y=]\frac{1}{2(2)+1}$ and $[y=]\frac{2-1}{5}$ **and** evaluates both expressions as $\frac{1}{5}$	**B2**	**B1** for $[y=]\frac{1}{2(2)+1}$ **and** $5y=2-1$ oe
	Alternative 1 $[y=]\frac{1}{2(2)+1}=\frac{1}{5}$ or $[y=]\frac{2-1}{5}=\frac{1}{5}$ **and** solves $5\times\frac{1}{5}=x-1$ oe to get $x = 2$ or $\frac{1}{5}=\frac{1}{2x+1}$ oe to get $x = 2$	**(B2)**	**B1** for $\frac{1}{2(2)+1}=\frac{1}{5}$ and $5\times\frac{1}{5}=x-1$ oe or $\frac{2-1}{5}=\frac{1}{5}$ and $\frac{1}{5}=\frac{1}{2x+1}$ oe
	Alternative 2 $2x^2 - x - 6 = 0$ **and** solves or factorises to get $(2x + 3)(x – 2)$ and states $x = 2$ OR shows $2(2^2) – 2 – 6 = 0$ oe	**(B2)**	**B1** for $(2x + 1)(x – 1) = 5$ or $2x^2 - x - 6 = 0$
	Alternative 3 $(2x + 1)(x – 1) = 5$ oe **and** shows $(2 \times 2 + 1)(2 – 1) = 5$	**(B2)**	**B1** for $(2x + 1)(x – 1) = 5$
12(b)	$\frac{1}{2}\times1\times0.2$ oe or $\frac{2^2}{5\times2}-\frac{2}{5}-\left(\frac{1^2}{5\times2}-\frac{1}{5}\right)$ oe	**B1**	
	$[F(x)=]\ \frac{1}{2}\ln(2x+1)$ $[+c]$ oe or $\frac{1}{2}\ln(x+0.5)$ $[+c]$ oe	**B2**	**B1** for $\frac{1}{2}\ln 2x+1$ or $\frac{1}{2}\ln x+0.5$ or $k\ln(2x+1)$ or $k\ln(x+0.5)$, $k \neq 0.5$ or 0
	F(2) – F(0) – *their* 0.1	**M1**	**FT** *their* F(x) providing at least **B1** for integration of curve awarded
	$0.5\ln5-0.1$ or exact equivalent	**A1**	

Question	Answer	Marks	Partial Marks
13(a)	$[\text{fg}(x) =] \dfrac{2\left(\dfrac{1}{x}\right)^2 - 1}{3\left(\dfrac{1}{x}\right)}$ oe	**M1**	
	$[\text{fg}(x) =] \dfrac{2-x^2}{3x}$ or $\dfrac{2}{3x} - \dfrac{x}{3}$	**A1**	mark final answer
13(b)(i)	$f^{-1} > 0$	**B1**	
13(b)(ii)	$2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$	**B1**	
	Correctly applies quadratic formula: $[x =] \dfrac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)}$ oe or $[y =] \dfrac{-(-3x) \pm \sqrt{(-3x)^2 - 4(2)(-1)}}{2(2)}$ oe	**M1**	**FT** *their* $2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	**B1**	
	$f^{-1}(x) = \dfrac{3x + \sqrt{9x^2 + 8}}{4}$ cao	**A1**	must be a function of x