Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

IGCSE™ is a registered trademark.

© UCLES 2018

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

© UCLES 2018

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

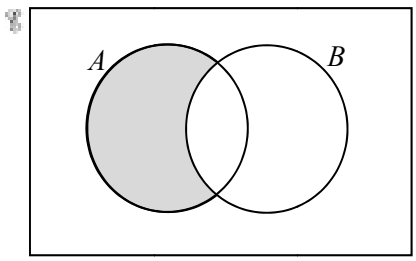
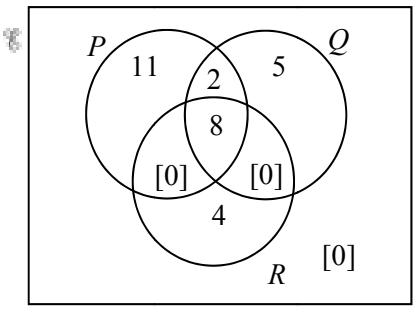
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

© UCLES 2018

Question	Answer	Marks	Partial Marks
1(i)	Uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ Uses $\cos^2\theta + \sin^2\theta = 1$ Completes to $\frac{1}{\sin\theta} = \text{cosec}\,\theta$	**B3**	**B1** for using $\cot\theta = \frac{\cos\theta}{\sin\theta}$ oe or $\tan\theta = \frac{\sin\theta}{\cos\theta}$ oe at some stage **B1** for use of $\cos^2\theta + \sin^2\theta = 1$ oe **B1** for common denominator of $\sin\theta$ oe either in a compound fraction or in two partial fractions or for writing $\frac{1-\sin^2\theta}{\sin\theta}$ as $\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$ oe **Maximum of 2 marks if not fully correct or does not complete to cosecθ**
1(ii)	$\sin\theta = \frac{1}{4}$	**M1**	
	14.5° or 14.47[751…] rot to 4 or more figures isw	**A1**	Not from wrong working
2(a)		**B1**	
2(b)		**B3**	**B1** for 8 correctly placed and all the empty regions correct **B1** for 11, 2, 5 correctly placed **B1** for 4 correctly placed **maximum of 2 marks if fully correct but other values such as 30, 21 and/or 15 present within the diagram**
	their 12	**B1**	**STRICT FT** *their* Venn diagram

Question	Answer	Marks	Partial Marks
3	p(–3) = 0 or p(2) = –15 stated or implied	**M1**	
	$-54 + 9a + 72 + b = 0$ or better	**A1**	finds one correct equation; implies **M1**
	$16 + 4a - 48 + b = -15$ or better	**A1**	finds another correct equation; implies **M1**
	Solves a pair of simultaneous equations in a and b	**M1**	**dep** on first **M1** condone one sign or arithmetic error in *their* solution; as far as finding one unknown
	$a = -7$, $b = 45$	**A1**	
	60 cao	**A1**	
4	Eliminates one of the unknowns	**M1**	
	Simplifies to a correct 3-term quadratic: $2x^2 + 4x - 16$ [= 0] oe or $2y^2 - 6y - 36$ [= 0] oe	**A1**	
	Factorises or solves $(x+4)(x-2) = 0$ oe or $(y+3)(y-6) = 0$ oe	**M1**	**FT** *their* 3-term quadratic in x or y;
	(2, 6) and (–4, –3) oe	**A2**	Not from wrong working **A1** for either (2, 6) or (–4, –3) or **A1** for $x = 2$ and $x = -4$ or $y = 6$ and $y = -3$
5(a)	${}^7P_4$ or $7 \times 6 \times 5 \times 4$ oe	**M1**	
	840	**A1**	
5(b)(i)	20	**B1**	
5(b)(ii)	${}^5C_1 \times {}^4C_1 \times {}^2C_1$ or $5 \times 4 \times 2$ oe	**M1**	
	40	**A1**	
5(b)(iii)	${}^5C_3 + {}^4C_3$ oe	**M1**	
	14	**A1**	

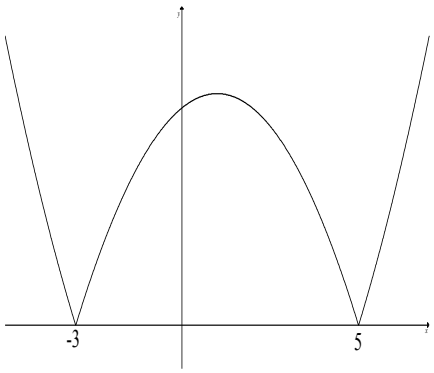
© UCLES 2018

Question	Answer	Marks	Partial Marks
6(i)	(Arc length =) 1.5 × 5 oe soi	**M1**	implied by 7.5
	(*DE* =) 10sin(0.75) oe soi	**M1**	implied by awrt 6.82
	34.3 or answer in range 34.31 to 34.32	**A1**	
6(ii)	(Area sector =) $\frac{1}{2} \times 5^2 \times 1.5$ oe	**M1**	implied by 18.75
	(Area triangle =) $\frac{1}{2} \times 5^2 \times \sin(1.5)$ oe	**M1**	implied by awrt 12.47
	31.2 or answer in range 31.21 to 31.22	**A1**	
7(i)	$\lvert their(\mathbf{a} + \mathbf{c})\rvert = \sqrt{their(5^2 + 14^2)}$	**M1**	
	$\sqrt{221}$	**A1**	mark final answer
7(ii)	[(2 + *m*)**i** + (3 – 5*m*)**j** therefore] *their* (2 + *m*) = 0	**M1**	for attempting to form **a** + *m***b** and equate the scalar of the **i** component to 0
	m = –2 only	**A1**	implies **M1**
7(iii)	[(2*n* – 1)**i** + (3*n* + 5)**j** = 3**i** + 11**j** or *n*(2**i** + 3**j**) = (3**i** + 11**j**) + (**i** – 5**j**) oe leading to] 2*n* – 1 = 3 or 3*n* + 5 = 11 oe, soi	**M1**	
	n = 2 only	**A1**	implies **M1**

© UCLES 2018

Question	Answer	Marks	Partial Marks
8(a)	$\begin{pmatrix} -2 & 6 \\ 1 & 12 \end{pmatrix}$	**B2**	**B1** for a 2 by 2 matrix with 2 or 3 correct elements
	their $\left[\frac{1}{-30}\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}\right]$ oe isw	**B2**	**FT** *their* non-singular **BA** **B1 FT** for either $\frac{1}{their(-30)}\begin{pmatrix} & \\ & \end{pmatrix}$ or $\ldots \times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ If *their* **BA** is singular, **B0** then **SC1** for $\ldots \times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ OR **Alternative method $\mathbf{A^{-1}B^{-1}}$:** **B2** for $\mathbf{A^{-1}} = \frac{1}{-5}\begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$ isw or $\mathbf{B^{-1}} = \frac{1}{6}\begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$ isw or **B1** for a multiplier of $\frac{1}{-5}$ or for $\begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$ or for a multipler of $\frac{1}{6}$ or for $\begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$ **B2 FT** for $\mathbf{A^{-1}}\ \mathbf{B^{-1}} = their\frac{1}{-30} \times their\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ or **B1 FT** for a 2 by 2 matrix with 2 or 3 correct elements **Maximum of 3 marks if not fully correct**
8(b)(i)	2×3	**B1**	
8(b)(ii)	$\begin{pmatrix} 2 & -\frac{1}{2} \end{pmatrix}$ oe isw	**B2**	**B1** for each correct element; must be in a 1 by 2 matrix or **M1** for a full method as far as finding values for the two elements

© UCLES 2018

Question	Answer	Marks	Partial Marks
9(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{\sin x}\right)=\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)$ oe	**B2**	**B1** for $\frac{1}{2}(\sin x)^{-\frac{1}{2}}\times...$ or for $\frac{1}{2}(\sin x)^{-\frac{1}{2}}$ or for $\frac{1}{2}(...)^{-\frac{1}{2}}\times\cos x$ or for $their\,\frac{1}{2}(\sin x)^{\left(their\frac{1}{2}\right)-1}\times\cos x$
	$their\left(4x^3\right)\sqrt{\sin x}$ $+x^4\left(their\,\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe	**M1**	Applies correct form of product rule
	$4x^3\sqrt{\sin x}+x^4\left(\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe isw	**A1**	Not from wrong working
9(ii)	$\int\left(4x^3\sqrt{\sin x}\right)\mathrm{d}x$ $+\int\left(x^4\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)\mathrm{d}x$ $=x^4\sqrt{\sin x}$ oe	**M1**	or $\int x\,\mathrm{d}x+2\int\left(\frac{x^4\cos x}{2\sqrt{\sin x}}+4x^3\sqrt{\sin x}\right)\mathrm{d}x$ oe **FT** *their* **(i)**
	$\frac{x^2}{2}+2x^4\sqrt{\sin x}\ [+c]$	**A2**	**A1** for $\int x\,\mathrm{d}x+2x^4\sqrt{\sin x}$
10(a)(i)		**B2**	**B1** for correct shape **B1** for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
10(a)(ii)	Any correct domain	**B1**	
10(b)(i)	$\frac{4}{3x-1}$	**B1**	mark final answer

Question	Answer	Marks	Partial Marks
10(b)(ii)	Correct method for finding inverse function e.g. swopping variables **and** changing subject or vice versa; or indicates $(\text{hg})^{-1}(x) = \text{g}^{-1}\text{h}^{-1}(x)$ and finds $\text{g}^{-1}(x) = \frac{x+1}{3}$ and $\text{h}^{-1}(x) = \frac{4}{x}$	**M1**	**FT** only if *their*hg(x) of the form $\frac{a}{bx+c}$ where a, b and c are integers
	$\left[(\text{hg})^{-1}(x) =\right] \frac{1}{3}\left(\frac{4}{x}+1\right)$ oe isw or $\left[(\text{hg})^{-1}(x) =\right] \frac{4+x}{3x}$ oe isw	**A1**	**FT** *their* $(\text{hg})^{-1}(x) = \frac{a-cx}{bx}$ oe If **M0** then **SC1** for *their* hg(x) of the form $y = \frac{a}{x} + b$ oe leading to *their* $(\text{hg})^{-1}(x)$ of the form $y = \frac{a}{x-b}$ isw
10(c)	a cao	**B1**	
11(a)	$\frac{(2x-1)^{\frac{4}{3}}}{\frac{4}{3}\times 2}$ $[+c]$ oe isw	**B2**	**B1** for $k \times \frac{(2x-1)^{\left(\frac{1}{3}+1\right)}}{\left(\frac{1}{3}+1\right)}$ where $k \neq 0$
11(b)(i)	$k\cos 4x\,[+c]$ where $k < 0$ or $k = \frac{1}{4}$	**M1**	
	$-\frac{1}{4}\cos 4x\,[+c]$	**A1**	
11(b)(ii)	Sight of correct substitution of limits: $-\frac{1}{4}\cos\frac{4\pi}{4} - \left(-\frac{1}{4}\cos\frac{4\pi}{8}\right)$ oe	**M1**	**FT** *their* $k\cos 4x$ from **(b)(i)** **dep** on **M1** awarded in **(b)(i)**
	$\frac{1}{4}$	**A1**	does **not** imply **M1**

Question	Answer	Marks	Partial Marks
11(c)	$\int e^{\frac{x}{3}}\,dx = ke^{\frac{x}{3}}\ [+c]$	**M1**	k any non-zero constant
	$k = 3$	**A1**	
	Sight of correct substitution of limits: $their\ ke^{\frac{\ln 8}{3}} - their\ ke^{0}$ oe	**M1**	**dep** on first **M1**
	Shows how to deal with the power of the first term e.g. $\frac{\ln 8}{3} = \ln 8^{\frac{1}{3}}$ or $\frac{\ln 8}{3} = \ln 2$ or $3\left(\sqrt[3]{8}\right)$ seen	**B1**	
	$6 - 3 = 3$	**A1**	Not from wrong working
12(i)	$\tan\frac{\pi}{12} = \frac{r}{h}$ oe	**M1**	
	$r = h(2-\sqrt{3})$ or $r = h\tan\frac{\pi}{12}$ oe	**A1**	
	$[V =]\frac{1}{3}\pi\left(2-\sqrt{3}\right)^2 h^2 \times h$ oe	**M1**	Correctly uses *their* expression for r in terms of h in formula for volume of a cone dependent on finding an expression connecting r and h
	$[V =]\frac{\pi\left(4-4\sqrt{3}+3\right)h^3}{3}$ oe correctly leading to $[V =]\frac{\pi\left(7-4\sqrt{3}\right)h^3}{3}$ **AG**	**A1**	
12(ii)	Correct derivative of V e.g. $\frac{3\pi\left(7-4\sqrt{3}\right)h^2}{3}$ oe isw	**B1**	
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ soi	**B1**	
	$\frac{1}{their\left(\frac{dV}{dh}\right)\Big\vert_{h=5}} \times 30$	**M1**	if correct implies **B1 B1;** if incorrect, a correct **FT** statement implies the second **B1**
	5.32	**A1**	