Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

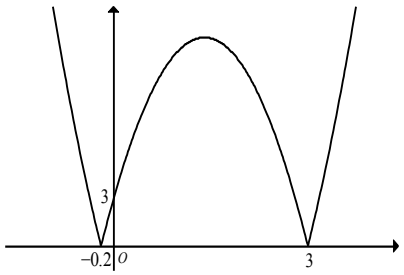
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

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Question	Answer	Marks	Partial Marks
1(i)(a)	A is not a [proper] subset of B oe	**B1**	
1(i)(b)	A and C are mutually exclusive oe or A intersection C is the empty set oe	**B1**	
1(ii)(a)	$\mathrm{n}(A \cup B) = 3$	**B1**	
1(ii)(b)	$x \in (A \cap C')$ oe	**B1**	
2(i)	$k \times \frac{1}{3x-1}$	**M1**	
	$3 \times \frac{1}{3x-1}$	**A1**	
2(ii)	$x = \frac{11}{15}$ soi	**B1**	
	$0.125 \approx \textit{their} \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\textit{their}\frac{11}{15}} \times \delta x$ oe	**M1**	
	0.05 nfww	**A1**	
3(i)	$({}^{12}P_7 =)\ 3\,991\,680$	**B1**	
3(ii)	$(4 \times {}^{11}P_6 =)\ 1\,330\,560$	**B1**	
3(iii)	$4! \times 4! \times 2$ oe	**M2**	**M1** for $4! \times 4!$ oe only or ${}^4P_4 \times {}^4P_3$ oe only
	1152	**A1**	

Question	Answer	Marks	Partial Marks
4(i)	$2(-4)^3+3(-4)^2-4a-12=0$ with one correct interim step leading to $a=-23$	**B1**	Note: $=0$ must be seen or may be implied by e.g. $-92=4a$ or $92=-4a$ or convincingly showing that $2(-4)^3+3(-4)^2-4(-23)-12=0$ or correct synthetic division at least as far as $\begin{array}{c\|cccc} -4 & 2 & 3 & a & -12 \\ & & -8 & 20 & -4a-80 \\ \hline & 2 & -5 & a+20 & 0 \end{array}$ then $a=-23$ or correct long division to, e.g. verify –23, at least as far as $\begin{array}{r} 2x^2-5x-3 \\ x+4\overline{)2x^3+3x^2-23x-12} \\ \underline{2x^3+8x^2} \\ -5x^2-23x \\ \underline{-5x^2-20x} \\ -3x-12 \\ \underline{-3x-12} \\ 0 \end{array}$
	$p(1)=2+3-23-12$ $b=-30$	**B1**	
4(ii)	finds a correct quadratic factor e.g. $(2x^2-5x-3)$	**B2**	**B1** for quadratic factor with 2 correct terms OR **B1** for finding $(x-3)$ using factor theorem **B1** for convincingly finding $(2x+1)$ as third factor
	Product of three linear factors $(2x+1)(x-3)(x+4)$	**M1**	
	$x=-\frac{1}{2}$, $x=3$, $x=-4$ nfww	**A1**	If **M0** then **SC1** if quadratic factorised correctly but does not show full factorisation but does give all 3 solutions correctly
5(i)	Putting $y=f(x)$, changing subject to x and swopping x and y or vice versa	**M1**	
	$f^{-1}(x)=\frac{1}{2}\left(\frac{1}{x}+5\right)$ or $\frac{5x+1}{2x}$ oe isw	**A1**	
5(ii)	$x>0$ oe	**B1**	

Question	Answer	Marks	Partial Marks
5(iii)	$\dfrac{1}{2\left(\dfrac{1}{2x-5}\right)-5}$	**B1**	
	$\dfrac{1}{\dfrac{2-5(2x-5)}{2x-5}}$ oe	**M1**	**FT** if expression of equivalent difficulty e.g. $\dfrac{1}{\left(\dfrac{1}{2x-5}\right)-5}$
	Completes to $\dfrac{2x-5}{-10x+27}$ oe final answer	**A1**	
6(i)	$16x = 40$ oe	**M1**	
	$x = 2.5$ oe (radians)	**A1**	
6(ii)	$\dfrac{1}{2}(16)^2(2.5)$ oe	**M1**	
	320	**A1**	
6(iii)	$\dfrac{1}{2}r^2(their 2.5) = (their 320) - 140$ oe	**M1**	**FT** provided *their* 320 > 140
	correct simplification to $r^2 = \ldots$	**M1**	**dep** on first **M1**
	12	**A1**	
7(i)	$4\tan x + 4x\sec^2 x$ isw	**B2**	Fully correct **B1** for one correct term as part of e.g. a sum of 2 terms
7(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{3x+1}) = 3\mathrm{e}^{3x+1}$	**B1**	
	$\dfrac{(x^2-1)(their 3\mathrm{e}^{3x+1}) - their(2x)\mathrm{e}^{3x+1}}{(x^2-1)^2}$	**M1**	
	$\dfrac{(x^2-1)(3\mathrm{e}^{3x+1}) - 2x\mathrm{e}^{3x+1}}{(x^2-1)^2}$ oe isw	**A1**	
8(i)	Takes logs of both sides	**M1**	
	$\ln y = \ln a + n\ln x$ or $\lg y = \lg a + n\lg x$	**A1**	

Question	Answer	Marks	Partial Marks
8(ii)	$n =$ –0.2 to –0.3 nfww	**B1**	
	attempts to equate y-intercept to ln a or forms *their* ln equation with *their* gradient and a point on the line or uses two points on the line to form a pair of simultaneous equations	**M1**	
	$a = e^{4.7}$ isw or 110 or 109.9[47…]	**A1**	maximum of 2 marks if no coordinates stated
8(iii)	use of ln(50) and ln$x$ = 3 to 3.2	**M1**	or for $\frac{50}{theira} = x^{their\ n}$ or better or for $\ln 50 = \ln(theira) + (their\ n)\ln x$ oe
	awrt 22 or 23 to 2 significant figures	**A1**	implies **M1**
9(i)	$5\left(x-\frac{7}{5}\right)^2 - \frac{64}{5}$	**B3**	**B1** for each of p, q, r correct in correct format; allow correct equivalent values. If **B0**, then **SC2** for $5\left(x-\frac{7}{5}\right) - \frac{64}{5}$ or **SC1** for correct values but incorrect format
9(ii)		**B4**	**B2** for fully correct shape in correct position or **B1** for fully correct shape translated parallel to the x-axis **B1** for y-intercept at (0, 3) marked on graph **B1** for roots marked on graph at –0.2 and 3
9(iii)	$0 < k < \left\lvert their\left(-\frac{64}{5}\right)\right\rvert$	**B2**	**FT** *their* **(i)** **B1** for any inequality using *their* $\frac{64}{5}$ or max y value is *their* 12.8soi
10(i)	$v = \frac{ds}{dt} = -3\sin 3t$	**B1**	
	When $v = 0$, $t = \frac{\pi}{3}$	**B1**	

Question	Answer	Marks	Partial Marks
10(ii)	Finding s when $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$	**M1**	
	Finding s when $t = their\frac{\pi}{3}$ and correct plan	**M1**	Using *their* **(i)** correctly
	1.29 nfww	**A1**	
10(iii)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -9\cos 3t$	**B1**	
	9	**B1**	**FT** *their* $k\cos 3t$
11(a)	$10(1-\sin^2 x)+3\sin x = 9$	**M1**	
	Solves $10\sin^2 x - 3\sin x - 1 = 0$ oe	**M1**	**dep** on first **M1** Solves *their* three term quadratic in $\sin x$
	$\sin x = \frac{1}{2}$, $\sin x = -\frac{1}{5}$	**A1**	
	30º, 150º and 191.5º, 348.5º awrt	**A2**	**A1** for any two correct solutions
11(b)	$3\frac{\sin 2y}{\cos 2y} = 4\sin 2y$ oe	**M1**	
	Solves $3\sin 2y - 4\sin 2y\cos 2y\ [=0]$	**M1**	**dep** on first **M1**
	$\sin 2y = 0$ $\cos 2y = \frac{3}{4}$	**A1**	
	Any two of π, 0.72273…, 5.56045… nfww	**A1**	
	$\frac{\pi}{2}$, 0.361, 2.78 awrt nfww	**A1**	**SC**: cancels out sin2y after **M1M0** allow **SC1** for 0.72273… and 5.56045... and **SC1**for 0.361 and 2.78
12(i)	$\tan 30 = \frac{h}{x/2}$ oe	**M1**	
	Correct completion to given answer	**A1**	
	$V = 5\sqrt{3}h^2$ isw	**B1**	

Question	Answer	Marks	Partial Marks
12(ii)(a)	$\frac{\mathrm{d}V}{\mathrm{d}h} = their 10\sqrt{3}\,h$ or $\frac{5\sqrt{3}}{2}$	**B1**	**FT** $their\, V = k\,h^2$
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ soi	**M1**	
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{their\left(\frac{\mathrm{d}V}{\mathrm{d}h}\right)} \times 0.5$	**M1**	
	0.115 or 0.11547 to 0.1155 oe	**A1**	
12(ii)(b)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \right) 2\sqrt{3} \times their \frac{1}{5\sqrt{3}}$	**M1**	
	$\frac{2}{5}$	**A1**	

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