Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

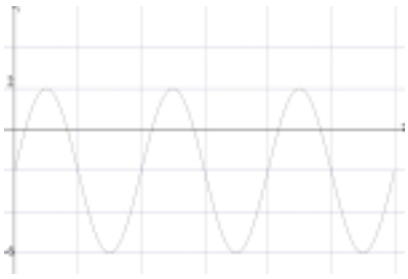
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

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Question	Answer	Marks	Partial Marks
1	Substitution and simplification to obtain a 3 term quadratic in one variable	**M1**	substitution of $y = x + 4$ or $x = y - 4$ and simplification to 3 terms.
	$x^2 - 2x - 8 = 0$ or $2x^2 - 4x - 16 = 0$ or $y^2 - 10y + 16 = 0$ or $y^2 - 10y + 16 = 0$	**A1**	correct equation of the form $ax^2 + bx + c = 0$ or $ay^2 + by + c = 0$
	Solution of quadratic equation	**M1**	**M1 dep**
	$x = 4,\ y = 8$ $x = -2,\ y = 2$	**A2**	**A1** for each pair
2	Midpoint $\left(\frac{5}{2}, -1\right)$	**B1**	
	Gradient of line $= -\frac{8}{3}$	**B1**	
	Gradient of perp $= \frac{3}{8}$	**M1**	
	Equation of perp bisector: $y + 1 = \frac{3}{8}\left(x - \frac{5}{2}\right)$	**M1**	**M1 dep** Using *their* perpendicular gradient and *their* midpoint
	$6x - 16y - 31 = 0$ or $-6x + 16y + 31 = 0$	**A1**	
3	<table><tr><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td></td><td>✓</td><td></td><td></td></tr><tr><td></td><td></td><td>✓</td><td>✓</td></tr><tr><td></td><td></td><td>✓</td><td></td></tr><tr><td>✓</td><td></td><td></td><td></td></tr></table>	**4**	**B1** for either each row correct or each column correct – mark to candidate's advantage.
4(i)	$b = 4$	**B1**	
	$c = 6$	**B1**	
	$2 = a + 4\sin\frac{\pi}{2}$	**M1**	Evaluation of a using *their b* and *their c* and the given point.
	$a = -2$	**A1**	

Question	Answer	Marks	Partial Marks
4(ii)		**3**	**B1** for $-6 \leqslant y \leqslant 2$ **B1** for 3 complete cycles **B1** for all correct
5(i)	The number of bacteria at the start of the experiment	**B1**	
5(ii)	$20\,000 = 800\mathrm{e}^{kt}$ so $\dfrac{20\,000}{800} = \mathrm{e}^{2k}$ or $\ln 20\,000 = \ln 800 + \ln(\mathrm{e}^{2k})$	**M1**	use of given equation and attempt to solve for e^{2k} or use logs correctly
	$2k = \ln 25$	**M1**	correct method to obtain $2k$
	1.61	**A1**	
5(iii)	$P = 800\mathrm{e}^{3\ln 5}$	**M1**	Substitution of $t = 3$ in formula using *their k*
	$= 100\,000$	**A1**	answer in range 99800 to 100200
6(a)	$\left(\dfrac{\log_3 p}{\log_3 2} \times \log_3 2\right) + \log_3 q$ or $\log_3 2^{\log_2 p} + \log_3 q$	**B1**	
	$\log_3 p + \log_3 q$ or $\log_3(2^{\log_2 p} \times q)$	**B1**	**B1 dep**
	$\log_3 pq$	**B1**	**B1 dep**
6(b)	$(\log_a 5 - 1)(\log_a 5 - 3) = 0$	**M1**	solution of quadratic equation
	$\log_a 5 = 1,\ a = 5$ $\log_a 5 = 3,\ a = \sqrt[3]{5}$ or 1.71 or $5^{\frac{1}{3}}$	**A2**	**A1** for $a = 5$ **A1** for $a = \sqrt[3]{5}$ or 1.71or $5^{\frac{1}{3}}$
7(i)	$\dfrac{1}{2}\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$	**2**	**B1** for $\dfrac{1}{2}$ **B1** for $\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$

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Question	Answer	Marks	Partial Marks
7(ii)	$4x-2y=\frac{5}{2}$ $-5x+3y=\frac{7}{2}$	**B1**	Relating solution of these equations to matrix in **(i)** B1 for adapted equation or $\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}$
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}2.5\\3.5\end{pmatrix}$ or $2\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}3&2\\5&4\end{pmatrix}\begin{pmatrix}5\\7\end{pmatrix}$	**M1**	Correct method for pre-multiplication by *their* inverse matrix.
	$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}7.25\\13.25\end{pmatrix}$ or $\begin{pmatrix}\frac{29}{4}\\\frac{53}{4}\end{pmatrix}$ $x=7.25,\ y=13.25$	**A2**	**A1** for each. Condone in matrix form.
8(a)	$3(2\mathbf{i}-5\mathbf{j})-4(\mathbf{i}-3\mathbf{j})$	**M1**	For expansion and collection of terms
	$3\mathbf{p}-4\mathbf{q}=2\mathbf{i}-3\mathbf{j}$	**A1**	
	Magnitude of their $2\mathbf{i}-3\mathbf{j}$ $\sqrt{2^2+(-3)^2}$	**M1**	For method to find magnitude
	Unit vector $=\frac{2\mathbf{i}-3\mathbf{j}}{\sqrt{13}}$	**A1**	
8(b)(i)	$v^2=2.73^2+1.25^2$	**B1**	Correct use of Pythagoras
	$v=3.00$	**B1**	
8(b)(ii)	$\tan\theta=\frac{1.25}{2.73}$ oe	**M1**	Use of a trig funtion to obtain a relevant angle
	Angle to AB= 24.6° or 0.429 radians	**A1**	
9(i)	$256x^8-64x^6+7x^4$	**3**	**B1** for each term

Question	Answer	Marks	Partial Marks
9(ii)	$\frac{1}{x^4}+\frac{2}{x^2}+1$	**B1**	
	$\left(256x^8-64x^6+7x^4\right)\left(\frac{1}{x^4}+\frac{2}{x^2}+1\right)$ $\left(256\times1-64\times2+7\times1\right)x^4$	**M1**	**M1** for three correctly obtained products leading to terms in x^4 using *their* $256x^8-64x^6+7x^4$ and *their* $\frac{1}{x^4}+\frac{2}{x^2}+1$
	Coefficient of x^4 is $256-128+7$ $=135$	**A1**	
10(a)	$\frac{5+6\sqrt{5}}{6+\sqrt{5}}\times\frac{6-\sqrt{5}}{6-\sqrt{5}}$	**M1**	for rationalisation
	$=\frac{30-5\sqrt{5}+36\sqrt{5}-30}{31}$	**M1**	**M1dep** for expanding the numerator to obtain four terms.
	$=\frac{31\sqrt{5}}{31}=\sqrt{5}$	**A1**	**A1** for $\sqrt{5}$ from correct working
10(b)	$\sqrt{3}\times\left(\sqrt{2}\right)^6\times\sqrt{2}=\sqrt{6}\times2^3$		
	$8\sqrt{6}$	**B2**	**B1** for $\sqrt{6}$ from $\sqrt{3}\times\sqrt{2}$ **B1** for 8 from $\left(\sqrt{2}\right)^6$ or 2^3

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Question	Answer	Marks	Partial Marks
10(c)	**EITHER**: $x^2 + \sqrt{2}x - 4 = 0$	**B1**	3 term quadratic equation equated to zero
	$x = \dfrac{-\sqrt{2} \pm \sqrt{18}}{2}$	**M1**	use of the quadratic formula
	for use of $\sqrt{18} = 3\sqrt{2}$	**M1**	**M1 dep**
	$\sqrt{2}, \ -2\sqrt{2}$	**A1**	For both from full working
	OR: $x^2 + \sqrt{2}x - 4 = 0$	**B1**	
	$\left(x + \dfrac{\sqrt{2}}{2}\right)^2 = 4 + \dfrac{1}{2}$ $x = \pm\dfrac{3}{\sqrt{2}} - \dfrac{\sqrt{2}}{2}$	**M1**	Correct use of completing the square method
	$x = -\dfrac{4}{\sqrt{2}}, \ \dfrac{2}{\sqrt{2}}$	**M1**	**M1dep** for dealing with $\sqrt{2}$ in denominator
	$x = \sqrt{2}, \ -2\sqrt{2}$	**A1**	
11(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 16 - \dfrac{54}{x^3}$	**M1**	for $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 16 \pm \dfrac{p}{x^3}$
	Equating to zero and obtaining x^3	**M1**	**M1dep**
	$x = \dfrac{3}{2}, \ y = 36$	**A2**	**A1** for each

Question	Answer	Marks	Partial Marks
11(ii)	**EITHER**: When $x = 1,\ y = 43$ When $x = 3,\ y = 51$	**B1**	**B1** for both
	$\left(\frac{1}{2}(43+51)\times 2\right) - \int_1^3 16x + \frac{27}{x^2}\,\mathrm{d}x$		
	Area of trapezium = $\left(\frac{1}{2}(43+51)\times 2\right)$ oe	**B1**	**FT** from *their P* and *their Q*
	Integration to find area under curve	**M1**	for $\left[px^2 + \frac{q}{x}\right]$
	$= \left[8x^2 - \frac{27}{x}\right]$	**A1**	Integration correct
	$= \left[8\times 3^2 - \frac{27}{3}\right] - \left[8\times 1^2 - \frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $= 94 - 82$ $= 12$	**A1**	
	OR: When $x = 1,\ y = 43$ When $x = 3,\ y = 51$	**B1**	**B1** for both
	Equation of PQ: $y = 4x + 39$	**B1**	Equation of line **FT** from *their P* and *their Q*
	Integration of their $4x + 39 - 16x - \frac{27}{x^2}$	**M1**	for $\left[px + qx^2 + \frac{r}{x}\right]$
	$= \left[39x - 6x^2 + \frac{27}{x}\right]$	**A1**	All correct
	$= \left[39\times 3 - 6\times 3^2 + \frac{27}{3}\right]$ $- \left[39\times 1 - 6\times 1^2 + \frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $= 72 - 60$ $= 12$	**A1**	

Question	Answer	Marks	Partial Marks
12	$\frac{dy}{dx} = (2x-5)^{\frac{1}{2}} \quad (+c)$	**M1**	for $k(2x-5)^{\frac{1}{2}}$,
	for $(2x-5)^{\frac{1}{2}}$	**A1**	
	Substitution to obtain arbitrary constant	**M1**	**M1 dep** Using $\frac{dy}{dx} = 6$ when $x = \frac{9}{2}$
	$\left(\frac{dy}{dx}\right) = (2x-5)^{\frac{1}{2}} + 4$	**A1**	for correct $\frac{dy}{dx}$
	Integration of *their* $k(2x-5)^{\frac{1}{2}} + c$	**M1**	**M1 dep** on first M1 for integration of $k(2x-5)^{\frac{1}{2}}$ to obtain $m(2x-5)^{\frac{3}{2}}$
	$y = \frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x \quad (+d)$	**A1**	for $\frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x$ **FT** *their* (non -zero) constant
	Finding constant	**M1**	**M1 dep** for obtaining arbitrary constant for $m(2x-5)^{\frac{3}{2}} + nx + d$ using $x = \frac{9}{2}, \quad y = \frac{2}{3}$
	$y = \frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x - 20$	**A1**	for correct equation

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