Cambridge
O Level

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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

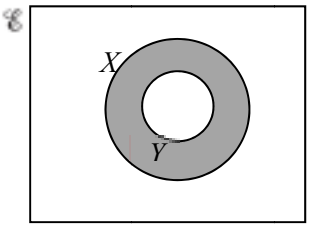
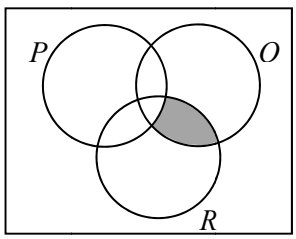
Abbreviations

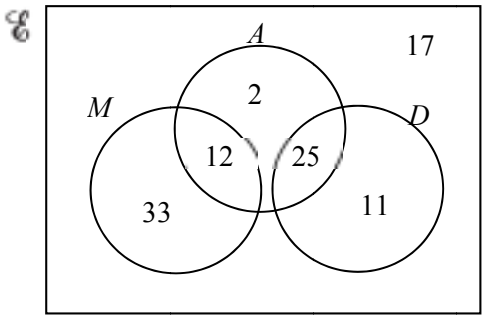
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

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Question	Answer	Marks	Guidance
1	Integrates	**M1**	must be clear attempt to integrate at least one term
	$[y =]x^4 + x\,(+c)$	**A1**	Both terms correct
	$17 = 2^4 + 2 + c$	**DM1**	Substitution of $x = 2$, $y = 17$ to find c
	$y = x^4 + x - 1$ cao	**A1**	must have $y =$
2(a)	$2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18}$ oe	**M1**	method must be shown – simplifies and combines product
	$18\sqrt{2}$	**A1**	If all over common denominator then consider the product for M1A1
	$9\sqrt{2}$ oe soi leading to final answer of $27\sqrt{2}$	**B1**	
2(b)	$[x =]\dfrac{6+\sqrt{3}}{2-\sqrt{3}}$	**M1**	Expanding and making x subject – condone slips but must be of equivalent difficulty
	$[x =]\dfrac{6+\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$ oe and multiplies out numerator and denominator	**M1**	numerator at least 3 terms; $12 + 2\sqrt{3} + 6\sqrt{3} + 3$
	$15 + 8\sqrt{3}$	**A1**	
3(i)	$\dfrac{2x}{x^2+1}$ final answer	**B2**	**B1** for $\dfrac{1}{x^2+1} \times (ax + b)$, a or b must be non-zero
3(ii)	$\delta y = their\left(\dfrac{2(3)}{(3)^2+1}\right) \times h$ or better	**M1**	Substitutes $x = 3$ into $their\dfrac{\mathrm{d}y}{\mathrm{d}x}$ **and** multiplies by h
	$\dfrac{6}{10}h$ oe	**A1**	
4(a)(i)	36	**B1**	
4(a)(ii)	7	**B1**	
4(b)	$[y =]\,5\sin 4x + 7$	**B4**	**B1** for each of 5, 4 and 7 and **B1** for sine Accept $a = 5$, $b = 4$, $c = 7$ for **B3**

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Question	Answer	Marks	Guidance
5(i)	$16+32ax+24a^2x^2+8a^3x^3+a^4x^4$	**B2**	**B1** for at most 2 terms incorrect or missing or for correct but unsimplified form **SC1** for $16+32ax+24ax^2+8ax^3+ax^4$ or all terms correct listed
5(ii)	$24a^2=8a^3$ and solves to given answer	**B1**	or verifies that $a = 3$ leads to coeff of 216 for both terms must be from correct terms in (i)
5(iii)	$x = –0.01$ or $ax = –0.03$ soi	**M1**	
	$16 + 32(3)(–0.01) +24(9)(–0.01)^2$ leading to $16 – 0.96 + 0.0216$ or 15.06…isw	**A1**	Must show clear substitution into their expansion for A1 and reach a value which rounds to 15.1
6(i)	$(\mathbf{M}=)\begin{pmatrix}90&10&30\\0&45&0\\25&0&15\\10&0&100\end{pmatrix}$	**B1**	columns and/or rows may be interchanged but must be consistent
6(ii)	$(\mathbf{LM}=)\begin{pmatrix}1&1&1&1\end{pmatrix}\begin{pmatrix}90&10&30\\0&45&0\\25&0&15\\10&0&100\end{pmatrix}=\begin{pmatrix}125&55&145\end{pmatrix}$	**B1**	Answer must be of correct order and must be consistent with a correct **M**
6(iii)	The total numbers of each type of ticket sold by all 4 cinemas oe	**B1**	
6(iv)	$(\mathbf{N}=)\begin{pmatrix}5\\4\\3\end{pmatrix}$	**B1**	Calculation not required
	The **total** income of **all** (4) cinemas or other valid comment e.g. **total** income from **all** ticket sales	**B1**	**Total** cost/value of tickets etc.
7(a)	 	**B2**	**B1** for each
7(b)(i)	$n(M \cap D)=0$ or $M \cap D=\varnothing$	**B1**	No additional brackets e.g. $M \cap D=\{\varnothing\}$ is **B0**

Question	Answer	Marks	Guidance
7(b)(ii)		**B3**	**B1 c**orrect intersection of circles with 12 and 25 correct **B1** 33, 2, 11 correctly placed **B1FT** 17; must be on the Venn diagram **and** identified as the required answer **FT** on 100– (sum of *their* 5 correctly positioned values)
8(a)	$\left[{}^{30}P_2 =\right] 870$	**B1**	
8(b)(i)	${}^{2}C_1 \times {}^{14}C_{10}$ oe (2 × 1001)	**M1**	Condone $\binom{14}{4}$ for $\binom{14}{10}$
	2002	**A1**	implies **M1**
8(b)(ii)	$\left({}^{2}C_1 \times {}^{5}C_4 \times {}^{9}C_6\right) + \left({}^{2}C_1 \times {}^{5}C_5 \times {}^{9}C_5\right)$ oe (840 + 252) or ${}^{2}C_1 \times {}^{14}C_{10} -$ $\left({}^{2}C_1 \times {}^{5}C_1 \times {}^{9}C_9 + {}^{2}C_1 \times {}^{5}C_2 \times {}^{9}C_8 + {}^{2}C_1 \times {}^{5}C_3 \times {}^{9}C_7\right)$ {2002 – (10 + 80 + 720)}	**M3**	**M3 f**or fully correct method soi **M2 f**or all necessary products but **n**ot summed with no extra products seen soi **M1 f**or one correct three term product soi
	1092	**A1**	implies **M3**
9(i)	Substitution of $y = 2(1-x)$	**M1**	Mus**t** be attempt at full subs**t**itution. Condone one sign error in substitution. Condone omission of = 0 or incorrect rhs
	$-3x^2 + 2x + 1 = 0$ oe $\left(3x^2 - 2x - 1 = 0\right)$	**A1**	Terms collected
	Solving *their* quadratic found from eliminating one variable $(3x+1)(1-x)$ or $(3x+1)(x-1)$	**M1**	can be implied by a correct pair of x values
	$\left(-\frac{1}{3}, \frac{8}{3}\right)$ oe and $(1, 0)$ oe isw nfww	**A2**	**A1 f**or each or **A1** for a correct pair of x-coo**r**dinates or a correct pair of y-coordinates

Question	Answer	Marks	Guidance
9(ii)	$[m=]\frac{1}{2}$ cao	**B1**	
	$\left(\frac{1}{3},\frac{4}{3}\right)$	**B1**	**FT**
	$y-their\frac{4}{3}=their\frac{1}{2}\left(x-their\frac{1}{3}\right)$	**M1**	or $y=their\frac{1}{2}x+c$ and substitutes their midpoint and reaches c = …
	$6y-3x=7$	**A1**	allow any equivalent form with integer coeffs/constant
10(i)	<table><tr><td>t</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td></tr><tr><td>ln$P$</td><td>1.48</td><td>2.12</td><td>2.76</td><td>3.4(0)</td></tr></table>	**M1**	allow ln$P$ values to 1 dp rounded or truncated (1.5, 2.1, 2.8, 3.4)
	single ruled line drawn within tolerance at least for t between 1 and 2.5	**A1**	All points within 1 square of line / must **not** pass through origin
10(ii)	e^{their3}	**M1**	
	18 to 22.2	**A1**	
10(iii)	(0, c) with $0.1 \leqslant c \leqslant 0.3$ (0.2)	**B1**	allow $y = c$ condone c =…
	m in the range $1.25 \leqslant m \leqslant 1.34$ (1.28)	**B1**	
10(iv)	$\ln P=(their1.28)t+their0.2$	**M1**	or $\ln P=(\ln b)t+\ln a$
	$P=e^{(their1.28)t+their0.2}$	**M1**	or $\ln b=m=their1.28$ and $\ln a=c=their0.2$
	$P=e^{their0.2}e^{(their1.28)t}$	**A1**	or $1.10 \leqslant a \leqslant 1.35$ $3.49 \leqslant b \leqslant 3.82$
10(v)	$1000*e^{their0.2}\times e^{their1.28t}$ or $1000*their\ a\times their\ b^t$	**M1**	A correct relationship e.g. $1.3t*\ln(1000)-0.2$ where * is = or an inequality sign
	5.3	**A1**	5.2 to 5.5 must be to 1dp

Question	Answer	Marks	Guidance
11(i)	$\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$ oe	**B2**	**B1** for either $\cot x=\frac{\cos x}{\sin x}$ or $\tan x=\frac{\sin x}{\cos x}$ used **B1** for correctly placing over a common denominator or for splitting into 3 correct terms **not** just for stating or working from both sides
	Valid use of Pythagorean identity e.g. $\cos^2 x+\sin^2 x=1$	**B1**	
	Simplification to sec$x$ (correct solution only)	**B1**	**not** if working from both sides
11(ii)	$\cos x=\frac{1}{2}$ soi	**M1**	
	60, 300	**A1**	Correct pair
	$\cos x=-\frac{1}{2}$ soi	**M1**	
	120, 240	**A1**	Correct pair
12(i)	$\left[v=\frac{\mathrm{d}\left(3t-\cos 5t+1\right)}{\mathrm{d}t}=\right]3+5\sin 5t$	**B2**	**B1** for either with no other terms or for both with 1 extra
	their $\left(3+5\sin 5t\right)=0$	**M1**	Must be from an attempt to differentiate
	awrt 0.76	**A1**	0.7570187525
	awrt 1.13	**A1**	1.12793684
	substitutes *their t* values into *s* (4.07…, 3.58…)	**DM1**	must be two values
	0.48 to 0.49 [m]	**A1**	Final **A1** may imply earlier **A1**s
12(ii)	25cos 5t	**M1**	Differentiating *their v* correctly providing at least 2 terms with one trig function
	–25	**A1**	Ignore +25 following –25

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