Cambridge O Level

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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Guidance
1 (i)	$(2k)^2 - 4\,(1)(4k-3)$ [< 0] Correct completion to given inequality $k^2 - 4k + 3 < 0$ isw	**M1** **A1**	clear attempt at $b^2 - 4ac$
(ii)	Critical values 1 and 3 soi $1 < k < 3$ as final answer	**M1** **A1**	May be implied by incorrect inequalities
2 (i)	Clear attempt at quotient rule or equivalent product rule $\left[\frac{dy}{dx} =\right] \frac{14}{(3-x)^2}$ or $\left[\frac{dy}{dx} =\right] \frac{14}{x^2 - 6x + 9}$ cao or correct simplified equivalent	**M1** **A1**	condone omission of brackets allow recovery from bracketing errors or omissions if implied in correct work to the correct answer
(ii)	$[y = 9]\ x = 2$ $\frac{0.07}{\delta x} \approx \left(their \left.\frac{dy}{dx}\right\|_{x=2} \right)$ oe 0.005 oe	**B1** **M1** **A1**	condone $\frac{0.07}{\delta x} = \left(their \left.\frac{dy}{dx}\right\|_{x=2} \right)$ not from wrong working; answer only does not score
3	Any one of: $\left[{}^6C_0 \times\right] {}^7C_3 + {}^6C_1 \times {}^7C_2$ or 35 + 126 or ${}^{13}C_3 - {}^6C_2 \times {}^7C_1 - {}^6C_3$ or 286 – 105 – 20 161	**M2** **A1**	**M1** for $\left[{}^6C_0 \times\right] {}^7C_3$ or ${}^6C_1 \times {}^7C_2$ or ${}^{13}C_3 - {}^6C_2 \times {}^7C_1$ or ${}^{13}C_3 - {}^6C_3$ or ${}^6C_2 \times {}^7C_1 + {}^6C_3$ or for the numerical equivalent of one of these calculations If **M0** then **B3** for answer only of 161

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Question	Answer	Marks	Guidance
4 (i)	$2(2)^3 - 3(2)^2 + 2q + 56 = 0$ with one correct interim step leading to $q = -30$	**B1**	allow for only $16 - 12 + 2q + 56 = 0$ $q = -30$ NB $= 0$ must be seen or may be implied by e.g. $-60 = 2q$ or $60 = -2q$; or convincingly showing $2(2)^3 - 3(2)^2 - 30(2) + 56 = 0$; allow for only $16 - 12 + 2(-30) + 56 = 0$ or correct synthetic division at least as far as $\begin{array}{c\|cccc} 2 & 2 & -3 & q & 56 \\ & & 4 & 2 & 2q+4 \\ \hline & 2 & 1 & q+2 & 0 \end{array}$ then $q = -30$
(ii)	$2x^2 + x - 28$ $(x-2)(2x-7)(x+4)$	**B2** **M1**	**B1** for any two terms correct For factorising the correct equation; condone $= 0$; condone $(2x-7)(x+4)$ only for **M1** but for **A1 must see** all 3 factors in this part; do not allow $\left(x - \frac{7}{2}\right)$
	$x = 2$, $x = -4$, $x = 3.5$ oe	**A1**	not from wrong working; answers only do not score
5 (i)	(2, 8)	**B1, B1**	
(ii)	$\frac{their 8 - 0}{their 2 - p} = -2$ or better	**M1**	Condone $\frac{their 8 - 0}{their 2 - p} = \frac{-1}{their \text{ gradient } AB}$ oe
	[p =] 6	**A1**	

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Question	Answer	Marks	Guidance
(iii)	$[MB =]\sqrt{(6-their2)^2+(10-their8)^2}$ soi or $\left[\frac{1}{2}AB=\right]\frac{1}{2}\sqrt{(6--2)^2+(10-6)^2}$ soi or $[MC =]\sqrt{(their2-their\ p)^2+(their8-0)^2}$ soi or $\tan[...]=\frac{8}{4}$ soi or $4.47^2=8.94^2+10^2-2(8.94)(10)\cos[...]$ or $8.94^2=10^2+10^2-2(10)(10)\cos[...]$	M1	implied by $[MB =]\sqrt{20}$ or $\left[\frac{1}{2}AB=\right]\frac{1}{2}\sqrt{80}$ e.g. 4.47, or $[MC =]\sqrt{80}$ or e.g. 8.94 or 63.4° or equivalents
	$\sin^{-1}\left(\frac{\sqrt{20}}{10}\right)$ oe soi	M1	or $\cos^{-1}\left(\frac{\sqrt{80}}{10}\right)$ or $\tan^{-1}\left(\frac{\sqrt{20}}{\sqrt{80}}\right)$ or $\tan^{-1}\left(\frac{4}{8}\right)$ or $90-\tan^{-1}\left(\frac{8}{4}\right)$ or equivalent complete correct method; implies first **M1**
	26.56 to 26.6° or 0.4636 to 0.464 rads cao	A1	Not from wrong working
6 (i)	Valid explanation	B1	e.g. arc length is greater than the radius or 7 is greater than 5
(ii)	$7=5\theta$	M1	
	$\theta=1.4$ oe	A1	implies **M1**
(iii)	$\frac{1}{2}\times5^2\times their1.4$ oe	M1	
	17.5oe	A1	

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Question	Answer	Marks	Guidance
(iv)	$[\text{triangle area} =]\ \frac{1}{2} \times 5^2 \times \sin \textit{their}1.4$ or 12.3 to 12.32 or for [$\frac{1}{2}$ × base × height=] $\frac{1}{2} \times 6.4[4...] \times 3.8[2...]$ oe	M1	may be embedded in a difference calculation
	5.18 to 5.2 inclusive	A1	implies **M1**
7 (i)	$\begin{pmatrix} 12 & 15 \\ 9 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ soi	M1	if no method shown, may be implied by their answer with at least 2 correct elements
	$\begin{pmatrix} 16 & 17 \\ 10 & 9 \end{pmatrix}$	A1	
(ii)	det**A** = 4 × 2 – 3 × 5 = –7 or det**B** = 4 × 3 – 2 × 1= 10	B1	allow for e.g. (4 × 2 – 3 × 5) × (4 × 3 – 2 × 1) = –70 or det**A** = 8 – 15 = –7 or det**B** = 12 – 2= 10
	AB = $\begin{pmatrix} 21 & 23 \\ 14 & 12 \end{pmatrix}$	B2	or **B1** for two elements correct
	det(**AB**) = 21 × 12 – 23 × 14 = –70	B1	allow for det(**AB**) = 252 – 322 = –70 For full marks must conclude that det**AB** = det**A** × det**B** or show the product –7 × 10 = –70 otherwise max 3 marks
(iii)	$\frac{1}{\textit{their}\det \mathbf{AB}} \times \textit{their}\begin{pmatrix} 12 & -23 \\ -14 & 21 \end{pmatrix}$ isw	B2	correct or correct **FT**; **FT** *their* **AB** and *their non-zero* det **AB**; *their* **AB** must be an attempt at a matrix product e.g. $\begin{pmatrix} 16 & 10 \\ 3 & 6 \end{pmatrix}$ **B1** for $\frac{1}{\textit{their}\det \mathbf{AB}} \times \textit{their}\begin{pmatrix} & \\ & \end{pmatrix}$ or for $k \times \textit{their}\begin{pmatrix} 12 & -23 \\ -14 & 21 \end{pmatrix}$

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Question	Answer	Marks	Guidance
8	Eliminates y e.g. $4+\frac{5}{15x+10}+\frac{3}{x}=0$ or eliminates x e.g. $4+\frac{5}{y}+\frac{3}{(y-10)/15}=0$	**M1**	allow even after incorrect rearrangement of the equation of the curve (dependent on resulting equation still in terms of x and y); condone substitution of e.g. $\frac{y+10}{15}$
	Rearrange to a 3-term quadratic $60x^2+90x+30=0$ oe or $4y^2+10y-50=0$ oe	**M1** **A1**	condone sign slips / arithmetic slips
	Factorise or solve 3-term quadratic $x=-\frac{1}{2}$, $x=-1$ isw $y=2\frac{1}{2}$, $y=-5$ isw	**M1** **A1** **A1**	or $y=2\frac{1}{2}$, $y=-5$ or $x=-\frac{1}{2}$, $x=-1$ If final A marks not awarded then **A1** for a correct x, y pair
9 (a)	$\frac{x^2}{2}+x-\frac{1}{x}(+c)$ isw	**B3**	**B1** for each term allow $\frac{x^2}{2}+x+\frac{x^{-1}}{-1}(+c)$ isw for **B3**
(b) (i)	$k\cos(5x+\pi)$ where $k<0$ or $\frac{\cos(5x+\pi)}{5}$ $\frac{-\cos(5x+\pi)}{5}(+c)$	**M1** **A1**	
(ii)	$\frac{-\cos(5(0)+\pi)}{5}-\frac{-\cos(5(-\pi/5)+\pi)}{5}$ or $\frac{-\cos(\pi)}{5}-\left(\frac{-\cos(0)}{5}\right)$ 0.4 oe	**M1** **A1**	correct substitution of the given limits into *their* expression of the form $k\cos(5x+\pi)$, dep on **M1** in (b)(i) answer only does not score
10 (a)	$2=p-q$ and $14=4p-2q$ oe $p=5$ $q=3$	**M1** **A1** **A1**	
(b)	Factorise $10^{2x}-2(10^x)-24\,[=0]$ or factorise $u^2-2u-24\,[=0]$	**M1**	or applies the formula or completes the square
	$10^x=6$ $x=\lg 6$ cao as final answer	**A1** **A1**	ignore $10^x=-4$ for this mark or exact equivalent

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Question	Answer	Marks	Guidance
(c)	$\frac{x+1}{x} = 2^3$ oe www	M2	combines logs and anti-logs or **B1** for one correct log move e.g. $\log_2\left(\frac{x+1}{x}\right) = 3$ or $\log_2(x+1) - \log_2(x) = \log_2 8$ or $\log_2(x+1) - \log_2(x) = 3\log_2 2$
	$x = \frac{1}{7}$ or 0.143 or 0.1428 to 0.1429	A1	
11 (a)	Valid method	M1	Completing the square as far as e.g. constant $-\left(x-\frac{1}{2}\right)^2$ or calculus as far as $1 - 2x = 0$ or finding roots $x = 0$ and $x = 1$and using symmetry soi
	when $x = \frac{1}{2}$	A1	Implies **M1** if not clearly from wrong working
	[greatest value =] $\frac{1}{4}$	B1	
(b)	Valid comment e.g. when $x \geq 1$, f′ is always decreasing	B1	Allow e.g. a sketch with a comment such as the curve is one-one [when $x \geq 1$] or e.g. the curve is one-one when $x > \frac{1}{2}$
(c) (i)	k(10) = 8 or $5 + \sqrt{10-1} = 8$ or stating h(8)	M1	or [hk(x) =] $\lg(7 + \sqrt{x-1})$
	h(8) = 1 or $\lg(8+2) = 1$ cao	A1	[hk(10) =] $\lg(7 + \sqrt{10-1}) = 1$
(ii)	$(y-5)^2 = x-1$	M1	or $(x-5)^2 = y-1$
	$k^{-1}(x) = (x-5)^2 + 1$ isw	A1	
	or $k^{-1}(x) = x^2 - 10x + 26$ isw		
	$5 < x < 15$	B1, B1	**B1**for $5 < x$ oe and **B1**for $x < 15$ oe allow (5, 15); one mark for each limit of the interval; if **B0** then **SC1** for $5 \leqslant x \leqslant 15$ or '5 to 15' or [5, 15] etc.
	$1 < k^{-1}(x) < 101$	B1	allow (1, 101)

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Question	Answer	Marks	Guidance
12 (i)	$8(1-\cos^2 A)+2\cos A=7$ or better	B1	
	Solves or factorises *their* 3-term quadratic in cos$A$	M1	
	60, 104.477... rounded or truncated to 1 dp or more;	A2	with no extras in range; not from clearly wrong working but allow recovery from minor slips or **A1** for either, ignoring extras
(ii)	$\sin(3B+1)=0.4$ soi	B1	may be implied by $\frac{1}{\sin(3B+1)}=2.5$
	$[3B+1=]$ 0.41 or better	M1	implies **B1**
	0.577, 1.9[0], 2.67 or 0.57669..., 1.89823... , 2.67108... rounded or truncated to 4 or more sf	A2	with no extras in range; or **A1** for any one correct ignoring extras If **M0** then **B2** for all 3 correct angles found or **B1** for 1 or 2 correct angles found

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