Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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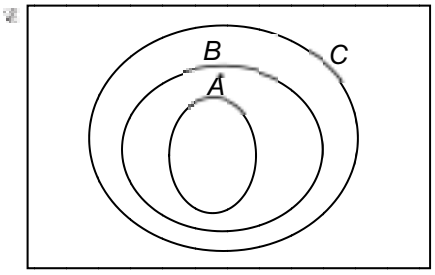
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Abbreviations

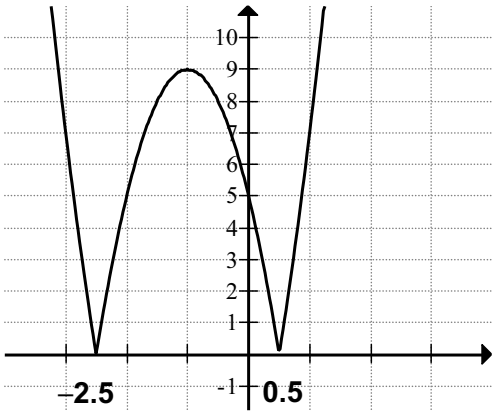
awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Guidance
1	$x^2 - 2x - 15$	**M1**	expands and rearranges to form a 3 term quadratic
	critical values −3 and 5	**A1**	not from wrong working
	$x < -3 \quad x > 5$	**A1**	mark final inequality; **A0** if spurious attempt to combine e.g. $5 < x < -3$
2 (a)		**B1**	It must be clear how the sets are nested
(b) (i)	$h \in P$	**B1**	Allow $\{m,a,t,h,s\}$ for P
(ii)	$n(P \cap Q) = 2$ cao	**B1**	
(iii)	{ t, h, s}	**B1**	
3 (i)	−2	**B1**	
(ii)	$-n$	**B1**	
(iii)	$\frac{\lg 5}{\log_5 10} = [(\lg y)^2]$ or $\frac{\lg 20 - \lg 4}{1 \big/ \lg 5} = [(\lg y)^2]$	**M1**	One log law used correctly
	correct completion to $(\lg 5)^2$ isw	**A1**	answer only does not score
(iv)	$[\log_r]6x^2 = [\log_r]600$	**B1**	Condone base missing
	$x = 10$ only	**B1**	

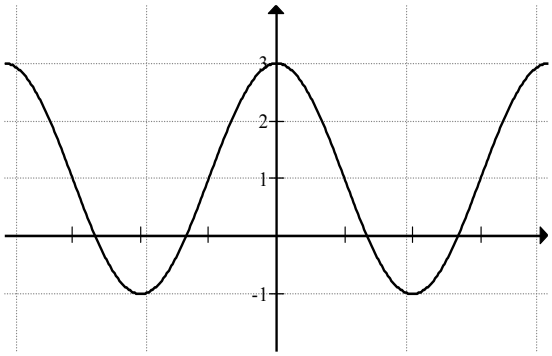
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Question	Answer	Marks	Guidance
4 (i)	$\frac{\pi}{3}$ isw	**B1**	
(ii)	[Area triangle ABC =] $\frac{1}{2}\times 10^2 \times \sin\left(their\ \frac{\pi}{3}\right)$ oe	**M1**	seen or implied by $25\sqrt{3}$ or 43.3(0...)
	[Area 1 sector =] $\frac{1}{2}\times 5^2 \times their\ \frac{\pi}{3}$ oe or $\pi \times 5^2 \times \frac{their 60°}{360}$	**M1**	seen or implied by $\frac{25\pi}{6}$ or 13.0(8...) or 13.09
	Complete correct plan	**M1**	e.g. *their* triangle – 3(*their* sector)
	4.03(1...) or $25\sqrt{3} - \frac{25\pi}{2}$ isw	**A1**	Units not required
5 (a)	$\frac{\sqrt{8}}{\left(\sqrt{7}-\sqrt{5}\right)} \times \frac{\left(\sqrt{7}+\sqrt{5}\right)}{\left(\sqrt{7}+\sqrt{5}\right)}$ and attempt to multiply	**M1**	
	$\frac{\sqrt{56}+\sqrt{40}}{2}$ oe	**A1**	not from wrong working
	$\sqrt{14}+\sqrt{10}$	**A1**	
(b)	$q^2 + 4q\sqrt{3} + 12$ soi	**B1**	
	$28 = q^2 + 12$ oe	**M1**	can be implied by 4 and 16 or –4 and –16
	$q =$ 4, –4 $\quad p =$ 16, –16	**A1**	all values
6 (i)	$4(x+1)^2 - 9$	**B3,2, 1,0**	one mark for each of p, q, r correct in a correctly formatted expression; allow correct equivalent values; If **B0** then **SC2** for $4(x+1)-9$ or **SC1** for correct 3 values seen in incorrect format e.g. $4(x+1x)-9$ or $4(x^2+1)-9$ or for a correct completed square form of the original expression in a different but correct format. e.g. $2(\sqrt{2}x+\sqrt{2})^2-9$

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Question	Answer	Marks	Guidance
(ii)	(–1, 9)	B2FT	B1FT $(-q\ ,-r)$ $r<0$ for each correct coordinate
(iii)		B1 B1 B1	Correct symmetric W shape with cusps on x-axis y-intercept marked at 5 only or coords indicated on graph x-intercepts marked at –2.5 and 0.5 only x-axis or coords indicated on graph or close by
7 (i) (a)	$\mathbf{q}-\mathbf{p}$	B1	
(b)	$2\mathbf{q}-2\mathbf{p}$ or $2(\mathbf{q}-\mathbf{p})$	B1	
(ii)	The points are collinear oe	B1	
	$\overrightarrow{PQ}$ is a (scalar) multiple of $\overrightarrow{QR}$ **and** they have a point in common. oe	B1	Condone $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ **and** …
(iii)	[$\overrightarrow{OR}$ =] $4\mathbf{i}-3\mathbf{j}$ oe soi	B1	
	$\sqrt{4^2+(-3)^2}$ (=5)	M1	condone $\sqrt{4^2+3^2}$; may be implied by correct answer or correct FT answer
	$\frac{1}{5}(4\mathbf{i}-3\mathbf{j})$ oe	A1	
8 (a) (i)	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$ final answer	B2,1,0	–1 each error/omission
(ii)	$6(2x)^2\left(\frac{1}{5x}\right)^2$ soi	M1	Could be in full expansion
	$\frac{24}{25}$ or 0.96 isw	A1	Must be explicitly identified
(b)	$\frac{1}{8}\left(\frac{n(n-1)(n-2)}{6}\right)=\frac{5n}{12}$ soi leading to a cubic or quadratic ($n^2-3n-18=0$)	M1	Must attempt to expand and remove fractions
	Solves *their* quadratic [$(n-6)(n+3)$]	M1	must have come from a valid attempt
	[n =] 6 only, not from wrong working	A1	Must be n if labelled

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Question	Answer	Marks	Guidance
9 (a)	$a = 2 \quad b = 4 \quad c = -2$	B3	**B1** for each correct value
(b) (i)		B3,2,1,0	sinusoidal curve symmetrical about y-axis clear intent to have amplitude of 2 2 cycles If not fully correct max **B2**
(ii)	$-\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}$ cao	B2	**B1** for any 4 correct
10 (a) (i)	$2 \times 4!$ or $\frac{2}{5} \times 5!$ oe	M1	
	48	A1	
(ii)	${}^5P_3$ or $\frac{5!}{2!}$ or $5 \times 4 \times 3$ oe	M1	
	60	A1	
(b) (i)	$4 \times 2[!] \times 3$oe	M1	Correct first step implied by a correct product of two elements
	24	A1	
(ii)	3! or 3×3 seen	M1	
	18	A1	
11 (i)	$\frac{3x^2}{2} - \frac{2x^{5/2}}{5}(+c)$ isw	B1+B1	
(ii)	(9, 0) oe	B1	Not just $x = 9$
(iii)	Substitute (3, 9) into **both** lines Or solves simultaneously ($6x = 27 - 3x$ oe) to get $x = 3$, $y = 9$	B1	$3 \times 3 = 9$ **and** $\frac{27 - 3 \times 3}{2} = 9$

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Question	Answer	Marks	Guidance
(iv)	$[\text{Area } AOB =]\ \frac{1}{2}\times 9\times 9$ oe ($\frac{81}{2}$ or 40.5)	M1	Uses *their* (ii). May split into 2 triangles (13.5 and 27). May integrate. Must be a complete method.
	$their\left[\frac{3(9)^2}{2}-\frac{2(9)^{\frac{5}{2}}}{5}\right]-[0]$ (= 24.3)	M1	lower limit may be omitted but must be correct if seen
	$their\frac{81}{2}-their\frac{243}{10}$	M1	must be from genuine attempts at area of triangle and area under curve
	16.2	A1	
12 (i)	$\left[\frac{dy}{dx}=\right]\frac{2(x-1)-(2x-5)}{(x-1)^2}$	M1A1	Allow slips in $\frac{du}{dx}$ and $\frac{dv}{dx}$ but must be explicit. Allow $(x-1)^2=x^2-2x+1$
	– 12 isw	B1	
	ALT using $y=\frac{-12x^2+14x-5}{x-1}$		
	$-24x+14$	B1	
	$\left[\frac{dy}{dx}=\right]\frac{(x-1)(-24x+14)-(-12x^2+14x-5)}{(x-1)^2}$	M1	
		A1FT	**FT** on their derivative of 3 term quadratic
(ii)	$\left[\frac{d^2y}{dx^2}=\right]k(x-1)^{-3}$	M1	No additional terms
	$k = -6$ isw	A1	

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Question	Answer	Marks	Guidance
(iii)	*their* $\left[\frac{3}{(x-1)^2}-12\right]=0$ and find a value for x	M1	$12x^2-24x+9=0$ oe $(2x-3)(2x-1)=0$ oe
	$x = 0.5$ and $x = 1.5$	A1	
	$y = 2$ and $y = -22$	A1	if **A0 A0** then **A1** for a correct (x, y) pair
	$\frac{-6}{(-0.5)^3}>0$ therefore min when $x = 0.5$ oe	B1	or $\left[\frac{-6}{(-0.5)^3}=\right]48$ therefore min when $x = 0.5$ oe
	$\frac{-6}{(0.5)^3}<0$ therefore max when $x = 1.5$ oe	B1	or $\left[\frac{-6}{(0.5)^3}=\right]-48$ therefore max when $x = 1.5$ oe **M1A1** is possible from other methods

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