Cambridge O Level

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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/11**

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MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Guidance
1 (i)	-27	B1	
(ii)	$9-8k=0$	M1	for use of discriminant with a complete method to get to $k=$
	$k=\frac{9}{8}$	A1	
	Or $\frac{dy}{dx}=4x-3$	M1	for a complete method to get to $k=$
	when $\frac{dy}{dx}=0$, $x=\frac{3}{4}$		
	so $k=\frac{9}{8}$	A1	
	Or completing the square		
	$y=2\left(x-\frac{3}{4}\right)^2+k-\frac{9}{8}$	M1	for a complete method to get to $k=$
	$k=\frac{9}{8}$	A1	
2 (a)	$2^{4(3x-1)}=2^{3(x+2)}$		
	or $4^{2(3x-1)}=4^{\frac{3}{2}(x+2)}$	B1	**B1** for a correct statement
	or $8^{\frac{4}{3}(3x-1)}=8^{x+2}$		
	or $16^{3x-1}=16^{\frac{3}{4}(x+2)}$		
		M1	for equating indices
	leading to $x=\frac{10}{9}$ cao	A1	
(b)	$p=\frac{5}{3}$	B1	
	$q=-2$	B1	

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Question	Answer	Marks	Guidance
3	On x-axis, $2x^2 - 7 = 1$ $x = 2$	**M1** **A1**	for equating to 1
	$\frac{dy}{dx} = \frac{4x}{2x^2 - 7}$	**B1**	
	When $x = 2$, $\frac{dy}{dx} = 8$		
	Gradient of normal = $-\frac{1}{8}$		
	Equation of normal $y = -\frac{1}{8}(x - 2)$	**M1**	for attempt at perpendicular through *their* $(2, 0)$, must be using $y = 0$
	Required form $x + 8y - 2 = 0$	**A1**	must be equated to zero with integer coefficients
4 (a)	$\mathbf{A}^2 = \begin{pmatrix} 7 & -2 \\ -3 & 6 \end{pmatrix}$	**B1**	
	$\mathbf{A}^2 - 2\mathbf{B} = \begin{pmatrix} 1 & -2 \\ -5 & 2 \end{pmatrix}$	**M1** **A1**	for their $\mathbf{A}^2 - 2\mathbf{B}$
(b)	$\begin{pmatrix} 4 & 1 \\ 10 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$		
	so $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -10 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	**M1**	for pre-multiplication by *their* inverse matrix
	leading to $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$	**DM1**	**DM1** for attempt at matrix multiplication
	$x = 1$	**A1**	Allow in matrix form
	$y = -3$	**A1**	
5 (i)	$\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = e^{4x} - \left(\left(x \times 4e^{4x}\right) + e^{4x}\right)$	**B1**	for $\frac{d}{dx}\left(\frac{e^{4x}}{4}\right) = e^{4x}$
		M1	for attempt to differentiate a product
		A1	for a correct product
	$= -4xe^{4x}$	**A1**	for correct final answer
(ii)	$\int_0^{\ln 2} xe^{4x}\, dx = -\frac{1}{4}\left[\frac{e^{4x}}{4} - xe^{4x}\right]_0^{\ln 2}$	**B1FT**	**FT** for use of *their* $\frac{1}{p} \times \left(\frac{e^{4x}}{4} - xe^{4x}\right)$, must be numerical p, but $\neq 0$
	$= -\frac{1}{4}\left(\left(\frac{16}{4} - 16\ln 2\right) - \frac{1}{4}\right)$	**B1** **M1**	for $e^{4\ln 2} = 16$ for correct use of limits, must be an integral of the correct form
	$= 4\ln 2 - \frac{15}{16}$	**A1**	

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Question	Answer	Marks	Guidance
6 (i)	$2-\sqrt{5} < \mathrm{f}(x) \leqslant 2$	**B2**	**B1** for $\leqslant 2$ **B1** for $2-\sqrt{5}<$ or awrt –0.24 Must be using f, f(x) or y, $2-\sqrt{5}<$, if not then **B1** max
(ii)	$\mathrm{f}^{-1}(x) = (2-x)^2 - 5$	**M1** **A1**	for a correct method to find the inverse
	Domain $2-\sqrt{5} < x \leqslant 2$	**B1**	Must be using the correct variables for the B marks
	Range y or $-5 \leqslant \mathrm{f}^{-1}(x) < 0$	**B1**	
(iii)	$\mathrm{fg}(x) = \mathrm{f}\left(\frac{4}{x}\right)$		
	$= 2 - \sqrt{\frac{4}{x} + 5}$	**M1** **DM1**	for correct order of functions for solution of equation
	leading to $x = -4$	**A1**	
7 (i)	Finding an angle of 68.2° or 21.8°	**B1**	
	$\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin \alpha}$	**B1**	for the sine rule
	leading to $\alpha = 29.7^\circ$ (allow ± 0.1)	**B1**	
	Direction is 82.1° to the bank, upstream(allow $\pm 0.1^\circ$)	**B1**	
(ii)	$\frac{4.5}{\sin 68.2} = \frac{2.4}{\sin 29.7} = \frac{v_r}{\sin 82.1}$	**B1**	for the sine rule
	leading to $v_r = 4.8$	**B1**	for resultant velocity
	time taken = $\frac{80.78}{4.8} = 16.8$	**M1** **A1**	for attempt to find AB and hence the time taken
	Alternative method: Finding an angle of 68.2° or 21.8°	**B1**	
	$4.5^2 = 2.4^2 + v_r^2 - (2 \times 2.4 \times v_r \cos 68.2)$	**B1**	for correct use of the cosine rule
	leading to $v_r = 4.8$	**B1**	for resultant velocity
	Use of sine rule to obtain angle and direction to obtain direction is 82.1° to the bank, upstream	**B1** **B1** **B1**	for use of the sine rule for $\alpha = 29.7^\circ$ for 82.1°
	Use of time taken = $\frac{80.78}{4.8} = 16.8$	**M1** **A1**	for attempt to find AB and hence the time taken

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Question		Answer	Marks	Guidance
8	(i)	$y-6=-\frac{4}{12}(x+8)$ $(\ 3y+x=10\ )$	**M1** **A1**	for a correct method allow unsimplified
	(ii)	$y-7=3(x+1)$ $(y=3x+10)$	**DM1** **A1**	for attempt at a perpendicular line using $(-1,\ 7)$ allow unsimplified
	(iii)	point of intersection $(-2,\ 4)$ which is the midpoint of AB	**M1** **M1** **A1**	for attempt to find the point of intersection using simultaneous equations for attempt to find midpoint for all correct
		Alternative method: Midpoint $(-2,\ 4)$ Verification that this point lies on CP.	**M1** **M1** **A1**	for attempt to find midpoint for full verification for all correct
	(iv)	$CP=\sqrt{10}$ or 3.16	**B1**	
	(v)	Area $=\frac{1}{2}\times\sqrt{10}\times4\sqrt{10}$ $=20$	**M1** **A1**	for correct method **using *CP*** for 19.9 – 20.1

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Question	Answer	Marks	Guidance
9 **(i)**	$2\cos x\cot x = \cot x + 2\cos x$		
	$2\cos x\dfrac{\cos x}{\sin x} + 1 = \dfrac{\cos x}{\sin x} + 2\cos x$	**M1**	for use of $\cot x = \dfrac{\cos x}{\sin x}$ for both terms
	$2\cos^2 x + \sin x = \cos x + 2\cos x\sin x$	**DM1**	for multiplication throughout by $\sin x$
	$2\cos^2 x - 2\cos x\sin x = \cos x - \sin x$		
	$2\cos x(\cos x - \sin x) = \cos x - \sin x$	**DM1**	for attempt to factorise
	$(2\cos x - 1)(\cos x - \sin x) = 0$	**A1**	for completely correct solution www
	Alternative method:		
	$a\cos^2 x - a\cos x\sin x - b\cos x + b\sin x = 0$	**M1**	for expansion of RHS
	$a\cos x\cot x - a\cos x - b\cot x + b = 0$	**DM1**	for division by $\sin x$
		DM1	for comparing like terms to obtain both a and b
	$a = 2,\quad b = 1$	**A1**	for both correct www
(ii)	$(2\cos x - 1)(\cos x - \sin x) = 0$		
	$\cos x = \dfrac{1}{2}$, $\tan x = 1$	**M1**	for either
	$x = \dfrac{\pi}{3}$, $x = \dfrac{\pi}{4}$	**A1,A1**	**A1** for each, penalise extra solutions within the range by withholding the last A mark
	Alternative method:		
	$(2\cos x - 1)(\cot x - 1) = 0$		
	Leading to $\cos x = \dfrac{1}{2}$, $\tan x = 1$	**M1**	for attempt to factorise the original equation and attempt to solve
	$x = \dfrac{\pi}{3}$, $x = \dfrac{\pi}{4}$	**A1,A1**	**A1** for each, penalise extra solutions within the range by withholding the last A mark
10 **(i)**	$f(-2) = -32 - 2k + p = 0$	**M1**	for attempt at $f(-2)$
	$f'\left(\dfrac{1}{2}\right) = \dfrac{12}{4} + k = 0$	**M1**	for attempt at $f'\left(\dfrac{1}{2}\right)$
	leading to $k = -3$ and $p = 26$	**A1,A1**	**A1** for each
(ii)		**B1FT**	**FT** for *their* $\dfrac{p}{2}$
	$(x+2)(4x^2 - 8x + 13)$	**B1**	all correct
(iii)	Showing that $4x^2 - 8x + 13 = 0$ has no real roots	**M1,**	M1 for a valid attempt at solution of equation leading to no solution or consideration of the discriminant
	so $x = -2$ only www	**A1**	

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Question	Answer	Marks	Guidance
11 (i)	$AB = 2r\sin\theta$ or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$ or $\dfrac{r\sin 2\theta}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$ or $\dfrac{r\sin 2\theta}{\cos\theta}$	B1	
(ii)	$2r\sin\theta + 2r\theta = 20$	M1	for use of (i) + arc length = 20, oe
	$r = \dfrac{10}{\theta + \sin\theta}$	A1	must be convinced
(iii)	$\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -\dfrac{10(1+\cos\theta)}{(\theta+\sin\theta)^2}$	M1 A2,1,0	for a correct attempt to differentiate –1 each error
	When $\theta = \dfrac{\pi}{6}$, $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -17.8$	A1	allow awrt -17.8
(iv)	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = 15$	B1	may be implied
	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{\mathrm{d}r}{\mathrm{d}t} \div \dfrac{\mathrm{d}r}{\mathrm{d}\theta}$	M1	for use of $\dfrac{15}{\textit{their}\ \text{(iii)}}$
	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -0.842$	A1	allow -0.84 or -0.843

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