CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/21 Paper 2, maximum raw mark 80

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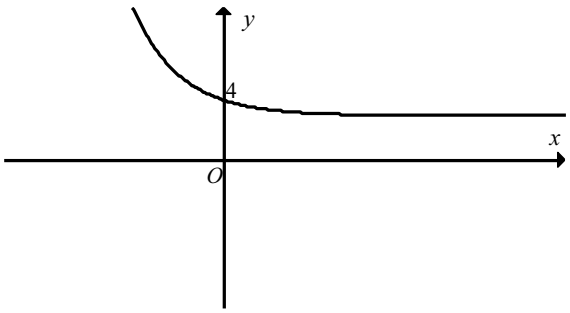
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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

1	**(a)**	$\frac{\log_3 x}{\log_3 27}$ $\frac{\log_3 x}{3}$ isw	M1 A1	Can use other interim bases if all correct but M1 when in base 3 only NOT $\log_3 x \div 3$
	(b)	$\log_a 15 - \log_a 3 = \log_a 5$ soi $\log_a 5^3$ or $\log_a a$ $\log_a y = \log_a 125a \Rightarrow y = 125a$	M1 M1 A1	
2	**(a)**	$[f(x) =] 2x - 4$ and $[f(x) =] -2x + 4$	B1,B1	Condone $y =$….
	(b)		B1 B1 B1	correct shape; y intercept marked or seen nearby; intent to tend to $y = 3$ (i.e. not tending to or cutting x-axis)
3	**(a)**	$\mathbf{A} = \frac{1}{4}\left[\begin{pmatrix} 51 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix} - \begin{pmatrix} 20 & 0 & -5 \\ 15 & -10 & 25 \end{pmatrix}\right]$ $\mathbf{A} = \begin{pmatrix} 8 & -2 & 6 \\ 4 & 3 & 10 \end{pmatrix}$	M1 A1	 Integer values
	(b) (i)	The (total) value of the stock in **each** of the 3 shops	B1	Must have "each" oe
	(ii)	The **total** value of the stock in all 3 shops	B1	Must have "total" oe

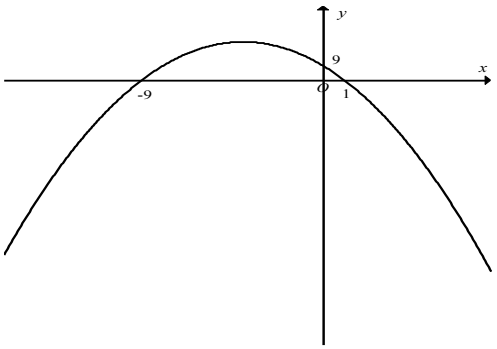
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4	**(i)**	$\frac{PT}{8}=\tan\left(\frac{3\pi}{8}\right)$ oe	M1	$\frac{PT}{\sin\frac{3\pi}{8}}=\frac{8}{\sin\frac{\pi}{8}}$
		$PT=19.3$	A1	awrt 19.3
	(ii)	$\frac{1}{2}\times8^2\times\frac{3\pi}{4}$ oe (75.4)	M1	or $\frac{1}{2}\times8^2\times\frac{3\pi}{8}$
		$8\tan\left(\frac{3\pi}{8}\right)\times8$ – *their* sector oe (=154.5-'75.4')	M1	$8\times$ *their PT* – *their* sector
		79.1	A1	awrt 79.1
	(iii)	$8\left(\frac{3\pi}{4}\right)$ oe (18.8)	M1	
		$\left[6\pi+16\tan\left(\frac{3\pi}{8}\right)\right]=57.5$	A1	Accept 57.4 to 57.5
5	**(a)**	Permutation because the order matters oe	B1	
	(b) (i)	${}^6C_4+{}^5C_4+{}^7C_4$	M1	3 correct terms added
		55	A1	
	(ii)	${}^2C_1\times{}^6C_1\times{}^5C_1\times{}^7C_1$	M1	4 correct terms multiplied
		420	A1	
	(iii)	${}^6C_3\times{}^2C_1$ or ${}^2C_2\times{}^5C_1\times{}^6C_1$	M1	for either correct product
		summation	M1	adding two correct products
		70	A1	
				If 0 scored, then SC1for 1,1,1,0 and 0,0,2,1 seen
6	**(i)**	$2t^2-14t+12=0$	M1	Can use formula, etc.
		$(t-1)(t-6)$ oe		
		$(t=)$ 1	A1	If t = 1 with no working, then M1A1
	(ii)	$\int(2t^2-14t+12)\,dt$	M1	
		$(s=)\ \frac{2t^3}{3}-\frac{14t^2}{2}+12t$	A2,1,0	−1 for each error or for +c left in or limits introduced
	(iii)	$(a=)\frac{dv}{dt}\ (4t-14)$	M1	
		[4(3) – 14 =] –2 cao	A1	

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7	**(a)**	$\overrightarrow{AB} = 15\mathbf{b} - 5\mathbf{a} = 5(3\mathbf{b} - \mathbf{a})$ or $\overrightarrow{BC} = 24\mathbf{b} - 3\mathbf{a} - 15\mathbf{b} = 3(3\mathbf{b} - \mathbf{a})$ or $\overrightarrow{AC} = 24\mathbf{b} - 3\mathbf{a} - 5\mathbf{a} = 8(3\mathbf{b} - \mathbf{a})$	B1 B1	Any correct simplified vector Any second simplified vector
		Comment: e.g. the vectors are scalar multiples of each other AND they have a common point (A, B or C as appropriate)	B1dep	Dep on both B marks being awarded.
	(b) (i)	$2\mathbf{i} + 11\mathbf{j}$ soi $\Rightarrow \sqrt{2^2 + 11^2}$	B1	
		$\sqrt{125}$ or $5\sqrt{5}$ or 11.2 (3 s.f.) or better)	B1fT	ft *their* $2\mathbf{i} + 11\mathbf{j}$ (not $\overrightarrow{OP}$ or $\overrightarrow{OQ}$)
	(ii)	$\frac{1}{5\sqrt{5}}(2\mathbf{i} + 11\mathbf{j})$ isw	B1fT	ft *their* answers from **(i)**
	(iii)	$\frac{\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7\mathbf{j}}{2}$ or $\mathbf{i} - 4\mathbf{j} + \frac{2\mathbf{i} + 11\mathbf{j}}{2}$ or $3\mathbf{i} + 7\mathbf{j} - \frac{2\mathbf{i} + 11\mathbf{j}}{2}$	M1	
		$2\mathbf{i} + 1.5\mathbf{j}$	A1	
8	**(a) (i)**	ke^{4x+3} $(+c)$ oe	M1	any constant, non-zero k
		$k = \frac{1}{4}$ oe	A1	
	(ii)	$\frac{1}{4}\left(e^{4(3)+3} - e^{4(2.5)+3}\right)$ or better	DM1	ft *their* integral attempt
		706 650.99… = 707 000 to 3 sf or better	A1	Accept $\frac{1}{4}\left(e^{15} - e^{13}\right)$
	(b) (i)	$k \sin\left(\frac{x}{3}\right)$ $(+ c)$	M1	any constant, non-zero k
		$k = 3$	A1	
	(ii)	$3\sin\left(\frac{\pi}{6} \times \frac{1}{3}\right) - 3\sin(0)$	DM1	Dep on *their* integral attempt in sin; condone omission of lower limit
		0.520 944… = 0.521 to 3 sf or better	A1	Accept $3\sin\left(\frac{\pi}{18}\right)$
	(c)	$\int\left(x^{-2} + 2 + x^2\right)dx = \frac{x^{-1}}{-1} + 2x + \frac{x^3}{3}$	B1 M1 A1	Expands – accept unsimplified integration of *their* 3 term expansion Fully correct
		$+ c$	B1	$+c$

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9	**(a)**	$(4x-1)(x+5)$ [$\leqslant 0$]	M1	Solves quadratic
		critical values $\frac{1}{4}$ and -5 soi	A1	
		$-5 \leqslant x \leqslant \frac{1}{4}$	A1	Accept: $\left[-5, \frac{1}{4}\right]$; $-5 \leq x$ AND $x \leq 0.25$
	(b) (i)	$(x+4)^2 - 25$ or $a = 4$ and $b = -25$	B1, B1	
	(ii)	(Greatest value =) 25	B1ft	Must be clear
		$x = -4$	B1ft	
	(iii)		B1	Correct shape with maximum in second quadrant and crossing positive and negative axes correctly
			B1	All 3 intercepts correctly shown on graph
10	**(i)**	$\ln y = \ln(Ab^x) \Rightarrow \ln y = \ln A + \ln b^x$	M1	
		$\Rightarrow \ln y = \ln A + x \ln b$	A1	
	(ii)	$\ln A = 11.4 \Rightarrow A = e^{their\ 11.4}$	M1	condone misread of scale for M1 (11.2 only)
		$A = 90\,000$ cao	A1	
		$\ln b = -1$	M1	Allow awrt -1
		$b = 0.4$ cao	A1	
	(iii)	$x = 2.5 \Rightarrow \ln y = 9$	M1	
		$y = e^9$ or 8000 to 1 sf	A1	Allow awrt 8100
11	**(i)**	$7 - x, x, 6 - x$ oe	B1	
		their attempt at $7 - x + x + 6 - x + 16 = 25$ oe	M1	
		$x = 4$	A1	Condone $x = 4$ for all 3 marks
	(ii)	$23 - y, y, 9 - y$ oe	B1	or $n(A \cup C) = 48 - 16 = 32$
		$48 = 30 + 25 + 15 - 7 - 6 - (their\ 4 + y) + their\ 4$ oe soi	M1	or $32 = 30 + 15 - (their\ 4 + y)$ or $48 = (23 - y) + 3 + 16 + y + 4 + 2 + (9 - y)$
		$y = 9$	A1	Condone $y = 9$ for all 3 marks
	(iii)	$n(C) = 15$ and $y + n(B \cap C) = 9 + 6 = 15$ [and so $A' \cap B' \cap C = \varnothing$].	B1	or equivalent deduction

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