**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**Cambridge Ordinary Level**

# MARK SCHEME for the May/June 2015 series

# 4037 ADDITIONAL MATHEMATICS

**4037/11** Paper 1, maximum raw mark 80

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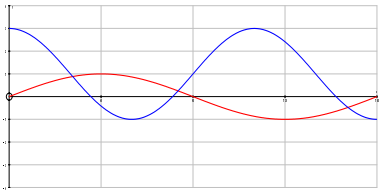
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**Abbreviations**

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

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|---|---|---|---|---|
| **1** | **(i)** | 180° or π radians or 3.14 radians ( or better) | **B1** | |
| | **(ii)** | 2 | **B1** | |
| | **(iii)** **(a)** |  | **B1** | $y = \sin 2x$ all correct |
| | **(b)** | | **B1** | for either<br>↑↓↑starting at their highest value and ending at their lowest value<br>Or<br>a curve with highest value at $y = 3$<br>and lowest value at $y = -1$ |
| | | | **B1** | completely correct graph |
| | **(iv)** | 3 | **B1** | |
| **2** | **(i)** | $\tan\theta = \dfrac{(8+5\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$<br>$= \dfrac{32-24\sqrt{2}+20\sqrt{2}-30}{16-18}$<br>$= 1+2\sqrt{2}$ cao | **M1**<br><br><br>**A1** | attempt to obtain $\tan\theta$ and rationalise. Must be convinced that no calculators are being used |

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| | **(ii)** | $\sec^2\theta = 1+\tan^2\theta$<br>$= 1+\left(-1+2\sqrt{2}\right)^2$<br>$= 1+1-4\sqrt{2}+8$<br>$= 10-4\sqrt{2}$<br>**Alternative solution:**<br>$AC^2 = \left(4+3\sqrt{2}\right)^2+\left(8+5\sqrt{2}\right)^2$<br>$= 148+104\sqrt{2}$<br>$\sec^2\theta = \dfrac{148+104\sqrt{2}}{\left(4+3\sqrt{2}\right)^2}$<br>$= \dfrac{148+104\sqrt{2}}{\left(4+3\sqrt{2}\right)^2}\times\dfrac{34-24\sqrt{2}}{34-24\sqrt{2}}$<br>$= 10-4\sqrt{2}$ | **M1**<br>**DM1**<br>**A1**<br><br><br><br><br>**M1**<br>**DM1**<br>**A1** | attempt to use $\sec^2\theta = 1+\tan^2\theta$, with *their* answer to **(i)**<br>attempt to simplify, must be convinced no calculators are being used.<br>Need to expand $\left(-1+2\sqrt{2}\right)^2$ as 3 terms |
| **3** | **(i)** | $64+192x^2+240x^4+160x^6$ | **B3,2,1,0** | –1 each error |
| | **(ii)** | $\left(64+192x^2+240x^4\right)\left(1-\dfrac{6}{x^2}+\dfrac{9}{x^4}\right)$<br>Terms needed $64-(192\times 6)+(240\times 9)$<br>$= 1072$ | **B1**<br>**M1**<br>**A1** | expansion of $\left(1-\dfrac{3}{x^2}\right)^2$<br>attempt to obtain 2 or 3 terms using *their* **(i)** |

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| **4** | **(a)** | $\mathbf{X}^2 = \begin{pmatrix} 4-4k & -8 \\ 2k & -4k \end{pmatrix}$ | **B2,1,0** | –1 each incorrect element |
| | **(b)** | Use of $\mathbf{AA}^{-1} = \mathbf{I}$<br>$\begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix}\begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ | **M1** | use of $\mathbf{AA}^{-1} = \mathbf{I}$ and an attempt to obtain at least one equation. |
| | | Any 2 equations will give $a = 2$, $b = 4$ | **A1,A1** | |
| | | **Alternative method 1:**<br>$\frac{1}{5a-b}\begin{pmatrix} 5 & -1 \\ b & a \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ | **M1** | correct attempt to obtain $\mathbf{A}^{-1}$ and comparison of at least one term. |
| | | Compare any 2 terms to give $a = 2$, $b = 4$ | **A1,A1** | |
| | | **Alternative method 2:**<br>Inverse of $\frac{1}{6}\begin{pmatrix} 5 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$ | **M1**<br>**A1,A1** | reasoning and attempt at inverse |
| **5** | | $3x-1 = x(3x-1)+x^2-4$ or<br>$y = \left(\frac{y+1}{3}\right)y + \left(\frac{y+1}{3}\right)^2 - 4$ | | |
| | | $4x^2-4x-3=0$ or $4y^2-4y-35=0$<br>$(2x-3)(2x+1)=0$ or $(2y-7)(2y+5)=0$ | **M1**<br>**DM1** | equate and attempt to obtain an equation in 1 variable<br>forming a 3 term quadratic equation and attempt to solve |
| | | leading to $x = \frac{3}{2}, x = -\frac{1}{2}$ and | **A1** | $x$ values |
| | | $y = \frac{7}{2}, y = -\frac{5}{2}$ | **A1** | $y$ values |
| | | Midpoint $\left(\frac{1}{2}, \frac{1}{2}\right)$ | **B1** | for midpoint, allow anywhere |
| | | Perpendicular gradient = $-\frac{1}{3}$ | **M1** | correct attempt to obtain the gradient of the perpendicular, using $AB$ |
| | | Perp bisector: $y - \frac{1}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ | **M1** | straight line equation through the midpoint; must be convinced it is a perpendicular gradient. |
| | | $(3y + x - 2 = 0)$ | **A1** | allow unsimplified |

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| **6** | **(i)** | $f\left(\frac{1}{2}\right)=\frac{a}{8}-\frac{15}{4}+\frac{b}{2}-2=0$<br>leading to $a+4b=46$<br>$f(1)=a-15+b-2=5$<br>leading to $a+b=22$ | **M1**<br><br><br>**A1** | correct use of either $f\left(\frac{1}{2}\right)$ or f(1)<br>paired correctly<br><br>both equations correct (allow unsimplified) |
| | | giving $b=8$ (AG), $a=14$ | **M1,A1** | M1 for solution of equations<br>A1 for both $a$ and $b$. **AG** for $b$. |
| | **(ii)** | $(2x-1)(7x^2-4x+2)$ | **M1,A1** | M1 for valid attempt to obtain g($x$), by either observation or by algebraic long division. |
| | **(iii)** | $7x^2-4x+2=0$ has no real solutions as<br>$b^2<4ac$<br>$16<56$ | **M1**<br><br>**A1** | use of $b^2-4ac$<br><br>correct conclusion; must be from a correct g($x$) or 2g($x$) www |
| **7** | **(i)** | $\frac{dy}{dx}=\frac{(x-1)\frac{8x}{(4x^2+2)}-\ln(4x^2+3)}{(x-1)^2}$ | **M1**<br>**B1**<br>**A1** | differentiation of a quotient (or product)<br>correct differentiation of $\ln(4x^2+3)$<br>all else correct |
| | | When $x=0$, $y=-\ln 3$ oe | **B1** | for $y$ value |
| | | $\frac{dy}{dx}=-\ln 3$ so gradient of normal is $\frac{1}{\ln 3}$<br>(allow numerical equivalent) | **M1** | valid attempt to obtain gradient of the normal |
| | | normal equation $y+\ln 3=\frac{1}{\ln 3}x$<br>or $y=0.910x-1.10$, or $y=\frac{10}{11}x-\frac{11}{10}$ cao<br>(Allow $y=0.91x-1.1$) | **M1**<br><br>**A1** | attempt at normal equation must be using a perpendicular |
| | **(ii)** | when $x=0$, $y=-\ln 3$<br>when $y=0$, $x=(\ln 3)^2$<br>Area = ±0.66 or ±0.67 or awrt these<br>or $\frac{1}{2}(\ln 3)^3$ | **M1**<br><br><br>**A1** | valid attempt at area |

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| **8** | **(i)** | Range for f: $y \geq 3$<br>Range for g: $y \geq 9$ | **B1**<br>**B1** | |
| | **(ii)** | $x = -2 + \sqrt{y-5}$ | **M1** | attempt to obtain the inverse function |
| | | $g^{-1}(x) = -2 + \sqrt{x-5}$ | **A1** | Must be correct form |
| | | Domain of $g^{-1}$: $x \geq 9$ | **B1** | for domain |
| | | **Alternative method:** | | |
| | | $y^2 + 4y + 9 - x = 0$ | **M1** | attempt to use quadratic formula and find inverse |
| | | $y = \dfrac{-4 + \sqrt{16 - 4(9-x)}}{2}$ | **A1** | must have + not ± |
| | **(iii)** | Need $g(3e^{2x})$ | **M1** | correct order |
| | | $(3e^{2x} + 2)^2 + 5 = 41$ | **DM1** | correct attempt to solve the equation |
| | | or $9e^{4x} + 12e^{2x} - 32 = 0$<br>$(3e^{2x} - 4)(3e^{2x} + 8) = 0$ | | |
| | | leading to $3e^{2x} + 2 = \pm 6$ so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$ | **M1** | dealing with the exponential correctly in order to reach a solution for $x$ |
| | | or $e^{2x} = \dfrac{4}{3}$ so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$ | **A1** | Allow equivalent logarithmic forms |
| | | **Alternative method:** | | |
| | | Using $f(x) = g^{-1}(41)$, $g^{-1}(41) = 4$ | **M1** | correct use of $g^{-1}$ |
| | | leading to $3e^{2x} = 4$, so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$ | **DM1** | dealing with $g^{-1}(41)$ to obtain an equation in terms of $e^{2x}$ |
| | | | **M1**<br>**A1** | dealing with the exponential correctly in order to reach a solution for $x$<br>Allow equivalent logarithmic forms |
| | **(iv)** | $g'(x) = 6e^{2x}$ | **B1** | B1 for each |
| | | $g'(\ln 4) = 96$ | **B1** | |

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| **9 (i)** | $\frac{dy}{dx} = 3x^2 - 10x + 3$ | | |
| | When $x = 0$, for curve $\frac{dy}{dx} = 3$, | | |
| | gradient of line also 3 so line is a tangent. | **A1** | comparing both gradients |
| | **Alternate method:** | | |
| | $3x + 10 = x^3 - 5x^2 + 3x + 10$ | **M1** | attempt to deal with simultaneous equations |
| | leading to $x^2 = 0$, so tangent at $x = 0$ | **A1** | obtaining $x = 0$ |
| **(ii)** | When $\frac{dy}{dx} = 0$, $(3x-1)(x-3) = 0$ | **M1** | equating gradient to zero and valid attempt to solve |
| | $x = \frac{1}{3}$, $x = 3$ | **A1,A1** | A1 for each |
| **(iii)** | Area $= \frac{1}{2}(10+19)3 - \int_0^3 x^3 - 5x^2 + 3x + 10\,dx$ | **B1** | area of the trapezium |
| | $= \frac{87}{2} - \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 10x\right]_0^3$ | **M1** | attempt to obtain the area enclosed by the curve and the coordinate axes, by integration |
| | $= \frac{87}{2} - \left(\frac{81}{4} - 45 + \frac{27}{2} + 30\right)$ | **A1** | integration all correct |
| | | **DM1** | correct application of limits (must be using *their* 3 from **(ii)** and 0) |
| | $= 24.7$ or $24.8$ | **A1** | |
| | **Alternative method**: | | |
| | Area $= \int_0^3 (3x+10) - \left(x^3 - 5x^2 + 3x + 10\right)dx$ | **B1** | correct use of '$Y$–$y$' |
| | $= \int_0^3 -x^3 + 5x^2\,dx$ | **M1** | attempt to integrate |
| | | **A1** | integration all correct |
| | $= \left[-\frac{x^4}{4} + \frac{5x^3}{3}\right]_0^3 = \frac{99}{4}$ | **DM1** | correct application of limits |
| | | **A1** | |
| **10 (a)** | $\sin^2 x = \frac{1}{4}$ | | |
| | $\sin x = (\pm)\frac{1}{2}$ | **M1** | using $\text{cosec}\, x = \frac{1}{\sin x}$ and obtaining $\sin x = \ldots$ |
| | $x = 30°, 150°, 210°, 330°$ | **A1,A1** | A1 for one correct pair, A1 for another correct pair with no extra solutions |

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| **(b)** | $\left(\sec^2 3y - 1\right) - 2\sec 3y - 2 = 0$ | **M1** | use of the correct identity |
| | $\sec^2 3y - 2\sec 3y - 3 = 0$<br>$(\sec 3y + 1)(\sec 3y - 3) = 0$ | **M1** | attempt to obtain a 3 term quadratic equation in sec 3*y* and attempt to solve |
| | leading to $\cos 3y = -1$, $\cos 3y = \frac{1}{3}$ | **M1** | dealing with sec and 3*y* correctly |
| | $3y = 180°, 540°$ $\quad 3y = 70.5°, 289.5°, 430.5°$<br>$y = 60°, 180°, 23.5°, 96.5°, 143.5°$ | **A1,A1**<br>**A1** | A1 for a correct pair, A1 for a second correct pair, A1 for correct 5th solution and no other within the range |
| | **Alternative 1:** | | |
| | $\sec^2 3y - 2\sec 3y - 3 = 0$ | **M1** | use of the correct identity |
| | leading to $3\cos^2 3y + 2\cos 3y - 1$ | **M1** | attempt to obtain a quadratic equation in cos 3*y* and attempt to solve |
| | $(3\cos y - 1)(\cos y + 1) = 0$ | **M1** | dealing with 3*y* correctly<br>**A** marks as above |
| | **Alternative 2:** | | |
| | $\frac{\sin^2 y}{\cos^2 y} - \frac{2}{\cos y} - 2 = 0$<br>$\left(1 - \cos^2 x\right) - 2\cos x - 2\cos^2 x = 0$ | **M1** | use of the correct identity,<br>$\tan y = \frac{\sin y}{\cos y}$ and $\sec y = \frac{1}{\cos y}$, then as before |
| **(c)** | $z - \frac{\pi}{3} = \frac{\pi}{3}, \frac{4\pi}{3}$ | **M1** | correct order of operations |
| | $z = \frac{2\pi}{3}, \frac{5\pi}{3}$ or 2.09 or 2.1, 5.24 | **A1,A1** | A1 for a correct solution<br>A1 for a second correct solution and no other within the range |

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