CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2013 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

© Cambridge International Examinations 2013

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

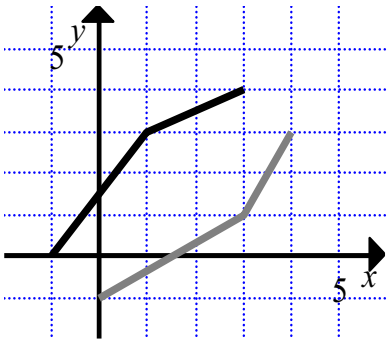
OW –1, 2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

© Cambridge International Examinations 2013

1		$m = \dfrac{18-3}{4-1}$ or 5 soi	**M1**	or $18 = 4m + c$ and $3 = m + c$ subtracting/substituting to solve for m or c, condone one error
		$Y - 3 = \textit{their}\ 5(X-1)$ or $Y - 18 = \textit{their}\ 5(X-4)$		
		or $3 = \textit{their}\ 5 + c$ or $18 = \textit{their}\ 5 \times 4 + c$	**M1**	or using *their m* or *their c* to find *their c* or *their m*, without further error
		$\sqrt{y} = (\textit{their}\ m)\ x^2 + (\textit{their}\ c)$ or $\sqrt{y} = (\textit{their}\ m)\ (x^2 - 1) + 3$ or $\sqrt{y} = (\textit{their}\ m)\ (x^2 - 4) + 18$	**M1**	their m and c must be validly obtained
		$y = (5x^2 - 2)^2$ or $y = (5(x^2 - 1) + 3)^2$ or $y = (5(x^2 - 4) + 18)^2$ cao, isw	**A1**	
2	**(a)**	$(p + 1) \ln 3 = \ln 0.7$	**M1**	or $p + 1 = \log_3 0.7$ or $p \ln 3 = \ln\left(\dfrac{0.7}{3}\right)$
		$p = \dfrac{\ln 0.7}{\ln 3} - 1$ or $p = \dfrac{\lg 0.7}{\lg 3} - 1$	**M1**	or $p = \log_3 0.7 - 1$ or $p \ln 3 = \ln\left(\dfrac{0.7}{3}\right) \div \ln 3$
		–1.32 cao	**A1**	allow **M2** for $p = \log_3\left(\dfrac{0.7}{3}\right)$ correct answer only scores **B3**
	(b)	$2^{\frac{5}{2}} \times x^6 \times y^{-\frac{1}{2}}$ or $a = \dfrac{5}{2}$, $b = 6$, $c = -\dfrac{1}{2}$	**B3**	**B1** for each component
3	**(a) (i)**	A and E	**B2**	1 mark for each **B1** for 1 extra, **B0** if 2 or more extras
	(ii)	C and D	**B2**	1 mark for each **B1** if 1 extra, **B0** if 2 or more extras
	(b)		**B2**	(–1, 0), (1, 3), (3, 4) or **B1** for two points correct and joined or for three points correct but clearly not joined

© Cambridge International Examinations 2013

4	**(i)**	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ or $\overrightarrow{OB} - \overrightarrow{OA} = 3(\overrightarrow{OC} - \overrightarrow{OA})$ soi	**B1**	or $3\overrightarrow{AC} = 3(c_1 - 4)\mathbf{i} + 3(c_2 + 21)\mathbf{j}$ o.e. soi
		$\pm(18\mathbf{i} - 9\mathbf{j})$ o.e. or $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA} + \frac{1}{3}\overrightarrow{OB}$	**B1**	
		$4\mathbf{i} - 21\mathbf{j} + \frac{1}{3}(their(18\mathbf{i} - 9\mathbf{j}))$ o.e. or $\frac{2}{3}(4\mathbf{i} - 21\mathbf{j}) + \frac{1}{3}(22\mathbf{i} - 30\mathbf{j})$	**M1**	or $3(c_1 - 4) = their\ 18$ and $3(c_2 + 21) = their\ (-9)$
		$10\mathbf{i} - 24\mathbf{j}$ cao	**A1**	
	(ii)	$\left\lvert\overrightarrow{OC}\right\rvert = \sqrt{their10^2 + their(-24)^2}$ soi	**M1**	condone $\left\lvert\overrightarrow{OC}\right\rvert = \sqrt{their10^2 + their(24)^2}$
		$\frac{1}{13}(5\mathbf{i} - 12\mathbf{j})$ or $\frac{1}{26}(10\mathbf{i} - 24\mathbf{j})$ isw	**A1 FT**	FT their $x\mathbf{i} + y\mathbf{j}$ o.e.
5		$AX = \sqrt{45}$	**B1**	may be implied by $3\sqrt{5}$
		$AX = 3\sqrt{5}$	**B1**	may be seen later
		$\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times their\ \sqrt{45}$ soi	**M1**	may be implied by e.g. summation of rectangle and two triangles
		$15(\sqrt{5} + 2) = \frac{1}{2}(4 + \sqrt{5} + 2 + x) \times their\ \sqrt{45}$ or better	**M1**	
		Correctly divide *their* equation by *their* $\sqrt{5}$ or *their* $\sqrt{45}$ and rationalise denominator	**M1**	or correctly multiply both sides of *their* equation by *their* $\sqrt{5}$ or *their* $\sqrt{45}$ and obtain a rational coefficient of x soi
		completion to $4 + 3\sqrt{5}$ www	**A1**	answer only does not score

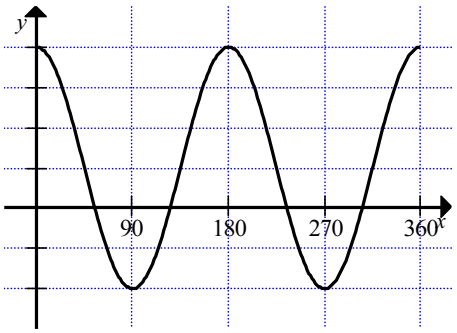
© Cambridge International Examinations 2013

6	**(i)**	arc $AB = r\left(\frac{\pi}{3}\right)$	**B1**	
		chord $AB = r$ with justification and summation and completion to given answer	**B1**	$r\left(\frac{3+\pi}{3}\right)$
	(ii)	$r = 12.7$	**B1**	must be seen; accept awrt 12.7
		$\frac{1}{2} \times their\ r^2 \times \left(\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right)\right)$	**M3**	may be implied for example 84.45…– 69.84… or **M1** for $\frac{1}{2} \times their\ r^2 \times \frac{\pi}{3}$ or 84.45... **and** **M1** for $\frac{1}{2} \times their\ r^2 \times \sin\frac{\pi}{3}$ o.e. or 69.84… **and** **M1** for Area Sector – Area triangle attempted
		awrt 14.6	**A1**	
7	**(i)**	$k(3-5x)^{11}$	**M1**	
		$5 \times 12(3-5x)^{11}$ or better, isw	**A1**	
	(ii)	$x^2(their \cos x) + (their\ 2x)\sin x$	**M1**	clearly applies correct form of product rule
		$x^2 \cos x + 2x \sin x$ isw	**A1**	
	(iii)	Quotient rule attempt:		Product rule attempt:
		$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$	**B1**	$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$
		$\frac{\mathrm{d}}{\mathrm{d}x}\left(1+\mathrm{e}^{2x}\right) = 2\mathrm{e}^{2x}$	**B1**	$\frac{\mathrm{d}}{\mathrm{d}x}(1+\mathrm{e}^{2x})^{-1} = -2\mathrm{e}^{2x}(1+\mathrm{e}^{2x})^{-2}$
		clearly applies correct form of quotient rule $\frac{(1+\mathrm{e}^{2x})(their\ \sec^2 x) - (their\ 2\mathrm{e}^{2x})\tan x}{(1+\mathrm{e}^{2x})^2}$	**M1**	$\tan x\ (their - 2\mathrm{e}^{2x}(1+\mathrm{e}^{2x})^{-2}) + (1+\mathrm{e}^{2x})^{-1}(their\ \sec^2 x)$
		$\frac{(1+\mathrm{e}^{2x})\sec^2 x - 2\mathrm{e}^{2x}\tan x}{(1+\mathrm{e}^{2x})^2}$ isw	**A1**	$\tan x\ (-2\mathrm{e}^{2x}(1+\mathrm{e}^{2x})^{-2}) + (1+\mathrm{e}^{2x})^{-1}(\sec^2 x)$

© Cambridge International Examinations 2013

8	**(i)**	$y-2=\left(\frac{6-2}{2+6}\right)(x+6)$ o.e. soi	**M1**	or $y-6=\left(\frac{6-2}{2+6}\right)(x-2)$
		$y=\frac{1}{2}x+5$ isw	**A1**	
	(ii)	Use of $m_1m_2=-1$	**M1**	
		$y-6=$ (*their* -2)$(x-2)$ or better, isw	**A1 FT**	or $y=$ (*their* -2)$x+c$, $c=$ *their* 10, isw
	(iii)	$(x+6)^2+(y-2)^2=10^2$ o.e.	**B1**	or $(x-2)^2+(y-6)^2=(\sqrt{20})^2$ o.e. or $(\sqrt{80})^2+\left((x-2)^2+(y-6)^2\right)=10^2$
		Substitute $y=$ *their* $(-2x+10)$	**M1***	or identifying one point by inspection from the length equation and testing it in the equation of *BC* or vice versa
		Solve their quadratic	**M1 dep***	or identifying the second point by inspection from the length equation and testing it in the equation of *BC* or vice versa
		(0, 10) and (4, 2) o.e. only	**A1**	answer only does not score
9	**(a)**	$14=k+c$ and $6=\frac{k}{9}+c$ o.e.	**M1**	for two equations in k and c; may be unsimplified; condone one slip in one equation
		$c=5$	**A1**	
		$k=9$	**A1**	
	(b) (i)	79.2 or 79.158574 … rot to 4 or more sf	**B1**	
	(ii)	$e^{2x}+5e^x-24(=0)$ or $(e^x)^2+5e^x-24(=0)$ o.e.	**M1**	condone one error, but must be three terms
		factorise *their* 3 term quadratic	**M1**	or correct/correct ft use of formula or completing the square
		$e^x=3$	**A1**	ignore $e^x=-8$
		$x=\ln 3$ or 1.1(0) or 1.0986122 … rot to 3 or more sf **as only answer from fully correct working**	**A1**	do not allow final mark if value given from $e^x=-8$ if **M0M0** then **SC2** if $e^x=3$ is seen www and leads to $x=\ln 3$ or 1.1(0) or 1.0986122… rot to 3 or more sf

© Cambridge International Examinations 2013

10	**(a)** **(i)**		**B1** **B1** **B1** **B1**	shape; cosine curve – ends must be approaching a turning point be centred on $y = 1$ clear intent to have min at –2 and max at 4 2 cycles
	(ii)	3	**B1**	
	(iii)	180	**B1**	
	(b)	$\text{cosec}\, x = \dfrac{1}{\sin x}$ soi	**B1**	or $1 + \tan^2 x = \dfrac{1}{\cos^2 x}$
		$\sin x = \sqrt{1 - \cos^2 x}$ or $\sqrt{1 - p^2}$	**B1**	or $\text{cosec}^2 x = 1 + \dfrac{1}{1 - p^2 \diagup p^2}$ soi
		$\dfrac{-1}{\sqrt{1 - p^2}}$ o.e.	**B1**	or $-\sqrt{1 + \dfrac{p^2}{1 - p^2}}$ or better

© Cambridge International Examinations 2013

11	**(i)**	$\frac{dy}{dx} = 3 - 3(x-4)^{-4}$ o.e. isw	**B1 + B1**	
		$\frac{d^2y}{dx^2} = (their\ 12)(x-4)^{their\ (-5)}$ o.e.	**M1**	
		$\frac{d^2y}{dx^2} = 12(x-4)^{-5}$ o.e. isw	**A1**	if **M0** then **SC1** for $12(x-4)^{-5}$ + one other term
	(ii)	Verifies $\frac{dy}{dx} = 0$ when $x = 3$ and $x = 5$	**M1**	if **M0** then **SC1** for verifying or correctly solving to find one x coordinate and showing that it gives rise to the corresponding y coordinate
		or solves $3 - \frac{3}{(x-4)^4} = 0$ to obtain 3 and 5		
		Shows that $x = 3 \Rightarrow y = 8$ and $x = 5 \Rightarrow y = 16$	**A1**	
	(iii)	$x = 5\ \frac{d^2y}{dx^2}\ (=12) > 0 \Rightarrow$ min or	**M1**	or, using first derivative e.g.
		$x = 3 \frac{d^2y}{dx^2}\ (=-12) < 0 \Rightarrow$ max		x | – | 5 | + $\frac{dy}{dx}$ | \ | 0 | / min at $x = 5$ or x | – | 3 | + $\frac{dy}{dx}$ | / | 0 | \ max at $x = 3$
		Both correct cao	**A1**	
	(iv)	$\frac{3x^2}{2} - \frac{(x-4)^{-2}}{2}(+c)$ o.e. isw	**B1 + B1**	may be unsimplified
	(v)	*their* $\left[\left(\frac{3(6)^2}{2} - \frac{1}{2(6-4)^2}\right) - \left(\frac{3(5)^2}{2} - \frac{1}{2(5-4)^2}\right)\right]$	**M1**	
		16.875 to 3 or more sf or $\frac{135}{8}$ or $16\frac{7}{8}$ cao	**A1**	

© Cambridge International Examinations 2013