CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2013 series

4037 ADDITIONAL MATHEMATICS

4037/21 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1, 2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1	$\dfrac{2+2\sin^2\theta}{\cos^2\theta}$	**B1**	For all methods look for: – correct simplified expression – correct use of Pythagoras – use of tan = $\dfrac{\sin}{\cos}$ – use of $\dfrac{1}{\cos} = \sec$
	$\dfrac{2}{\cos^2\theta} = 2\sec\theta$	**B1**	
	$\dfrac{\sin^2\theta}{\cos^2\theta} = 2\tan^2\theta$	**B1**	
	$2\sec^2\theta = 2 + 2\tan^2\theta$ and completion	**B1**	Award first 3 then last B1 for final expression from fully correct method. Inconsistent no angle used then –1 (can recover). If start from RHS award similarly.
	Or $(\sec\theta + \tan\theta)^2 + (\sec\theta - \tan\theta)^2$	**[B1, B1**	
	$2\sec^2\theta + 2\tan^2\theta$	**B1**	
	$2(1 + \tan^2\theta) + 2\tan^2\theta$ and completion	**B1]**	
	Or $\dfrac{2+2\sin^2\theta}{\cos^2\theta}$	**[B1**	
	$\dfrac{2(\sin^2\theta + \cos^2\theta) + 2\sin^2\theta}{\cos^2\theta}$	**B1**	
	$\dfrac{4\sin^2\theta}{\cos^2\theta} = 4\tan^2\theta$	**B1**	
	$\dfrac{2\cos^2\theta}{\cos^2\theta} = 2$ and completion	**B1]**	
2 **(i)**	3.2	**B1**	
(ii)	15	**B1**	
(iii)	uses area to find distance	**M1**	If split 2 or 3 correct formulae and must be attempting total area
	two of 40, 240 and 32	**A1**	
	312	**A1**	or **A2** for 312 from trapezium

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3		$\frac{dy}{dx} = k\sin x \cos x$	**M1**	
		$k = -8$	**A1**	
		Attempt to find x when $y = 8$	**M1**	Must get to x = numerical value
		$x = \frac{\pi}{4}\ (0.785)$	**A1**	45° = **A0** (but can still gain next 2 marks)
		Uses $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$	**M1**	Must use numerical value for x and 0.2 for $\frac{dx}{dt}$
		–0.8 (not rounded)	**A1**	(condone poor notation if correct terms multiplied)
4	**(i)**	Idea of modulus correct	**B1**	Two straight lines above and touching x-axis
		$\frac{1}{2}$ indicated on x-axis	**B1**	Must be a sketch
		2 indicated on y-axis	**B1**	Must be a sketch
	(ii)	$\frac{2}{3}$ (0.667)	**B1**	0.67 is **B0**
		Solve $4x - 2 = -x$ or $(4x-2)^2 = x^2$	**M1**	As far as x = numerical value
		$\frac{2}{5}$	**A1**	SC: If drawn then **B1**, **B2** for exact answers only
5	**(i)**	$(QR = PS =) \frac{96 - 3x}{2}$	**B1**	Can be implied by next statement
		Area $= \left(\frac{96-3x}{2}\right) \times x$	**B1**	**AG**
	(ii)	$\frac{dA}{dx} = \frac{96 - 6x}{2}$ or $48 - 3x$ o.e.	**B1**	
		Solving $\frac{dA}{dx} = \frac{96-6x}{2} = 0$	**M1**	As far as x = numerical value
		$x = 16$	**A1**	
		$A = 384$ and state maximum	**A1**	

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6		Applies quotient rule correctly	**M1**	or product rule
		$\frac{(x-2)2x-(x^2+8)}{(x-2)^2}$	**A1**	$2x\,(x-2)^{-1} - (x^2+8)\,(x-2)^{-2}$
		$y = 12$	**B1**	
		Uses $m_1m_2 = -1$	**M1**	
		(Gradient normal = $\frac{1}{2}$)		
		Uses equation of line for **normal**	**M1**	If uses $y = mx + c$ must find c for **M1**
		$y-12=\frac{1}{2}(x-4)$ or $y=\frac{1}{2}x+10$	**A1**	
7	**(i)**	$64 + 192x + 240x^2 + 160x^3$ mark final answer	**B3, 2, 1, 0**	3 terms correct earn **B2**; 2 terms correct earn **B1** Can be earned in **(ii)**; **SC2** correct but unsimplified
	(ii)	Multiply out $(1 + 3x)\,(1 - x)$	**M1**	
		$1 + 2x - 3x^2$ o.e.	**A1**	
		$(1) \times (160) + (2) \times (240) + (-3) \times (192)$ o.e.	**M1**	3 terms
		64	**A1**	
		Or Multiply out $(1 - x)\,(64 + 192x + 240x^2 + 160x^3)$	**[M1**	May be other variations: for first **M1** find x^2 term or x^3 term
		$...48x^2 - 80x^3...$o.e.	**A1**	
		Multiply by $1 + 3x$	**M1**	for second **M1** must produce all relevant terms
		64	**A1]**	
		Or $(1 + 3x)\,(64 + 192x + 240x^2 + 160x^3)$	**[M1**	
		$...816x^2 + 880x^3...$o.e.	**A1**	
		Multiply by $1 - x$	**M1**	
		64	**A1]**	

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8		Eliminates y (or x) and full attempt at expansion	**M1**	
		$4x^2 - 8x - 96 = 0$ or $y^2 + 12y - 64 = 0$	**A1**	
		Factorise 3 term relevant quadratic	**M1**	Or use correct formula
		$x = -4$ and 6 or $y = -16$ and 4	**A1**	
		$y = -16$ and 4 or $x = -4$ and 6	**A1√**	
		Uses Pythagoras for relevant points	**M1**	
		22.4 or $\sqrt{500}$ or $10\sqrt{5}$	**A1**	cao
9	**(i)**	Attempt to solve 3 term quadratic	**M1**	
		-3 and 8	**A1**	
		$-3 < x < 8$	**A1**	Condone $-3 < x$ AND $x < 8$
	(ii)	$4 < x$ (< 12)	**B1**	
		$S \cup T = -3 < x < 12$	**B1**	
	(iii)	$S \cap T = 4 < x < 8$ or $S' = -5 < x \leqslant -3,\ 8 \leqslant x < 12$ and $T' = -5 < x \leqslant 4$	**B1**	Penalise confusion over $<$ and $\leqslant$ (or $>$ and $\geqslant$) once only
		$-5 < x \leqslant 4$	**B1√**	*their* 4
		$8 \leqslant x < 12$	**B1√**	*their* 8 (Ignore AND/OR etc.)

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10	**(i)**	$\frac{\sin\alpha}{50} = \frac{\sin 58}{240}$	**M1 A1**	Use of sin rule/cosine rule/resolving with 50, 240 and 58/32/122/148. Must be correct for **A1**
		$\alpha = 10.2$	**A1**	
		Bearing (0)21.8 or (0)22	**A1√**	√ for $32 - \alpha$
	(ii)	$V^2 = 240^2 + 50^2 - 2 \times 240 \times 50 \times \cos(122 - \alpha)$	**M1**	Correct use of sin rule/cosine rule/resolving
		$V = 263$ awt	**A1**	Can be in **(i)**
		$T = \frac{500}{V}$	**M1**	Only allow if V calculated from non right-angled triangle
		114 or 1 hour 54 mins	**A1**	Do not allow incorrect units
		Or		
		$T = \frac{500\cos 32}{240\cos 21.8}$	**[M1**	Alternative for part **(ii)** only Also can find distance for 240 (457) then 457/240
		500 cos 32	**B1**	
		240 cos 21.8	**B1**	
		114 or 1 hour 54 mins	**A1]**	
11	**(i)**	1	**B1**	Not a range for k, but condone $x = 1$ and $x \geqslant 1$
	(ii)	$f \geqslant -5$	**B1**	Not x, but condone y
	(iii)	Method of inverse	**M1**	Do not reward poor algebra but allow slips
		$1 + \sqrt{x+5}$	**A1**	Must be $f^{-1} = ...$or $y = ...$
	(iv)	f: Positive quadratic curve correct range and domain	**B1**	Must cross x-axis
		f^{-1}: Reflection of f in $y = x$	**B1√**	√*their* f*(x)* sketch Condone slight inaccuracies unless clear contradiction.
	(v)	Arrange $f(x) = x$ or $f^{-1}(x) = x$ to 3 term quadratic = 0	**M1**	
		4 only www	**A1**	Allow $x = 4$ with no working. Condone (4, 4). Do not allow final **A** mark if –1 also given in answer

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12	**(i)**	f(3) = (27 + 9 + 3a + b) = 0 or 3a + b = –36	**M1**	Equate f(3) to 0
		f(–1) = (–1 + 1 – a + b) = 20 or –a + b = 20	**M1**	Equate f(–1) to 20
		Solve equations	**M1**	
		a = –14, b = 6	**A1**	If uses b = 6 then **M0, A0** Need both values for **A1**
	(ii)	Find quadratic factor	**M1**	If division, must be complete with first 2 terms correct If writes down, must be (x^2 + kx – 2)
		$x^2 - 4x - 2$	**A1**	
		Use quadratic formula or completing square on relevant 3 term quadratic	**M1**	If completing square, must reach $\left(x+\frac{k}{2}\right)^2 = 2 \pm \left(\frac{k}{2}\right)^2$
		$\frac{-4 \pm \sqrt{16+8}}{2}$ or better	**A1**√	
		$-2 \pm \sqrt{6}$ isw	**A1**	cao

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