CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2013 series

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

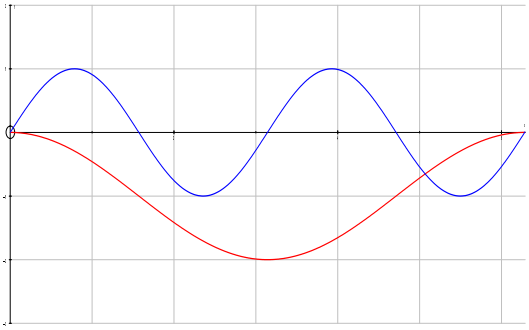
OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1	**(i)**		**B1**	correct shape for $y = \cos x - 1$
	(ii)		**B1**	all correct
			B1	correct shape for $y = \sin 2x$
			B1	all correct
	(iii)	3	**B1**	
2		**Either** gradient = 1	**B1**	
		intercept = 2	**B1**	
		$\ln b$ = gradient or $\ln A$ = intercept	**M1**	M1, need to equate either gradient to $\ln b$ or intercept to $\ln A$
		$b = \text{e}$ or 2.72	**A1**	
		$A = \text{e}^2$, $A = 7.39$	**A1**	
		Or $\text{e}^4 = Ab^2$ and $\text{e}^{10} = Ab^8$	**[B1 B1**	**B1** for each equation
		leading to $b^6 = \text{e}^6$ or $\text{e}^4 = \text{e}^2 A$ or $\text{e}^{10} = \text{e}^8 A$	**M1**	**M1** for attempt to solve for either A or b
		$b = \text{e}$ or 2.72	**A1**	
		$A = \text{e}^2$, $A = 7.39$	**A1]**	
		Or $10 = 8 \ln b + \ln A$	**[B1**	
		$4 = 2 \ln b + \ln A$	**B1**	
		leading to $\ln b = 1$ or $6 = 3 \ln A$	**M1**	**M1** for attempt to solve for either A or b
		$b = \text{e}$ or 2.72	**A1**	
		$A = \text{e}^2$, $A = 7.39$	**A1]**	

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3	**(i)**	${}^{14}C_6 = 3003$	**B1**	
	(ii)	${}^5C_3 \times {}^9C_3 = 840$	**M1 A1**	**M1** for product of 2 combinations
	(iii)	**Either** $3003 - {}^9C_6 = 2919$	**M1 B1 A1**	**M1** for 3003 – number of committees containing no men **B1** for ${}^9C_6$
		Or 1M + 5W: $5 \times {}^9C_5 = 630$ 2M + 4W: ${}^5C_2 \times {}^9C_4 = 1260$ 3M + 3W: 840 (part **(ii)**) 4M + 2W: ${}^5C_4 \times {}^9C_2 = 180$ 5M + 1W: $1 \times {}^9C_1 = 9$ Total: 2919	**[B2 1 0** **B1]**	 –1 each error **B1** for correct final answer
4	**(i)**	2	**B1**	
	(ii)	$\log_4 y^2 - \log_4 (5y - 12)$ $(=\log_4 2)$	**B1**	**B1** for power
		$\log_4\left(\frac{y^2}{5y-12}\right) = (=\log_4 2)$	**M1**	correct division
		$y^2 - 10y + 24 = 0$	**M1**	attempt at solution of a 3 term quadratic
		$y = 4, 6$	**A1**	**A1** for both
5	**(i)**	$x + \frac{6}{x}(+c)$	**B1 B1**	**B1** for each term
	(ii)	$\left(3k + \frac{6}{3k}\right) - \left(k + \frac{6}{k}\right)(=2)$	**M1**	correct use of limits
		$2k^2 - 2k - 4 = 0$	**M1**	attempt to obtain a 3 term quadratic from 2 brackets equated to 2
			DM1	**DM1** or solution of quadratic dependent on 2nd **M1**
		leading to $k = 2$	**A1**	

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6	**(i)**	$A^{-1} = \frac{1}{13}\begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}$	**B1 B1**	**B1** for matrix, **B1** for multiplying by a correct determinant
	(ii)	**Either**		
		$\begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}$	**M1**	evidence of multiplication of both sides by A^{-1}
		$= \frac{1}{13}\begin{pmatrix} 52 & 25+d \\ 13 & -15+2d \end{pmatrix}$		
		leading to $a = 4, c = 1$	**DM1**	**DM1** for attempt to equate like elements
		and $b = 2, d = 1$	**A3,2,1,0**	–1 each error
		Or		
		$\begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}$	**[M1**	**M1** for evidence of matrix multiplication
		$2a - c = 7,\ 3a + 5c = 17,\ a = 4,\ c = 1$	**DM1**	**DM1** for attempt to equate like elements –1 each error
		$2b + 1 = 5,\ 3b - 5 = d,\ b = 2,\ d = 1$	**A3,2,1,0]**	
7	**(i)**	$\tan B = \frac{\sqrt{5+1}}{\sqrt{5-2}}$	**B1**	
		$= \frac{\sqrt{5+1}}{\sqrt{5-2}} \times \frac{\sqrt{5+2}}{\sqrt{5+2}}$	**M1**	attempt at rationalisation (Allow if inverse is used)
		$= 7 + 3\sqrt{5}$	**A1**	
	(ii)	$(7 + 3\sqrt{5})^2 + 1 = \sec^2 B$	**M1 M1**	**M1** for attempt to use the correct identity **M1** for simplification to give 3 or 4 terms
		$\sec^2 B = 95 + 42\sqrt{5}$	√**A1** √**A1**	cao **A1** for 95, **A1** for $42\sqrt{5}$
		Or		
		$\sec^2 B = \frac{1}{\cos^2 B} = \frac{\left(\sqrt{5+1}\right)^2 + \left(\sqrt{5-2}\right)^2}{\left(\sqrt{5-2}\right)^2}$	**[M1**	**M1** for attempt to use to find BC^2
		$\sec^2 B = \frac{15 - 2\sqrt{5}}{9 - 4\sqrt{5}} \times \frac{9 + 4\sqrt{5}}{9 + 4\sqrt{5}}$	**M1**	**M1** for use of $\sec B = \frac{1}{\cos B}$
		$\sec^2 B = 95 + 42\sqrt{5}$	**A1 A1]**	**A1** for 95, **A1** for $52\sqrt{5}$

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8	**(i)**	**Either** $\tan\frac{\theta}{2}=\frac{8}{6}$	**M1**	**M1** for use of trig to obtain half angle
		$\frac{\theta}{2}=0.927...$		Can use $\sin\frac{\theta}{2}=\frac{8}{10}$ or $\cos\frac{\theta}{2}=\frac{6}{10}$
		$\theta=1.855$	**A1**	**A1** Allow if done in degrees and converted
		Or Area of triangle $MEF = 48$	**[M1**	**M1** for a complete method to find the obtuse angle
		$\frac{1}{2}\times10^2\times\sin\theta=48$		
		$\theta=1.287, \pi-1.287$		
		$\theta=1.855$	**A1]**	
		Or $16^2=10^2+10^2-(2\times10\times\cos\theta)$	**[M1**	**M1** for use of the cosine rule, need to see working as answer given
		$\theta=1.855$	**A1]**	
	(ii)	radius = 10	**B1**	**B1** for the radius, allow anywhere
		$P = (10\times1.855)+10+10+16$	**M1 M1**	**M1** for use of arc length **M1** for method, must be arc +3 sides
		= 54.6 or 54.5 or 54.55	**A1**	
	(iii)	$A = 256-2\left(\frac{1}{2}\times8\times6\right)-\frac{1}{2}10^2(1.855)$	**M1** **M1**	**M1** for area of sector **M1** for a correct plan to obtain the required area
		= 115.25 or 115.3 or 115	**A1**	
		awrt 115		

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9	**(i)**	$\overrightarrow{AP} = \frac{3}{4}(\mathbf{b}-\mathbf{a})$	**B1**	
		$\overrightarrow{OP} = \mathbf{a} + \frac{3}{4}(\mathbf{b}-\mathbf{a})$, or	**M1**	**M1** for attempt at vector addition
		$\overrightarrow{OP} = \mathbf{a} - \frac{1}{4}(\mathbf{b}-\mathbf{a})$,		
		$= \frac{1}{4}(\mathbf{a}+3\mathbf{b})$	**A1**	Answer given
	(ii)	$\overrightarrow{OQ} = \frac{2}{5}\mathbf{c}$, or $\overrightarrow{QC} = \frac{3}{5}\mathbf{c}$ or $\overrightarrow{CQ} = -\frac{3}{5}\mathbf{c}$	**B1**	**B1** for $\overrightarrow{OQ}$, $\overrightarrow{QC}$ or $\overrightarrow{CQ}$
		$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	**M1**	**M1** for correct vector addition/subtraction
		$= \frac{2}{5}\mathbf{c} - \frac{\mathbf{a}}{4} - \frac{3\mathbf{b}}{4}$	**A1**	
	(iii)	$2\mathbf{c} - \frac{5\mathbf{a}}{4} - \frac{15\mathbf{b}}{4} = 6(\mathbf{c}-\mathbf{b})$	**M1**	**M1** for use of *their* vectors and attempt to get k **c**
		$\mathbf{c} = \frac{9\mathbf{b}-5\mathbf{a}}{16}$	**A1**	
10	**(i)**	When $x = 2$, $y = -5$	**B1**	**B1** for $y = -5$
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 8x + 1$	**M1**	**M1** for attempt to differentiate
		when $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -3$	**DM1**	**DM1** for attempt at tangent equation – must be tangent with use of $x = 2$
		Tangent: $y + 5 = -3\,(x - 2)$ $(y = 1 - 3x)$	**A1**	allow unsimplified
	(ii)	$1 - 3x = x^3 - 4x^2 + x + 1$	**M1**	**M1** for equating tangent and curve equations
		$x\,(x-2)^2 = 0$	**DM1**	**DM1** for attempt to solve resulting cubic equation
		Meets at (0, 1)	**A1 A1**	**A1** for each coordinate

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(iii)	Grad of perp = $\frac{1}{3}$	**√B1**	**√B1** on *their* gradient in **(i)** only
	Midpoint (1, –2)	**M1**	**M1** for attempt to find the midpoint
	Perp bisector $y + 2 = \frac{1}{3}(x - 1)$	**M1 A1**	**M1** for attempt at line equation – must be perp bisector **A1** allow unsimplified
11 (a)	$\sin\left(x + \frac{\pi}{3}\right) = -\frac{1}{2}$	**B1**	
	$x + \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}$	**B1**	**B1** for $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$
	$x = \frac{5\pi}{6}, \frac{3\pi}{2}$	**B1 B1**	**B1** for first correct solution **B1** for a second correct solution with all solutions in radians and with no excess solutions within the range
(b)	$\tan y - 2 = \frac{1}{\tan y}$	**B1**	**B1** for a correct equation
	$\tan^2 y - 2 \tan y - 1 = 0$	**M1 A1**	**M1** for attempt to obtain a 3 term quadratic equation **A1** for a correct equation equated to zero
	$\tan y = 1 \pm \sqrt{2}$	**DM1**	**DM1** for solution of quadratic
	$y = 67.5°,\ 157.5°$	**A1 A1**	**A1** for first correct solution **A1** for a second correct solution with all solutions in degrees and with no excess solutions within the range.

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