**UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Ordinary Level**

# MARK SCHEME for the May/June 2011 question paper

# for the guidance of teachers

## 4037 ADDITIONAL MATHEMATICS

**4037/21** Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\surd$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1, 2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

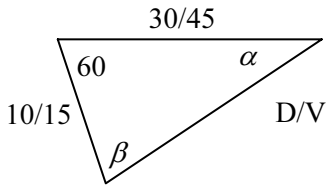
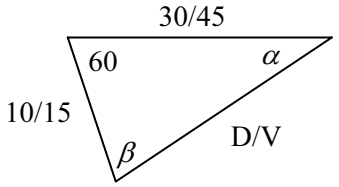
S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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| | | | |
|---|---|---|---|
| **1** | $\left(5+2\sqrt{3}\right)^2 = 37+20\sqrt{3}$ | B1 | |
| | $\frac{\left(37+20\sqrt{3}\right)}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ | M1 | |
| | $14+3\sqrt{3}$ | A1 + A1√ | **[4]** |
| **2 (i)** | 220 or $\pm\frac{1}{8}$ | B1 | |
| | –27.5 oe | B1 | |
| **(ii)** | 16.5($x^2$) | B1 | |
| | Correct method for collecting terms | M1 | |
| | (66 + **(i)**) 38.5 oe | A1√ | **[5]** |
| **3** | $\overrightarrow{AB} = 6\mathbf{i}+24\mathbf{j}$ (or $\overrightarrow{AC} = 4\mathbf{i}+16\mathbf{j}$) | B1 | |
| | $\overrightarrow{OC} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$ $\left(\mathbf{i}-4\mathbf{j}+\frac{2}{3}(6\mathbf{i}+24\mathbf{j})\right)$ | M1 | |
| | $\overrightarrow{OC} = 5\mathbf{i}+12\mathbf{j}$ | A1 | |
| | $\left\lvert\overrightarrow{OC}\right\rvert = \sqrt{5^2+12^2}$ | M1 | |
| | 13 | A1 | **[5]** |
| **4** | Eliminates $y$ | M1 | |
| | $x^2 + kx - 2x + 16$ (= 0) | A1 | |
| | Uses $b^2 - 4ac$ | M1 | |
| | $k^2 - 4k - 60$*0 or $(k-2)^2 \pm 8$ | A1 | |
| | $k = -6$ or 10 | A1 | |
| | $k < -6$ or $k > 10$ | A1 | **[6]** |
| **5 (i)** | f(1) = 1 + 8 + $p$ – 25 (= $p$ – 16) | B1 | |
| | f(–2) = –8 + 32 – 2$p$ – 25 (= –2$p$ – 1) | B1 | |
| | $p - 16 = 2p + 1$ oe | M1 | |
| | $p = -17$ | A1 | |
| **(ii)** | Evaluates f(–3) or divides by ($x$ + 3) to remainder | M1 | |
| | 71 (= 20 – 3$p$) | A1√ | **[6]** |
| **6 (a)** | **(i)** Evidence of 8, 7, 6, 5, 4, 3, 2, 1 or 8! | M1 | |
| | 40320 | A1 | |
| | **(ii)** Evidence of 5! (120) or 4! (or 24) | B1 | |
| | 2880 | B1 | |
| **(b)** | $\frac{7\times6\times5}{3\times2(\times1)}(=35)$ and $\frac{5\times4}{2(\times1)}(=10)$ | B1 | |
| | Multiply | M1 | |
| | 350 | A1 | **[7]** |
| **7 (i)** | $m = 2.5$ | B1 | |
| | $c = 2$ | B1 | |
| | $\lg y = 2.5\lg x + 2$ | M1 | |
| | $2 = \lg 100$ or $\lg 10^2$ | B1√ | |
| | $2.5\lg x = \lg x^{2.5}$ | B1√ | |
| | $y = 100\,x^{2.5}$ | A1 | |
| **(ii)** | Solve $2.5\lg x = \lg 3$ or $x^{2.5} = 3$ correctly | M1 | |
| | 1.55 | A1 | **[8]** |

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| | | | |
|---|---|---|---|
| **8 (i)** | 70 | B1 | |
| **(ii)** | 39.7 | B1 | |
| **(iii)** | $55e^{-0.1t} = 25 - 15$ oe | B1 | |
| | $0.1t = \ln\left(\frac{55}{10}\right)$ oe | M1 | |
| | 17(.0) | A1 | |
| **(iv)** | $\left(\frac{dT}{dt}\right) = ke^{-0.1t}$ | M1 | |
| | $k = -5.5$ oe | A1 | |
| | –1.11 | A1 | **[8]** |
| **9 (i)** | Either | | |
| |  | B1 | |
| | 10 or 45 found | B1 | |
| | Uses cosine rule | M1 | |
| | $D^2 = 10^2 + 30^2 - 2 \times 10 \times 30 \times \cos 60$ | A1 | |
| | or $V^2 = 15^2 + 45^2 - 2 \times 15 \times 45 \times \cos 60$ | | |
| | 39.7 or 39.8 or $15\sqrt{7}$ | A1 | |
| **(ii)** | $\frac{\sin\alpha}{10/15} = \frac{\sin 60}{D/V}$ (or $\frac{\sin\beta}{30} = \frac{\sin 60}{D}$ **and use** $\beta$) | M1 | |
| | $\alpha = 19.1$ or $\beta = 101$ | A1 | |
| | 251 | A1√ | **[8]** |
| **9 (i)** | Or | | |
| |  | B1 | |
| | 10 | B1 | |
| | $D\sin\alpha = 10\sin 60$ and $D\cos\alpha = 25$ | B1 | |
| | or $V\sin\alpha = 15\sin 60$ and $V\cos\alpha = 37.5$ | | |
| | Solve equations | M1 | |
| | $V = 39.7$ or 39.8 | A1 | |
| **(ii)** | $\tan\alpha = \frac{10\sin 60}{25}$ | M1 | |
| | $\alpha = 19.1$ | A1 | |
| | 251 | A1√ | **[8]** |

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|---|---|---|---|
| **10 (i)** | $\tan x = -1.33$ | B1 | |
| | 126.9 | B1 | |
| | 306.9 | B1√ | |
| **(ii)** | $6\cos y + \dfrac{6}{\cos y} = 13$ or $\dfrac{6}{\sec y} + 6\sec y = 13$ | B1 | |
| | Forms quadratic in cos$y$ or sec$y$ ($6\cos^2 y - 13\cos y + 6 = 0$) | M1 | |
| | Solve 3 term quadratic | M1 | |
| | 48.2 | A1 | |
| | 311.8 | A1√ | |
| **(iii)** | $2z - 3 = 0.775$ (or 2.37) radians | B1 | |
| | Solves for $z$ using radians | M1 | |
| | 1.89 and 2.68 | A1 | **[11]** |
| **11** | **EITHER** | | |
| **(i)** | $OA = \dfrac{12}{\cos 0.9}$ | M1 | |
| | $AC = 19.3 - 12 = 7.3$ | A1 | |
| **(ii)** | Complete method for major arc $(2\pi - 1.8) \times 12$ | M1 | |
| | 53.8 | A1 | |
| | $AB = 2 \times 12\tan 0.9$ or cosine rule | M1 | |
| | 30.2 | A1 | |
| | Complete plan $(53.8 + 30.2 + 2 \times 7.3)$ | M1 | |
| | 98.6 | A1 | |
| **(iii)** | Complete method for major sector $\dfrac{1}{2} \times 12^2 \times (2\pi - 1.8)$ | M1 | |
| | $\dfrac{1}{2} \times 19.3^2 \times \sin 1.8$ or $\dfrac{1}{2} \times 30.2 \times 12$ | M1 | |
| | 323 or 181 | A1 | |
| | 504 | A1 | **[12]** |
| **11** | **OR** | | |
| **(i)** | Uses product rule $\left(\dfrac{dy}{dx} = \sin x + x\cos x\right)$ | M1 | |
| | At $x = \dfrac{\pi}{2}$ gradient = 1 | A1 | |
| | Uses $m_1 m_2 = -1$ | M1 | |
| | Correctly reaches conclusion. e.g. $y - \dfrac{\pi}{2} = -1\left(x - \dfrac{\pi}{2}\right)$ with $y = 0$ | A1 | |
| **(ii)** | $\int \cos x dx - \int x \sin x dx = x\cos x$ | M1 | |
| | $\int x \sin x dx = \int \cos x dx - x\cos x$ | M1 | |
| | $\sin x - x\cos x$ | A1 | |
| **(iii)** | Uses limits of $\pi$ and $\dfrac{\pi}{2}$ | M1 | |
| | 2.14 or $\pi - 1$ | A1 | |
| | Area triangle = $\dfrac{1}{2} \times \dfrac{\pi}{2} \times \dfrac{\pi}{2}$ or $\dfrac{\pi^2}{8}$ or 1.23 | B1 | |
| | Subtracts area of triangle | M1 | |
| | 0.908 (allow 0.906 or 0.907) or 0.91 or $\pi - 1 - \dfrac{\pi^2}{8}$ | A1 | **[12]** |

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