UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2010**

2 hours

Additional Materials: Answer Booklet/Paper

* 7 6 2 9 6 6 3 9 9 1 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **2** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

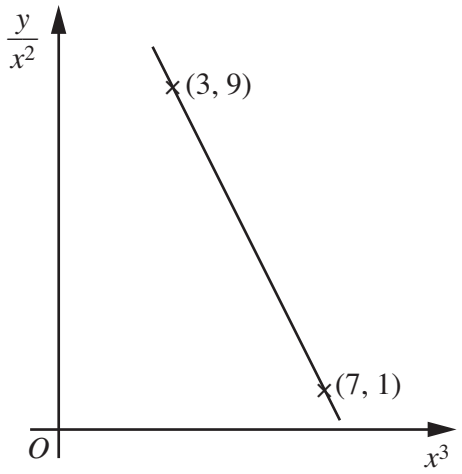
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

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1



The variables x and y are related so that, when $\frac{y}{x^2}$ is plotted against x^3, a straight line graph passing through (3, 9) and (7, 1) is obtained. Express y in terms of x. [4]

2 In a singing competition there are 8 contestants. Each contestant sings in the first round of this competition.

(i) In how many different orders could the contestants sing? [1]

After the first round 5 contestants are chosen.

(ii) In how many different ways can these 5 contestants be chosen? [2]

These 5 contestants sing again and then First, Second and Third prizes are awarded to three of them.

(iii) In how many different ways can the prizes be awarded? [2]

3 It is given that $x - 1$ is a factor of f(x), where $\text{f}(x) = x^3 - 6x^2 + ax + b$.

(i) Express b in terms of a. [2]

(ii) Show that the remainder when f(x) is divided by $x - 3$ is twice the remainder when f(x) is divided by $x - 2$. [4]

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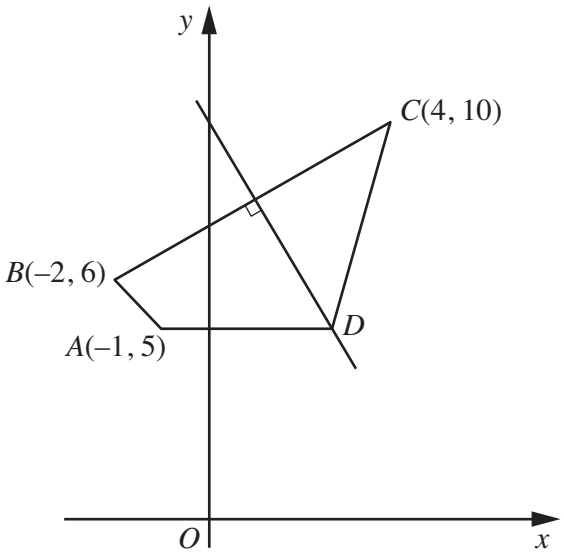
4 **(a)** Given that $\sin x = p$ and $\cos x = 2p$, where x is acute, find the exact value of p and the exact value of $\operatorname{cosec} x$. [3]

(b) Prove that $(\cot x + \tan x)(\cot x - \tan x) = \dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x}$. [3]

5 Given that a curve has equation $y = x^2 + 64\sqrt{x}$, find the coordinates of the point on the curve where $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$. [7]

6 The line $y = x + 4$ intersects the curve $2x^2 + 3xy - y^2 + 1 = 0$ at the points A and B. Find the length of the line AB. [7]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram the points $A(-1, 5)$, $B(-2, 6)$, $C(4, 10)$ and D are the vertices of a quadrilateral in which AD is parallel to the x-axis. The perpendicular bisector of BC passes through D. Find the area of the quadrilateral $ABCD$. [8]

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8 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 & 7 \\ 1 & -5 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 8 & 6 \end{pmatrix}$, calculate

(i) $2\mathbf{A}$, [1]

(ii) $\mathbf{B}^2$, [2]

(iii) $\mathbf{BA}$. [2]

(b) **(i)** Given that $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix}$, find $\mathbf{C}^{-1}$. [2]

(ii) Given also that $\mathbf{D} = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$, find the matrix $\mathbf{X}$ such that $\mathbf{XC} = \mathbf{D}$. [2]

9 A particle starts from rest and moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by

$$v = 4 \sin 2t.$$

(i) Find the distance travelled by the particle before it first comes to instantaneous rest. [5]

(ii) Find the acceleration of the particle when $t = 3$. [3]

10 In this question, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a unit vector due north.

A lighthouse has position vector $\begin{pmatrix} 27 \\ 48 \end{pmatrix}$ km relative to an origin O. A boat moves in such a way that its position vector is given by $\begin{pmatrix} 4 + 8t \\ 12 + 6t \end{pmatrix}$ km, where t is the time, in hours, after 1200.

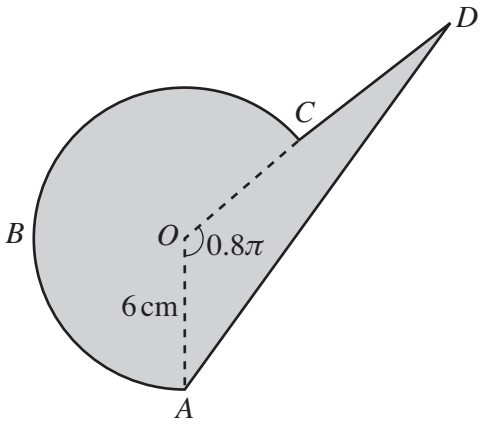
(i) Show that at 1400 the boat is 25 km from the lighthouse. [4]

(ii) Find the length of time for which the boat is less than 25 km from the lighthouse. [4]

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11 Answer only **one** of the following two alternatives.

EITHER

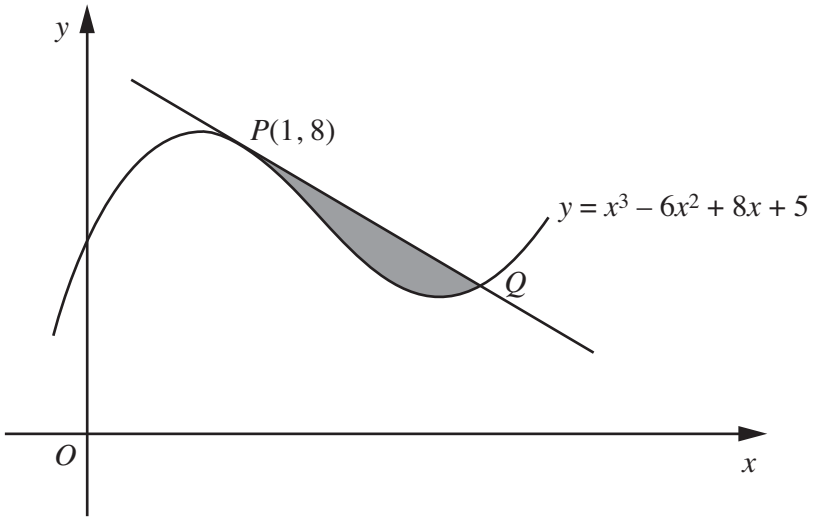


The diagram represents a company logo *ABCDA*, consisting of a sector *OABCO* of a circle, centre *O* and radius 6 cm, and a triangle *AOD*. Angle $AOC = 0.8\pi$ radians and *C* is the mid-point of *OD*. Find

(i) the perimeter of the logo, [7]

(ii) the area of the logo. [5]

OR



The diagram shows part of the curve $y = x^3 - 6x^2 + 8x + 5$. The tangent to the curve at the point $P(1, 8)$ cuts the curve at the point *Q*.

(i) Show that the *x*-coordinate of *Q* is 4. [6]

(ii) Find the area of the shaded region. [6]

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