UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 12, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

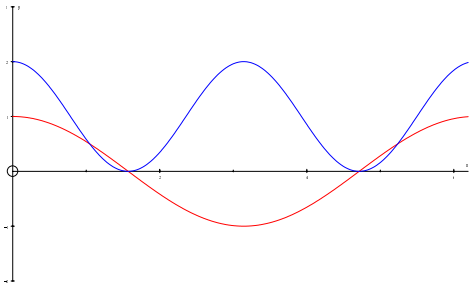
S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1	$24x^2 - 6x = 0$ (or $y^2 + 3y + 2 = 0$) leading to (0, 1) and $\left(\frac{1}{4}, -2\right)$	M1 M1 DM1 A1,A1 [5]	M1 for attempt to get an equation in one variable. M1 for attempt to get 2 or 3 term quadratic = 0 DM1 for attempt to solve A1 for each pair of values
2	$6(-2)^3 + a(-2)^2 - (a + 1)(-2) + b = 15$ $6a + b = 61$ when $x = -1$, $2a + b = 29$ leading to $a = 8$ and $b = 13$	M1 A1 A1 M1 A1 [5]	M1 for substitution of $x = -2$ or -1, or verification A1 for each correct (allow unsimplified) M1 for attempt to solve A1 for $a = 8$, $b = 13$
3	**(i)** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -17 \\ 25 \end{pmatrix}$ $= \begin{pmatrix} 21 \\ -20 \end{pmatrix}$ unit vector = $\begin{pmatrix} 21/29 \\ -20/29 \end{pmatrix}$ or equivalent **(ii)** $\overrightarrow{OC} - \begin{pmatrix} -17 \\ 25 \end{pmatrix} = 3\begin{pmatrix} 21 \\ -20 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 46 \\ -35 \end{pmatrix}$	B1 M1, A1 [3] M1 A1 [2]	B1 for $\overrightarrow{AB}$ M1 for magnitude of $\overrightarrow{AB}$ M1 for $\begin{pmatrix} -17 \\ 25 \end{pmatrix} + 3\overrightarrow{AB}$
4	**(i)** gradient = –2 $y^2 = -2\sec x + c$ leading to $y^2 = -2\sec x + 6.4$ **(ii)** when $y = 2$, $\cos x = \frac{5}{6}$	B1 M1 A1 [3] DM1 A1 [2]	B1 for gradient M1 for correct attempt to link y^2 and sec x DM1 for attempt to solve their equation using $y = 2$
5	$\frac{dy}{dx} = \frac{3}{x^2}$, gradient at $A = \frac{1}{3}$, normal grad = –3 coords of A (3, 5) normal $y - 5 = -3(x - 3)$ when $y = 0$, $x = \frac{14}{3}$	M1 DM1 B1 DM1 A1 [5]	M1 for attempt to differentiate DM1 for use of perp grads DM1 for attempt at normal

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6 **(a)** **(i)** 	B1 B1 B1 [3]	B1 for $y = \cos x$ B1 for either a translation of $\begin{pmatrix}0\\1\end{pmatrix}$ or 2 cycles B1 for correct curve
(ii) 4	B1 [1]	
(b) **(i)** 5	B1 [1]	
(ii) $\frac{2\pi}{3}$	B1 [1]	
7 **(i)** <table><tr><td>lg$v$</td><td>1</td><td>1.70</td><td>2.04</td><td>2.36</td></tr><tr><td>lg$p$</td><td>3.15</td><td>2.18</td><td>1.72</td><td>1.28</td></tr></table>	M1 A2,1,0 [3]	M1 for attempt to take logs and plot graph –1 for each error either in table or on graph.
(ii) gradient = n = –1.37 (allow 1.32 to 1.42)	M1 A1 [2]	M1 for use of gradient
(iii) $p = 30$ (allow 28 to 32)	M1 A1 [2]	M1 for use of graph or their equation
8 **(i)** $\begin{pmatrix}16 & 9\\1 & -2\end{pmatrix}$	B1 B1 [2]	B1 at least 2 correct B1 all correct
(ii) $\frac{1}{8-3}\begin{pmatrix}2 & -3\\-1 & 4\end{pmatrix}$	B1 B1 [2]	B1 for determinant B1 for matrix
(iii) $\mathbf{X} = \mathbf{AB}$ $= \begin{pmatrix}-5 & 12\\0 & 8\end{pmatrix}$	M1 A2,1,0 [3]	M1 for attempt at valid method –1 each error

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9	**(i)** $5 + 5 + 3\theta + 8\theta = 15.5$	M1, DM1	M1 for use of arc length DM1 for attempt to find perimeter
	$\theta = 0.5$	A1 [3]	
	(ii) $\frac{1}{2}(3)^2\theta : \frac{1}{2}(8)^2\theta - \frac{1}{2}(3)^2\theta$	M1	M1 for a sector area
		DM1	M1 for attempt to find area of *XABY*
	$= 9 : 55$	DM1, A1 [4]	M1 for attempt to obtain ratio
10	**(i)** ${}^{10}C_7 = 120$	B1 [1]	
	(ii) ${}^6C_5 \times {}^4C_2 = 36$	B1, B1 [2]	B1 for ${}^6C_5 \times {}^4C_2$, B1 for 36
	(iii) Need (6C + 1M) + (5C + 2M) + (4C + 3M)	M1	M1 for a correct method
	4 + **(ii)** + $({}^6C_4 \times {}^4C_3)$	B1, B1	B1 for 4, B1 for 60
	$= 100$	A1 [4]	
11	**(i)** $48 = 12 \ln (2t + 3)$	M1	M1 for attempt to deal with logs
	$2t + 3 = e^4$	DM1	DM1 for attempt to solve
	$t = 25.8$	A1 [3]	
	(ii) $x = 12 \ln (2t + 3)$	B1	B1 $\frac{1}{2t+3}$
	$v = \frac{24}{2t+3}$	B1	B1 24
	when $t = 1$, $v = 4.8$	B1 [3]	B1 for 4.8
	(iii) $a = -\frac{48}{(2t+3)^2}$	B1	B1 for $\frac{1}{(2t+3)^2}$
	when $t = 1$, $a = -1.92$	√B1	√B1 on '24'
		B1 [3]	B1 for –1.92

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12 EITHER		
(i) $y = 4 \sin 2x + c$	M1	M1 for attempt to integrate
	M1	M1 for attempt to get c provided a function of sin $2x$ is used
passes through $\left(\frac{\pi}{4}, 7\right)$, $c = 3$	A1	
	[3]	
(ii) $5 = 4 \sin 2x + 3$	M1	M1 for attempt to equate to 5 and solve
$0.5 = \sin 2x$	M1	M1 for a correct method to find x
$x = \frac{\pi}{12}, \frac{5\pi}{12}$	A1	
	√A1	√A1 on first solution
	[4]	
(iii) $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 4 \sin 2x + 3 \mathrm{d}x$	M1	M1 for attempt to integrate
$\left[-2\cos 2x + 3x\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$	A1	
	DM1	DM1 for correct use of limits
$= \pi + 2\sqrt{3}$		
Shaded area $= \pi + 2\sqrt{3} - \frac{5\pi}{3}$	M1	M1 for area of rectangle
$(= 1.37)$	A1	
	[5]	
12 OR		
(i) $y = 2e^{3x} - 12x + c$	M1, A1	M1 for attempt to integrate, condone omission of c
Passes through (0, 1), so $c = -1$	M1, A1	M1 for attempt to obtain c
	[4]	
(ii) $6e^{3x} - 12 = 0$	M1	M1 for attempt to solve
leading to $x = \frac{1}{3} \ln 2$ and $y = 3 - 4 \ln 2$	A1, A1	
(allow (0.231, 0.227)	[3]	
(iii) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 18e^{3x}$, always +ve so min	M1, A1	M1 for a complete, correct method
	[2]	
(iv) at (0, 1), gradient = –6		
tangent : $y - 1 = -6(x - 0)$	M1	M1 for attempt to get equation of tangent at (0, 1)
when $y = 0$, $x = \frac{1}{6}$	DM1	DM1 for substitution of $y = 0$
	A1	
	[3]	

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