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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **October/November 2024**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 9).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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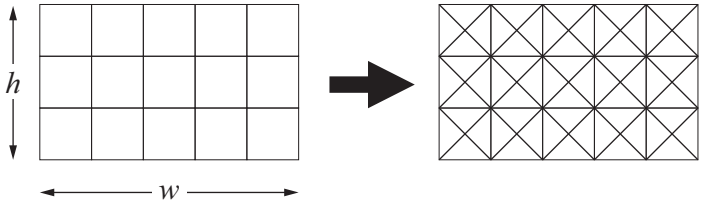
Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 6)

DIAGONAL SQUARES (30 marks)

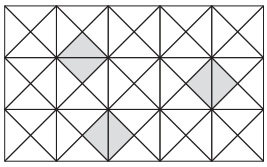
You are advised to spend no more than 50 minutes on this part.

This investigation looks at the number of *diagonal squares* in a grid where the diagonals of each square have been drawn.
This grid is h by w, where h is the number of rows in the grid and w is the number of columns in the grid.



Example

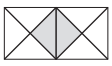
This grid is 3 by 5.
It has 3 rows and 5 columns, so $h = 3$ and $w = 5$.
Some of the diagonal squares are shaded.



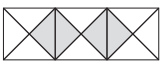
Throughout the investigation the number of diagonal squares in an h by w grid is called D(h, w).

1 This question is about grids with $h = 1$.

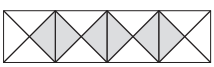
A 1 by 2 grid has one diagonal square, so D(1, 2) = 1.



A 1 by 3 grid has two diagonal squares, so D(1, 3) = 2.



A 1 by 4 grid has three diagonal squares, so D(1, 4) = 3.



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(a) Write down D(1, 7).

..... [1]

(b) Find an expression for D(1, w).

..... [1]

(c) D(1, w) = 20.

Write down the value of w.

..... [1]

(d) Explain why D(1, n) = D(n, 1).

..... [1]

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2 This question is about grids with $h = 2$.

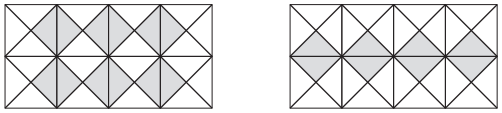
A 2 by 2 grid has 2 rows each with 1 diagonal square and 1 row of 2 diagonal squares.



The table shows this.

2 by 2 grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
2	1	$2 \times 1 = 2$
1	2	$1 \times 2 = 2$
		$D(2, 2) = 2 + 2 = 4$

A 2 by 4 grid has 2 rows of 3 diagonal squares and 1 row of 4 diagonal squares.



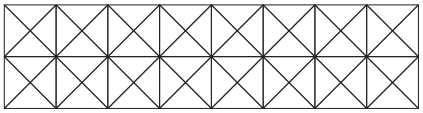
The table shows this.

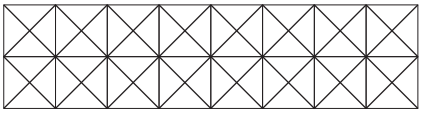
2 by 4 grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
2	3	$2 \times 3 = 6$
1	4	$1 \times 4 = 4$
		$D(2, 4) = 6 + 4 = 10$

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(a) Complete the table.
You may use the grids to help you.

2 by 8 grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
2		
1		
		D(2, 8) = + = 22





[3]

(b) A 2 by w grid has $h = 2$ and width w.

Complete the table with expressions in terms of w.

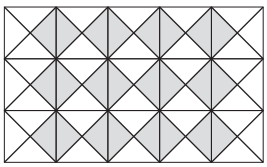
2 by w grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
2		
1		
		D(2, w) =

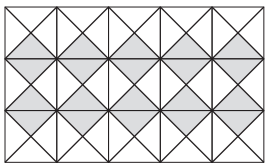
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3 This question is about grids with $h = 3$.

A 3 by 5 grid has 3 rows each with 4 diagonal squares and 2 rows each with 5 diagonal squares.

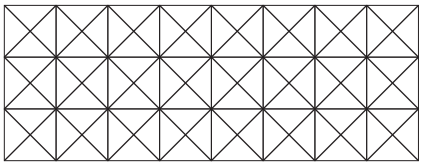


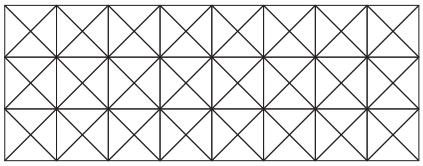


The table shows this.

3 by 5 grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
3	4	$3 \times 4 = 12$
2	5	$2 \times 5 = 10$
		$D(3, 5) = 12 + 10 = 22$

(a) Work out D(3, 8).





..... [2]

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(b) A 3 by w grid has $h = 3$ and width w.

Complete the table with expressions in terms of w.

3 by w grid		
Number of rows of diagonal squares	Number of diagonal squares in each row	Total
3		
2		
		D(3, w) =

[3]

(c) In a 3 by w grid, D(3, w) = 97.

Find the value of w.

..... [2]

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4 In an h by w grid there are:

- h rows with $(w-1)$ diagonal squares
- $(h-1)$ rows with w diagonal squares.

Show that $\mathrm{D}(h, w) = 2hw-(h+w)$.

[2]

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5 This question is about a square grid with n rows and n columns.
This is an n by n grid.

(a) $D(h, w) = 2hw - (h + w)$

Show that the number of diagonal squares in an n by n grid, $D(n, n)$, is always an even number.

[2]

(b) The width of square grid A is one more than the width of square grid B.
The width of square grid B is n.
The difference between the number of diagonal squares in the two square grids is 36.

Find the value of n.

..... [4]

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6 A grid is a rectangle.
It has 31 diagonal squares.

$D(h, w) = 2hw - (h + w)$

Find the sizes of the possible rectangular grids when $h < w$.

..... [5]

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B MODELLING (QUESTIONS 7 TO 9)

SAFE OVERTAKING (30 marks)

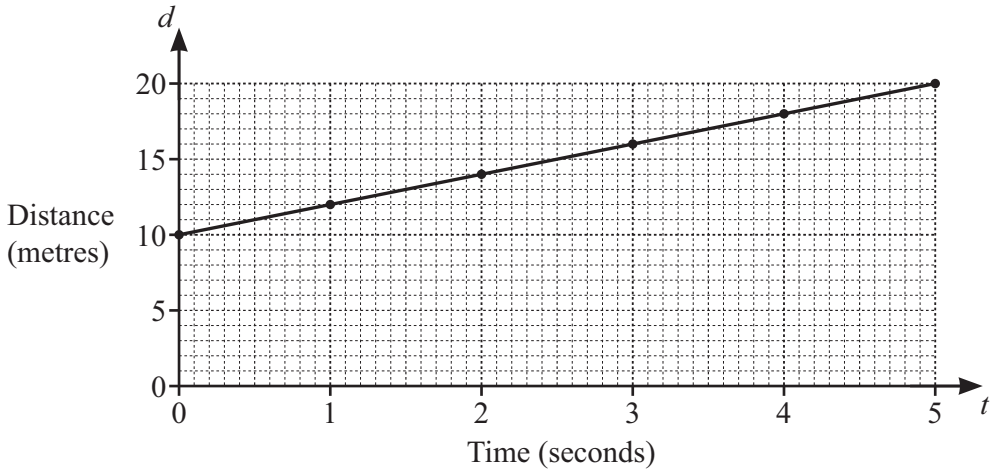
You are advised to spend no more than 50 minutes on this part.

This task looks at the distance travelled by a car when overtaking a truck.

7 The table shows the distance of a car from a fixed point on the road at time t seconds.

Time (t seconds)	0	1	2	3	4	5
Distance (d metres)	10	12	14	16	18	20

The graph shows this information.



(a) Find the speed of the car.

..... [2]

(b) Find a model for d in terms of t.

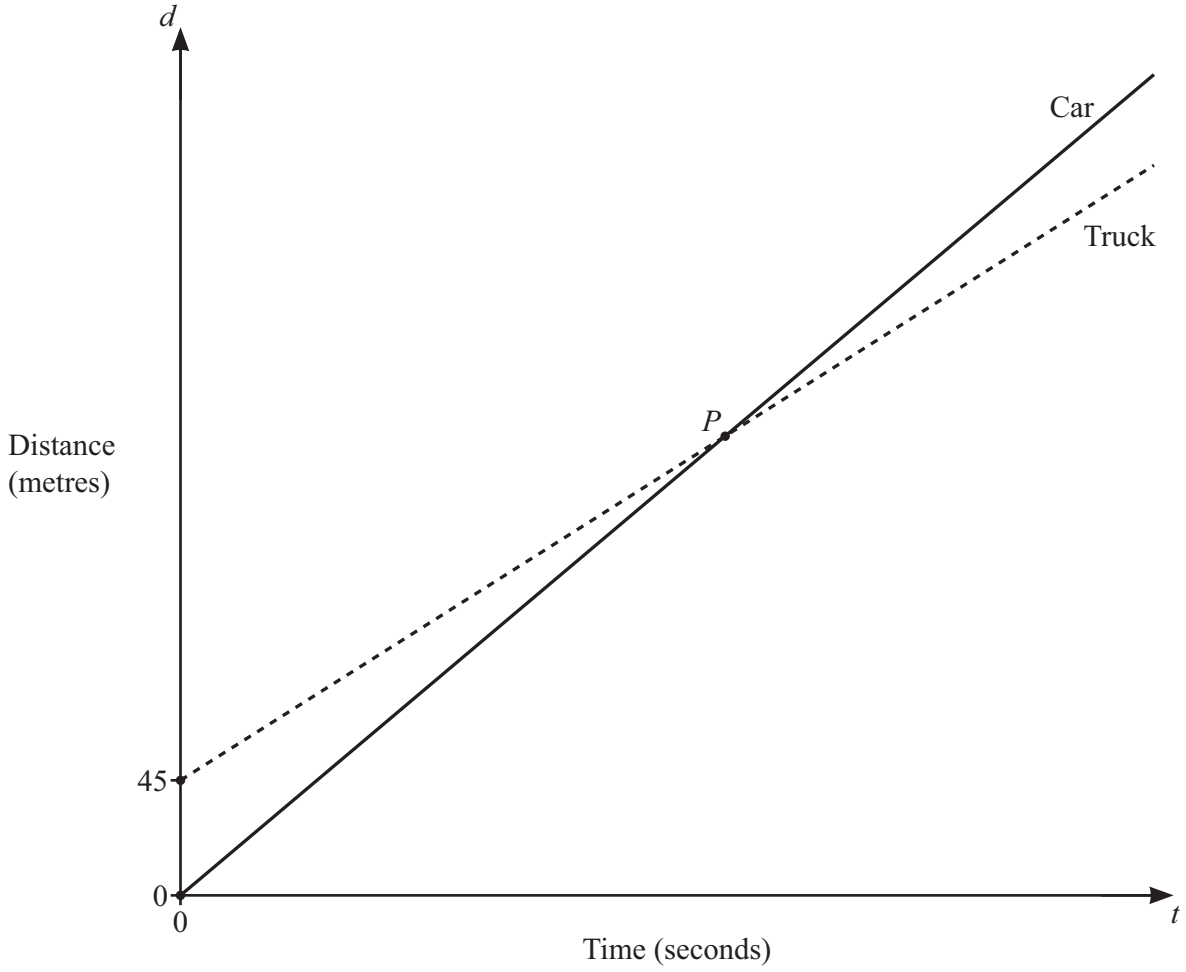
..... [2]

(c) Find the distance of the car from the fixed point when $t = 7$ seconds.

..... [1]

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8 A car and a truck travel in the same direction along a road.
The sketch graph shows their distances from a fixed point on the road.



(a) Write down the number of metres the truck is in front of the car when $t = 0$.

..... [1]

(b) A model for the distance the car travels, in terms of t, is $d = 25t$.
The speed of the truck is 20 m/s.

Find a model for the distance the truck travels in terms of t.

..... [1]

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(c) The car overtakes the truck at point P.

Use algebra to find the coordinates of point P.

(..... , ...) [3]

(d) When $t = 5$ the car changes its speed to 30 m/s.
The two lines now meet at a different point.

(i) On the axes, sketch the new graph for the car. [1]

(ii) Find the number of seconds the car has travelled from the fixed point when it reaches the truck.

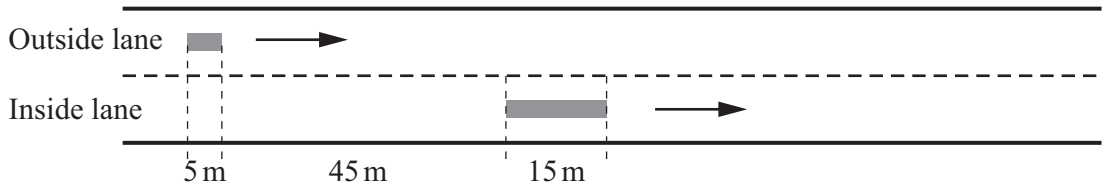
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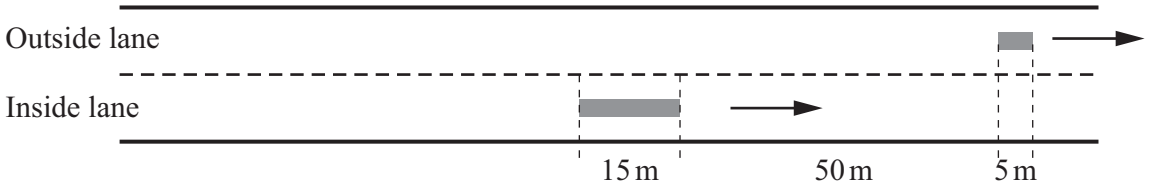
9 This question is about what happens when a car overtakes a truck.
The diagrams are not to scale.
In these diagrams, the direction of travel is from left to right.

The car has length 5 metres.
The truck has length 15 metres.

The car moves into the outside lane to overtake the truck when there is a gap of 45 metres between the front of the car and the back of the truck.



The car moves back into the inside lane when there is a gap of 50 metres between the back of the car and the front of the truck.



(a) When the car starts overtaking the distance between the front of the car and the front of the truck is 60 m.

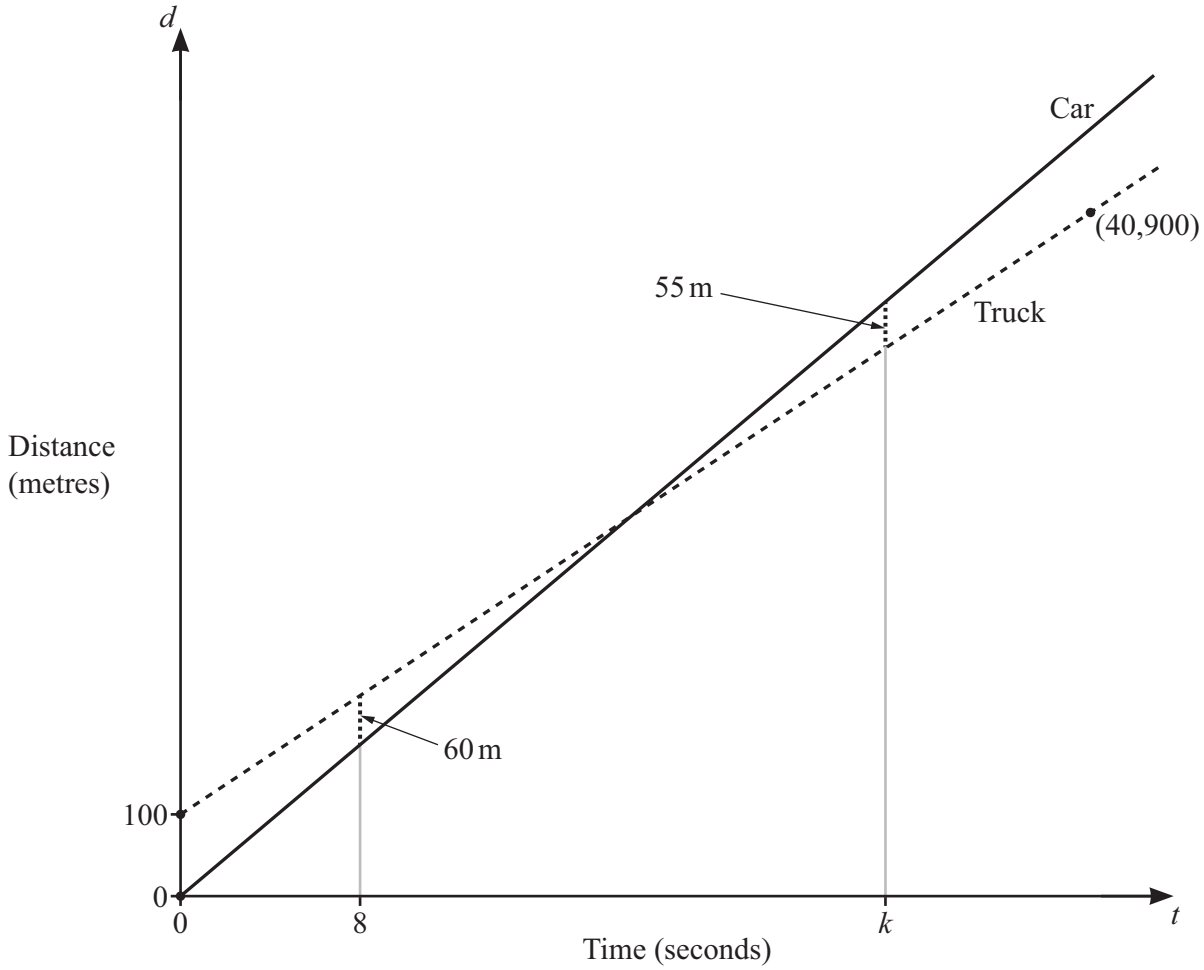
Show that when the car finishes overtaking the distance between the front of the car and the front of the truck is 55 m.

[1]

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(b) The car starts overtaking the truck at time $t = 8$ seconds.
The car finishes overtaking the truck at time $t = k$ seconds.

The sketch graph shows their distances from a fixed point on the road.



The distance travelled, d metres, and the time taken, t seconds, are measured from a fixed point on the road.
A model for the distance travelled by the car, in terms of t, is $d = 25t$.

Find the speed of the truck and write down a model for the distance travelled by the truck.

Speed .....

Model ..... [3]

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(c) When $t = 8$, the truck is 60 metres in front of the car.

Find the time, k seconds, when the car is 55 metres in front of the truck.

..... [2]

(d) Work out the total distance that the car travels when overtaking the truck.

..... [2]

(e) When the car overtakes the truck it travels further than the truck.

(i) Show that the difference between the distances travelled by the car and the truck is 115 metres.

[1]

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(ii) The speed of the car is now x m/s and the speed of the truck is 20 m/s.

Write down an expression for the difference in speed between the car and the truck.

..... [1]

(iii) Show that a model for the total distance, D metres, the car travels when overtaking the truck is $D = \frac{115x}{x-20}$.

[2]

(iv) The car travels 690 metres when overtaking the truck.

Find the speed of the car when overtaking the truck.

..... [3]

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