Cambridge Assessment International Education

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3586687226

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 Investigation (Core) **October/November 2023**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

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Answer **all** the questions.

INVESTIGATION **F-TYPE SEQUENCES**

This investigation explores patterns in a special type of sequence of positive integers.

In an *F-type sequence*:
- the first two terms are any two positive integers
- after the first two terms, each term is equal to the sum of the previous two terms.

1 Here is a table of the first seven terms of an F-type sequence.

The first term F_1 is 5.
The second term F_2 is 3.

F_1	F_2	F_3	F_4	F_5	F_6	F_7
5	3	8	11	19	30	49

In the table, $F_1 + F_2 = 5 + 3 = 8 = F_3$

$F_2 + F_3 = 3 + 8 = 11 = F_4$

$F_3 + F_4 = 8 + 11 = 19 = F_5$

and so on.

(a) Calculate the 8th term (F_8) and the 9th term (F_9).

$F_8 =$ .....

$F_9 =$ ..... [3]

(b) Complete the table.

F_1	$= 5$		$F_3 - F_2 = 8 - 3 = 5$
$F_1 + F_2$	$= 5 + 3$	$= 8$	$F_4 - F_2 =$ ..... $=$ .....
$F_1 + F_2 + F_3$	$= 5 + 3 + 8$	$= 16$	$F_5 - F_2 =$ ..... $=$ .....
$F_1 + F_2 + F_3 + F_4$	$=$	$=$	..... $=$ ..... $= 27$

[5]

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(c) Use what you notice in the table in **part (b)** to complete this statement.

$$F_1 + F_2 + F_3 + F_4 + F_5 = F_{\ldots\ldots} - F_{\ldots\ldots}$$ [1]

(d) Use the statement in **part (c)** to complete this general statement.

$$F_1 + F_2 + F_3 + \cdots + F_n = F_{n+\ldots\ldots} - F_{\ldots\ldots}$$ [1]

(e) Show that your statement in **part (d)** is correct when $n = 7$.

[3]

2 In another F-type sequence the first term is 3 and the second term is 1.

(a) Complete the first five terms.

3, 1, ..., ..., ... [2]

(b) Is your statement in **Question 1(c)** correct for the sum of the first five terms in this sequence?

..... [3]

3 In another F-type sequence the 2nd term is 3 and the 12th term is 652.

Use your answer to **Question 1(d)** to find the sum of the first 10 terms.

..... [2]

4 In another F-type sequence the first term is 6 and the second term is k.

(a) Find an expression, in terms of k, for the 4th term.
Write your answer in its simplest form.

..... [2]

(b) The 4th term of this sequence is 14.

Find k and complete the first four terms.

6, ... , ... , 14 [3]

5 An F-type sequence has five terms.

(a) There is a relationship between the middle term and the sum of the 1st term and the 5th term.

Investigate this relationship by making up at least three numerical examples of F-type sequences. Write down this relationship.

..... [6]

(b) In another F-type sequence the first term is x and the second term is y.

(i) Find, in its simplest form, an expression in terms of x and y for the 5th term.

..... [3]

(ii) Use algebra to show that the relationship in **part (a)** is correct.

[2]

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