Cambridge Assessment International Education

# Cambridge IGCSE™

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*8140996719*

**CAMBRIDGE INTERNATIONAL MATHEMATICS** **0607/63**

Paper 6 Investigation and Modelling (Extended) **October/November 2020**

**1 hour 40 minutes**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 8).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

**INFORMATION**

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

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**[Turn over**

Answer **both** parts **A** and **B**.

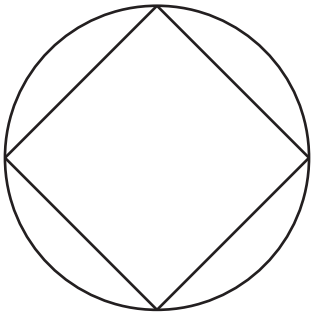
## A INVESTIGATION (QUESTIONS 1 TO 4)

### AREAS OF POLYGONS INSIDE AND OUTSIDE A CIRCLE (30 marks)

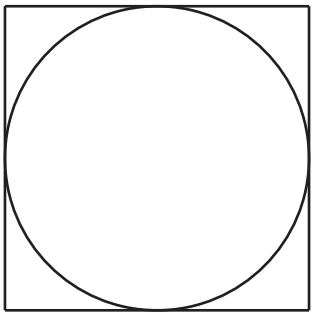
You are advised to spend no more than 50 minutes on this part.

This investigation looks at the areas of polygons drawn inside and outside a circle of radius 10 cm.

An inscribed polygon is a polygon in which all the vertices lie on a circle.
This is an inscribed square.



A circumscribed polygon is a polygon in which each side is a tangent to a circle.
This is a circumscribed square.



You may find some of these formulas useful.

Area, $A$, of circle, radius $r$ $\quad A = \pi r^2$

Area, $A$, of triangle, base $b$, height $h$ $\quad A = \frac{1}{2}bh$

In a right-angled triangle,

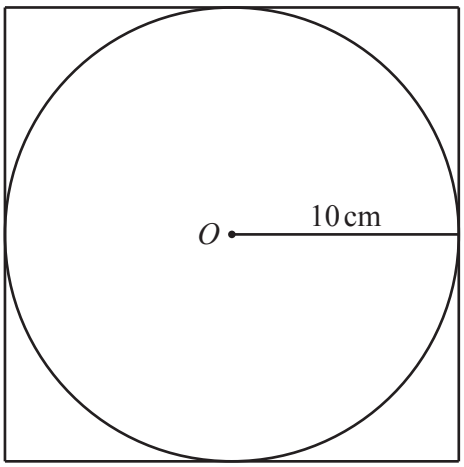
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}},$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}},$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$

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**1 (a)**



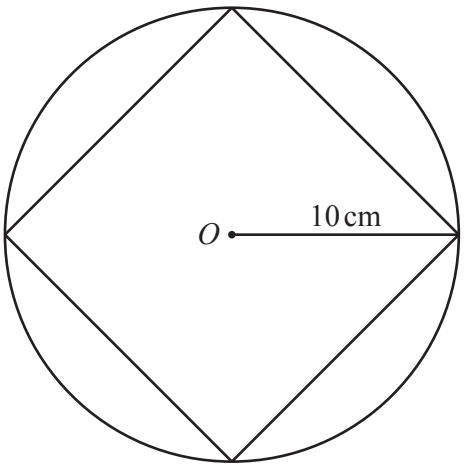

NOT TO SCALE

A square circumscribes a circle, centre $O$, radius 10 cm.

Work out the area of the square.

..... [1]

**(b)**




NOT TO SCALE

A square is inscribed in a circle, centre $O$, radius 10 cm.

Work out the area of the square.

..... [2]

**(c)** Show that the area of a circle, radius 10 cm, is $100\pi$ cm$^2$.

[1]

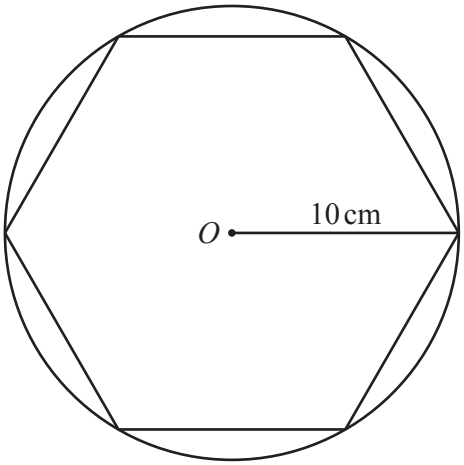
**(d)** Area of inscribed square $<$ Area of circle $<$ Area of circumscribed square

Use this statement to complete the inequality below.

..... $<\ \pi\ <$ ..... [1]

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**2 (a)**



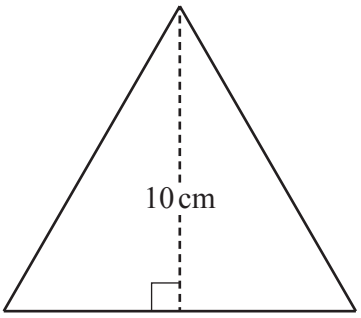

NOT TO SCALE

A regular hexagon is inscribed in a circle, centre $O$, radius 10 cm.

Find the area of the hexagon.

..... [3]

**(b) (i)**



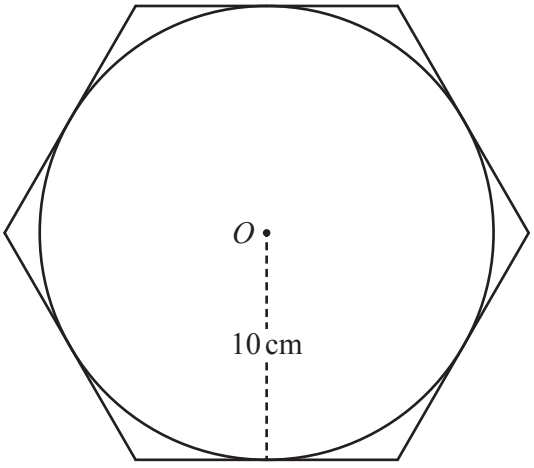

NOT TO SCALE

An equilateral triangle has height 10 cm.

Find the area of the triangle.

..... [3]

**(ii)**




NOT TO SCALE

A regular hexagon circumscribes a circle, centre $O$, radius 10 cm.

Using your answer to **part (i)**, find the area of the hexagon.

..... [2]

**(c) (i)** Use **Question 1(c)**, **Question 2(a)** and **Question 2(b)(ii)** to complete the inequality.

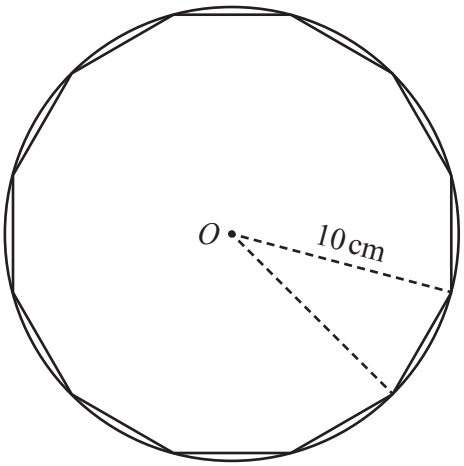
..... $<$ $\pi$ $<$ ..... [1]

**(ii)** Give a geometric reason why the range in the inequality in **Question 2(c)(i)** is smaller than the range in the inequality in **Question 1(d)**.

.....

..... [1]

**3 (a)**



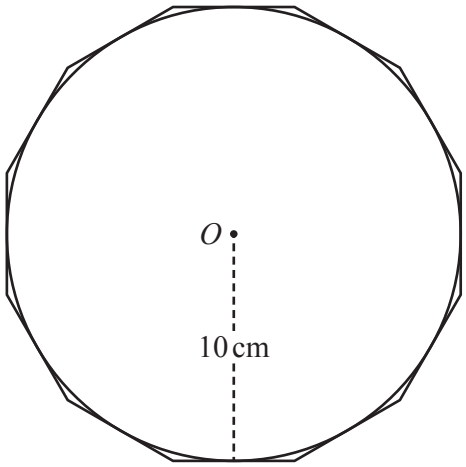

NOT TO SCALE

A regular 12-sided polygon is inscribed in a circle, centre $O$, radius 10 cm.

Find the area of this polygon.

..... [2]

**(b)**




NOT TO SCALE

A regular 12-sided polygon circumscribes a circle, centre $O$, radius 10 cm.

Find the area of this polygon.

..... [3]

**(c)** Use the answers to **part (a)** and **part (b)** to complete the inequality.

..... $<$ $\pi$ $<$ ..... [1]

**4** **(a)** **(i)** Show that a formula for the area, $A\,\text{cm}^2$, of a regular polygon with $n$ sides **inscribed** in a circle, radius 10 cm, is

$$A = 50n\sin\left(\frac{360}{n}\right)^\circ .$$

[2]

**(ii)** Show that a formula for the area, $B\,\text{cm}^2$, of a regular polygon with $n$ sides that **circumscribes** a circle, radius 10 cm, is

$$B = 100n\tan\left(\frac{180}{n}\right)^\circ .$$

[2]

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**(b) (i)** Work out the area of a regular polygon with 100 sides that is **inscribed** in a circle, radius 10 cm.
Give your answer correct to 4 significant figures.

..... [2]

**(ii)** Work out the area of a regular polygon with 100 sides that **circumscribes** a circle, radius 10 cm.
Give your answer correct to 4 significant figures.

..... [2]

**(c)** Use your answers to **part (b)** to explain how you can find the value of $\pi$ correct to 3 significant figures.

.....

..... [1]

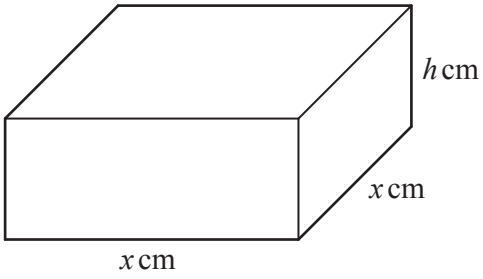
## B MODELLING (QUESTIONS 5 TO 8)

### MODELLING CONTAINERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

Olivia wants to design a closed container with a volume of $1000\,\text{cm}^3$ and minimum surface area.

**5** Olivia uses a square-based cuboid to model the container.




NOT TO SCALE

**(a) (i)** Write down a formula for the volume of the cuboid, $V\,\text{cm}^3$, in terms of $x$ and $h$.

..... [1]

**(ii)** Find a formula for the surface area, $S\,\text{cm}^2$, of the cuboid, in terms of $x$ and $h$.
Give your answer in its simplest form.

..... [2]

**(b) (i)** $V = 1000$.

Write $h$ in terms of $x$.

..... [1]

**(ii)** Show that $S = 2x^2 + \frac{4000}{x}$.

[1]

**(iii)** Work out the value of $S$ when $x = 25$.

..... [1]

**(c)** Sketch the graph of $S = 2x^2 + \frac{4000}{x}$ for $0 < x \leqslant 25$.




[3]

**(d) (i)** Find the minimum surface area of the cuboid.

..... [1]

**(ii)** Describe the container that gives the minimum surface area for Olivia's model.

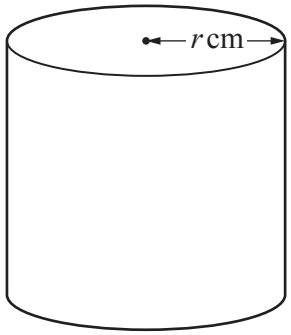
.....

..... [2]

**6**

| | |
|---|---|
| Volume, $V$, of a cylinder of radius $r$, height $h$ | $V = \pi r^2 h$ |
| Curved surface area, $A$, of a cylinder of radius $r$, height $h$ | $A = 2\pi rh$ |

Olivia now uses a cylinder to model the container.




NOT TO SCALE

The total surface area of this model is $T\,\text{cm}^2$.

**(a)** $V = 1000$.

Show that $T = 2\pi r^2 + \dfrac{2000}{r}$.

[3]

**(b) (i)** Find the minimum surface area of the cylinder.

..... [2]

**(ii)** Find the dimensions of the cylinder with the minimum surface area.

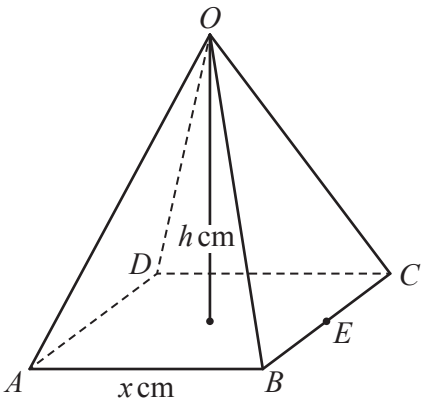
$r =$ .....

$h =$ ..... [2]

**7**

| | |
|---|---|
| Volume, $V$, of a pyramid, base area $A$, height $h$ | $V = \frac{1}{3}Ah$ |

Olivia now uses a square-based pyramid to model the container.




NOT TO SCALE

The pyramid, $OABCD$, has a square base of side $x$ cm and height $h$ cm.
The vertex of the pyramid, $O$, is directly above the centre of the square base.
$E$ is the mid-point of $BC$.

**(a)** Find an expression for $OE$ in terms of $h$ and $x$.

..... [2]

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**(b)** The total surface area of this model is $P\,\text{cm}^2$.

$V = 1000$.

Show that $P = x^2 + \dfrac{\sqrt{x^6 + 36\,000\,000}}{x}$ .

[4]

**(c) (i)** Find the minimum surface area of the pyramid.

..... [2]

**(ii)** Find the dimensions of the pyramid with the minimum surface area.

$x =$ .....

$h =$ ..... [2]

**8** Olivia recommends the container with the smallest surface area to a company.

Give a geometric reason why the company might not accept Olivia's recommendation.

Olivia recommends the .....

Geometric reason .....

..... [1]

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