Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

1289084149

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 (Extended) **October/November 2018**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 4) and **B** (Questions 5 to 8).
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **14** printed pages and **2** blank pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 4)

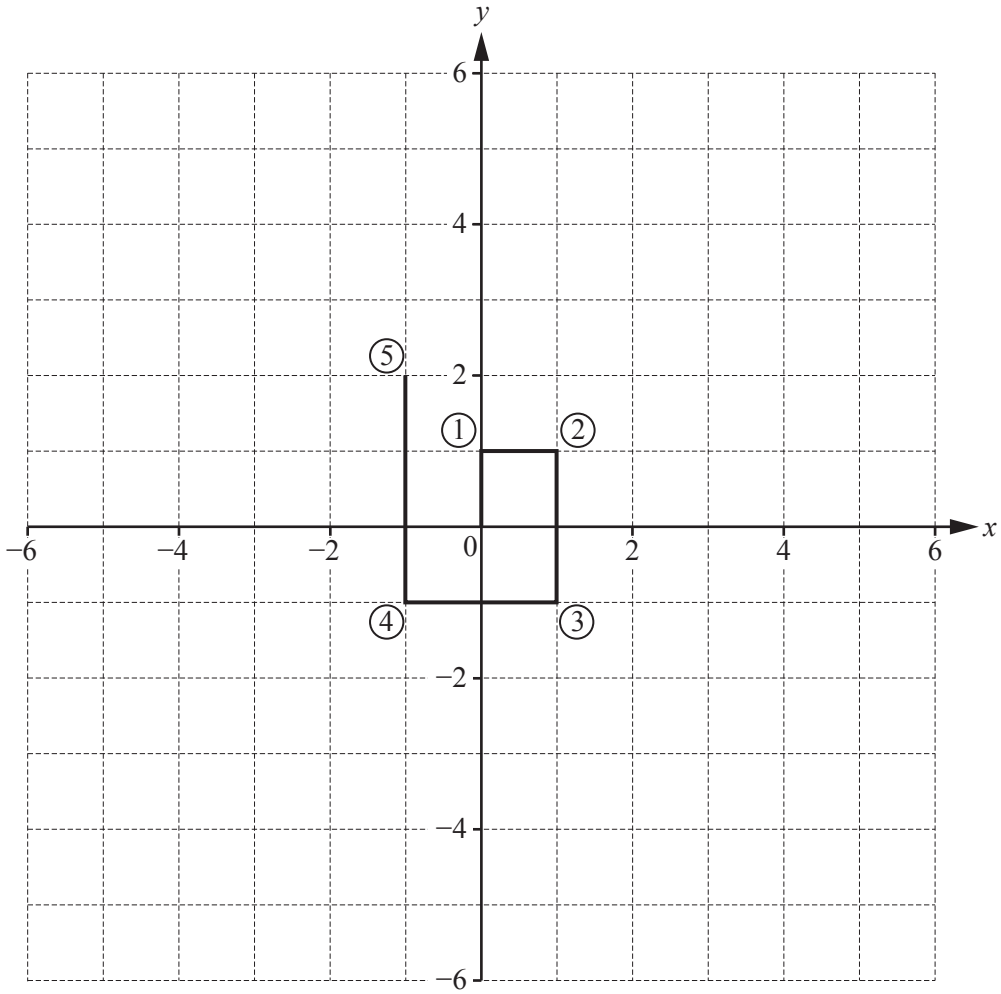
RIGHT SPIRALS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation is about the lengths of spirals drawn on a square co-ordinate grid.

A robot starts from (0, 0) and moves 1 unit to Corner (1).
It then turns right and moves 1 unit to Corner (2).
It then turns right and moves 2 units to Corner (3).
It then turns right and moves 2 units to Corner (4).
It then turns right and moves 3 units to Corner (5).

This forms a spiral, shown on the grid below.



The robot continues to turn and move in the same way.

1 **(a)** By continuing the spiral, show that Corner (10) is at (3, 3).

(b) The length of the spiral from (0, 0) to Corner 4 is 6 units.

Find the length of the spiral from (0, 0) to Corner (10).

.....

(c) Use your spiral to complete this table.

Corner number	Lengths added	Length from (0, 0)
1	1	1
2	1 + 1	2
3	1 + 1 + 2	4
4		6
5		
6	1 + 1 + 2 + 2 + 3 + 3	12
7	1 + 1 + 2 + 2 + 3 + 3 + 4	16
8		20
9		
10	1 + 1 + 2 + 2 + 3 + 3 + 4 + 4 + 5 + 5	

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[Turn over

2 The table shows the length, L, of the spiral from (0, 0) to Corner (k), where k is an even number.

Term number (n)	k	Length (L)
1	2	2
2	4	6
3	6	12
4	8	20
n	k	$n(n+1)$

(a) Find a formula for n in terms of k.

.....

(b) Use **part (a)** to show that the formula for the length, L, of the spiral from (0, 0) to Corner (k) is

$$L = \frac{k}{4}(k+2).$$

(c) Show that the formula gives the correct length of the spiral from (0, 0) to Corner (14).

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3 **(a)** When k is an even number, find an expression, in terms of k, for the length of the spiral

(i) from Corner $(k-1)$ to Corner (k),

.....

(ii) from Corner (k) to Corner $(k+1)$.

.....

(b) **(i)** Using **question 2(b)**, find a formula, in terms of k, for the length, L, of the spiral from (0, 0) to Corner $(k+1)$.

.....

(ii) Use your formula to show that the length of the spiral from (0, 0) to Corner (7) is 16.

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4 A corner on the spiral has co-ordinates (x, y).
Horizontal lengths of the spiral are added to give the total **horizontal** length, H.

x co-ordinate (x)	Horizontal lengths	Total horizontal length (H)
1	1	1
2	1 + 2 + 3	6
3	1 + 2 + 3 + 4 + 5	15
4	1 + 2 + 3 + 4 + 5 + 6 + 7	28
5		

(a) Complete the table.

(b) Find a formula, in terms of x, for the total **horizontal** length, H, of the spiral from (0, 0) to the corner with co-ordinates (x, y).

.....

(c) Write down a formula, in terms of y, for the total **vertical** length, V, of the spiral from (0, 0) to the corner with co-ordinates (x, y).

.....

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(d) k is an even number.

(i) Use your answers to **part (b)** and **part (c)** to show that a formula for the length, L, of the spiral from (0, 0) to Corner (k) with co-ordinates (x, y) is

$$L = 2x(2x - 1).$$

(ii) The spiral has length 1560 from (0, 0) to the corner with co-ordinates (x, y).

Use the formula in **part (i)** to find the value of x.

.....

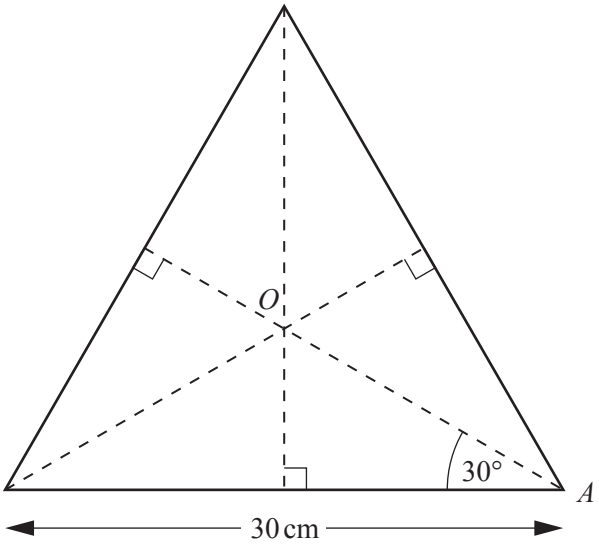
B MODELLING (QUESTIONS 5 to 8)

OPEN BOXES (20 marks)

You are advised to spend no more than 45 minutes on this part.

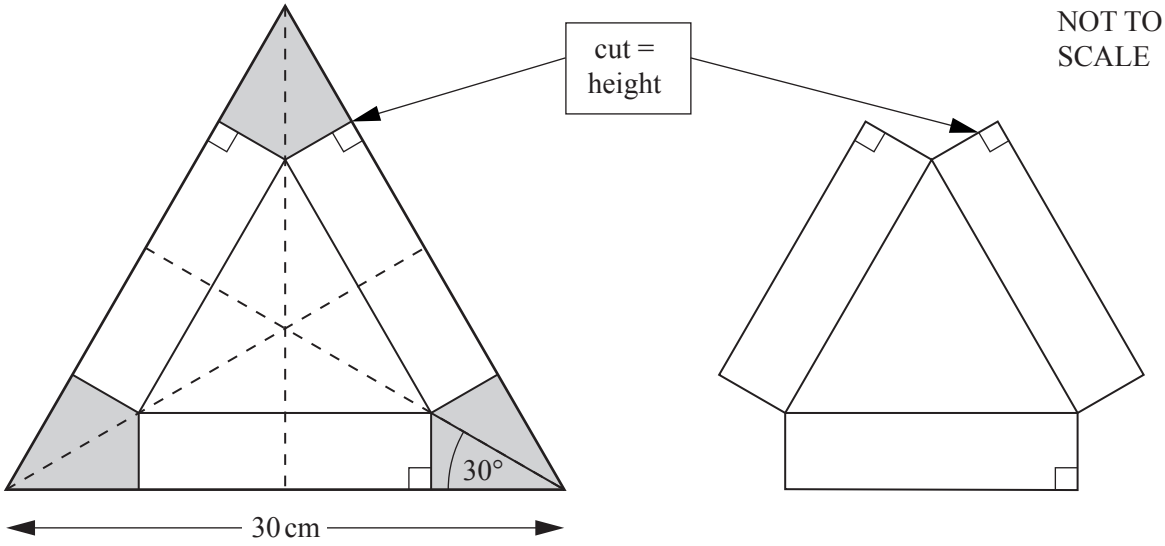
This task looks at maximum volumes when open boxes are made using regular shaped pieces of metal.

Jenny makes open triangular-based boxes from a piece of metal in the shape of an equilateral triangle.



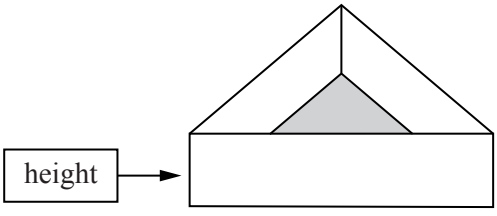
NOT TO SCALE

She cuts equal sized pieces from each corner of the equilateral triangle.
Each cut is at right angles to the side of the shape.



NOT TO SCALE

The sides are folded up to form the vertical sides of an open triangular box.



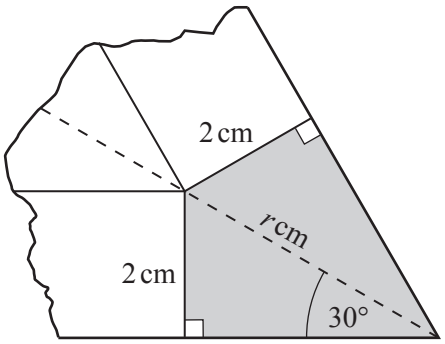
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5 **(a)** The length of one side of the metal equilateral triangle is 30 cm.

Use trigonometry to show that OA is 17.32 cm, correct to 4 significant figures.

(b) Here is an enlargement of one corner cut from the metal triangle.

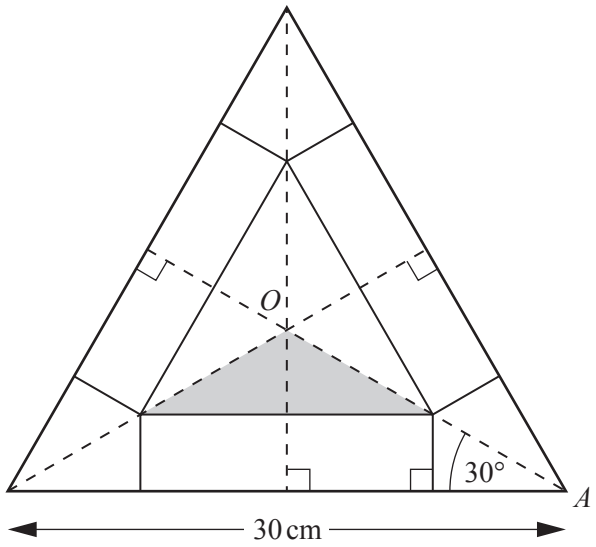
NOT TO SCALE



(i) Jenny makes a cut of 2 cm at right angles to the side of the equilateral triangle.

Show that $r = 4$.

(ii)

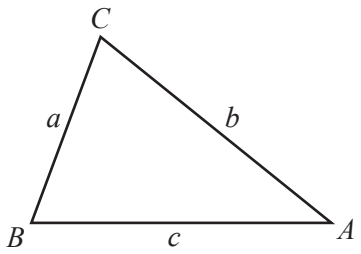


NOT TO SCALE

Using $OA = 17.32\,\text{cm}$ and $r = 4$, find the area of the shaded triangle.

You should use this formula.

$$\text{Area} = \frac{1}{2}bc\sin A$$



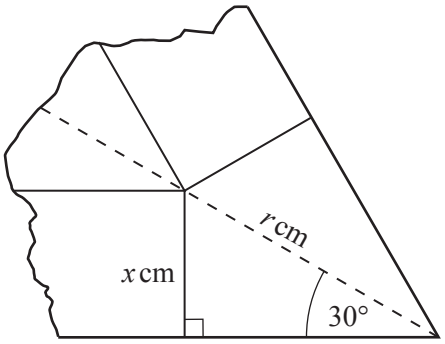
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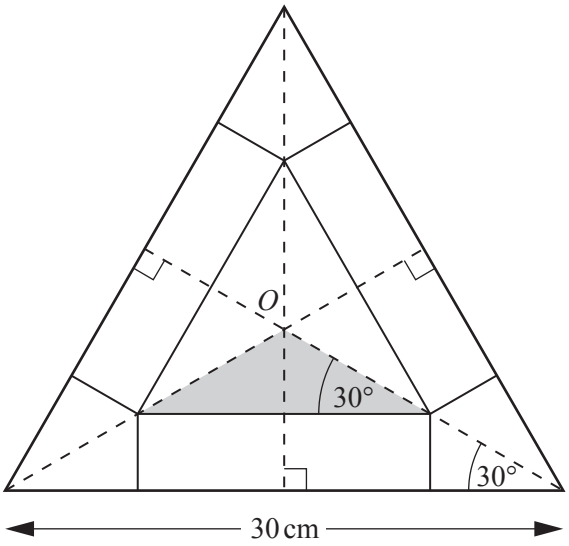
(iii) Jenny cuts the corner shown in **part (b)** from each corner of the equilateral triangle. She folds the sides up to make an open box.

Show that the volume of the box is approximately $461\,\text{cm}^3$.

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6 Jenny wants a model for the volume, $V\,\text{cm}^3$, of the open box made from an equilateral triangle of side 30 cm. She makes a cut of length x cm at right angles to the side of the equilateral triangle.





NOT TO SCALE

(a) (i) Find an expression for r, in terms of x.

.....

(ii) Find an expression, in terms of x, for the area of the shaded isosceles triangle.

.....

(iii) The height of the open box is x cm.

Explain why the model for the volume, $V\,\text{cm}^3$, of the open box is

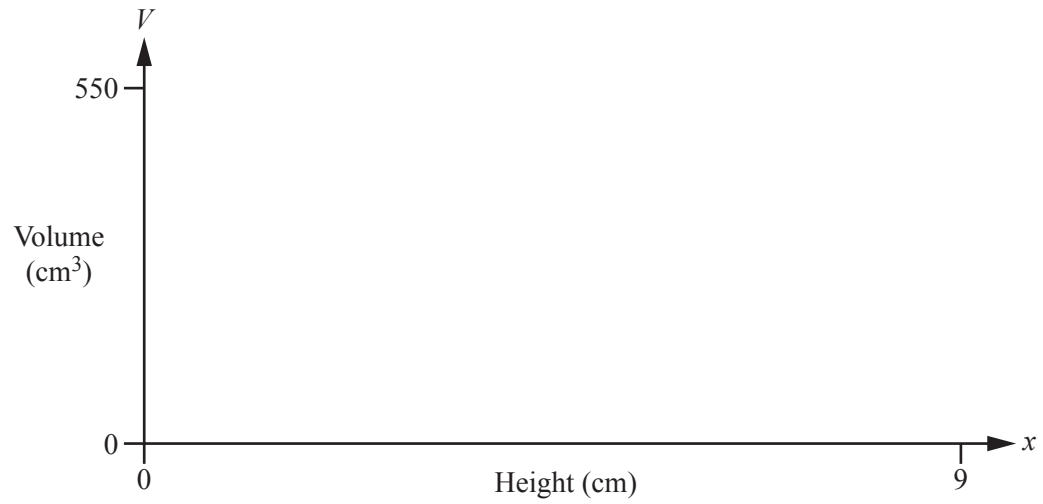
$$V = \frac{3}{2}x \sin 120° \left(\frac{15}{\cos 30°} - \frac{x}{\sin 30°} \right)^2.$$

.....

.....

.....

(iv) Sketch the graph of V against x on the axes below.



(b) For what values of x is the model valid?

.....

(c) Find the possible heights of the open box when $V = 400$.

.....

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7 The equilateral triangle now has side E cm.

(a) Give a reason why the model in **question 6(a)(iii)** becomes

$$V = \frac{3}{2}x \sin 120° \left(\frac{E}{2\cos 30°} - \frac{x}{\sin 30°} \right)^2.$$

.....

.....

(b) Use this model to find the height that gives the greatest volume when $E = 60$.

.....

8 Jenny makes an open square-based box from a square piece of metal of side E cm.

(a) Change the model in **question 7(a)** so that it gives the volume of this box.

.....

(b) The model in **part (a)** simplifies to $V = x(E - 2x)^2$.

Find the relationship between E and x which gives the maximum volume of the box.

.....

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