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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **May/June 2024**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 11).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Answer **both** parts **A** and **B**.

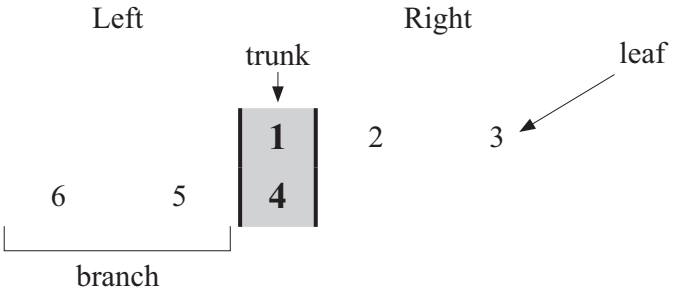
A INVESTIGATION (QUESTIONS 1 to 6)

INTEGER TREES (30 marks)

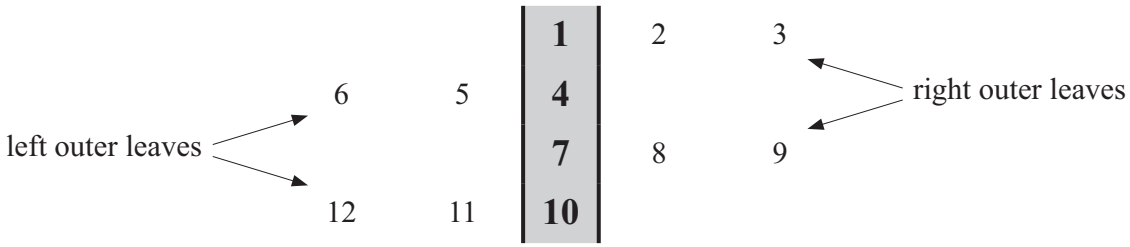
You are advised to spend no more than 50 minutes on this part.

This investigation looks at patterns when integers are arranged in the shape of a tree.
A tree uses consecutive integers starting with 1 at the top.
Each tree has a *trunk*, *branches* and *leaves*.
The integers always increase going away from the trunk, on both the right and the left.
The first branch is always on the right.

This tree uses the integers 1 to 6.
It has 2 branches with 2 leaves on each branch.



This tree uses the integers 1 to 12.
It has 4 branches with 2 leaves on each branch.
The leaves furthest from the trunk are the *outer leaves*.



A is the sum of the left outer leaves.

B is the sum of the right outer leaves.

T is the sum of the integers in the trunk.

Example

For the tree with 4 branches,

$A = 6 + 12 = 18$

$B = 3 + 9 = 12$

$T = 1 + 4 + 7 + 10 = 22$.

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In **Questions 1** to **4**, the number of branches, n, is always even.

1 This tree has 6 branches with 2 leaves on each branch.

		1	2	3
6	5	**4**		
		7	8	9
12	11	**10**		
		13	14	15
18	17	**16**		

Find the value of A, the value of B and the value of T.

$A =$ .....

$B =$ .....

$T =$ ..... [3]

2 **(a)** Complete the table for trees with two leaves on each branch.
Use your answers to **Question 1** to help you.

Number of branches (n)	A	B	T	$A-B$	$A+B-T$
2	6	3	5	3	4
4	18	12	22	6	8
6				9	
8	60				16

[3]

(b) **(i)** Use values from the table to give an example to show that $A-B=1.5n$.

..... [1]

(ii) Write an expression for $A+B-T$ in terms of n.

..... [1]

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3 In this question trees have 3 leaves on each branch.
This tree has 6 branches.

			1	2	3	4
8	7	6	**5**			
			9	10	11	12
16	15	14	**13**			
			17	18	19	20
24	23	22	**21**			

(a) Complete the table.

Number of branches (n)	A	B	T	$A-B$	$A+B-T$
2	8	4	6	4	6
4	24	16	28	8	12
6			66		18
8	80				

[2]

(b) (i) Write an expression for $A-B$ in terms of n.

..... [1]

(ii) Write an expression for $A+B-T$ in terms of n.

..... [1]

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4 The table shows values of $A-B$ and $A+B-T$ for trees with 4 leaves on each branch.

Number of branches (n)	$A-B$	$A+B-T$
2	5	8
4	10	16
6	15	24
8	20	32

(a) Complete the table below.
Use the table above and your answers from **Questions 2(b)(ii)** and **3(b)** to help you.

Number of branches	Number of leaves on each branch (x)	$A-B$	$A+B-T$
n	2	$1.5n$	
n	3		
n	4		

[2]

(b) A tree has an even number of branches, n, with x leaves on each branch.

Complete the expressions for $A-B$ and $A+B-T$ in terms of n and x.

$A-B = \frac{n}{2}$(.....)

$A+B-T =$ ..... [2]

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5 In this question trees have an **odd** number of branches.
Trees have more than one branch.

This tree has 5 branches with 2 leaves on each branch.

		1	2	3
6	5	**4**		
		7	8	9
12	11	**10**		
		13	14	15

(a) Find the value of $A-B$ and the value of $A+B-T$ for this tree.

$A-B =$ .....

$A+B-T =$ ..... [2]

(b) In **Question 4(b)** you found expressions for trees with an even number of branches.

Do your expressions in **Question 4(b)** give correct values for this tree?
Show how you decide.

$A-B$ .....

.....

$A+B-T$ .....

..... [2]

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(c) The table shows some values of $A-B$ and $A+B-T$ for trees with odd numbers of branches.

Number of leaves on each branch (x)	Number of branches (n)	$A-B$	$A+B-T$
2	3	-6	6
	5		
	7	-12	14
3	3	-8	9
	5	-12	15
	7	-16	21
4	3	-10	12
	5	-15	20
	7	-20	28

A tree has an odd number of branches, n, with x leaves on each branch.

Find expressions for $A-B$ and $A+B-T$ in terms of n and x.

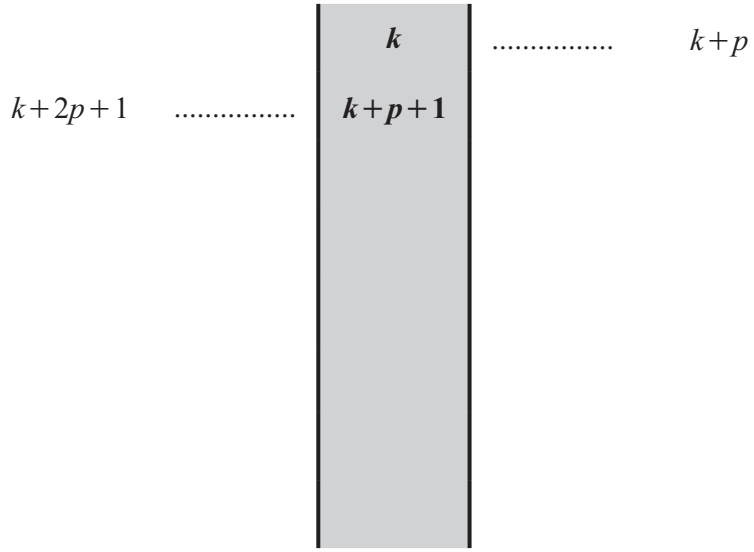
$A-B =$ .....

$A+B-T =$ ..... [3]

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6 In this question a tree can have either an even number or an odd number of branches.
A tree can now start with any integer, k.
A tree has p leaves on each branch.

This is the start of a tree.
It shows expressions for the integers on the trunk and the outer leaves only.



(a) An expression for the outer leaf on the second branch is $k + 2p + 1$.
Show that this statement is correct.

[1]

(b) Find expressions for the third integer on the trunk and the outer leaf on the third branch.
Write your expressions, in their simplest forms, on the tree.

[2]

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(c) (i) Find expressions for $A + B - T$ when a tree has 2 branches, 3 branches and 4 branches. Give each expression in its simplest form.

.....

.....

..... [3]

(ii) Find an expression for $A + B - T$ when a tree has n branches.

..... [1]

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B MODELLING (QUESTIONS 7 to 11)

CARBON FOOTPRINT OF AIR TRAVEL (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at modelling the personal carbon footprint of someone travelling by air.

Carbon footprint is the amount of carbon dioxide that an activity puts into the atmosphere.

Nikita wants to estimate her carbon footprint when she travels by air.
Her carbon footprint depends on the amount of fuel used by the aircraft, the length of the journey and the number of people on the aircraft.

Average fuel use is the amount of fuel, in kilograms, used for each kilometre travelled.

7 An aircraft uses T kg of fuel for a journey of length L km.
A formula for the average fuel use, F kg/km, is

$$F = \frac{T}{L} .$$

An aircraft makes a journey of 850 km and uses 4760 kg of fuel.

Find the average fuel use for this journey.

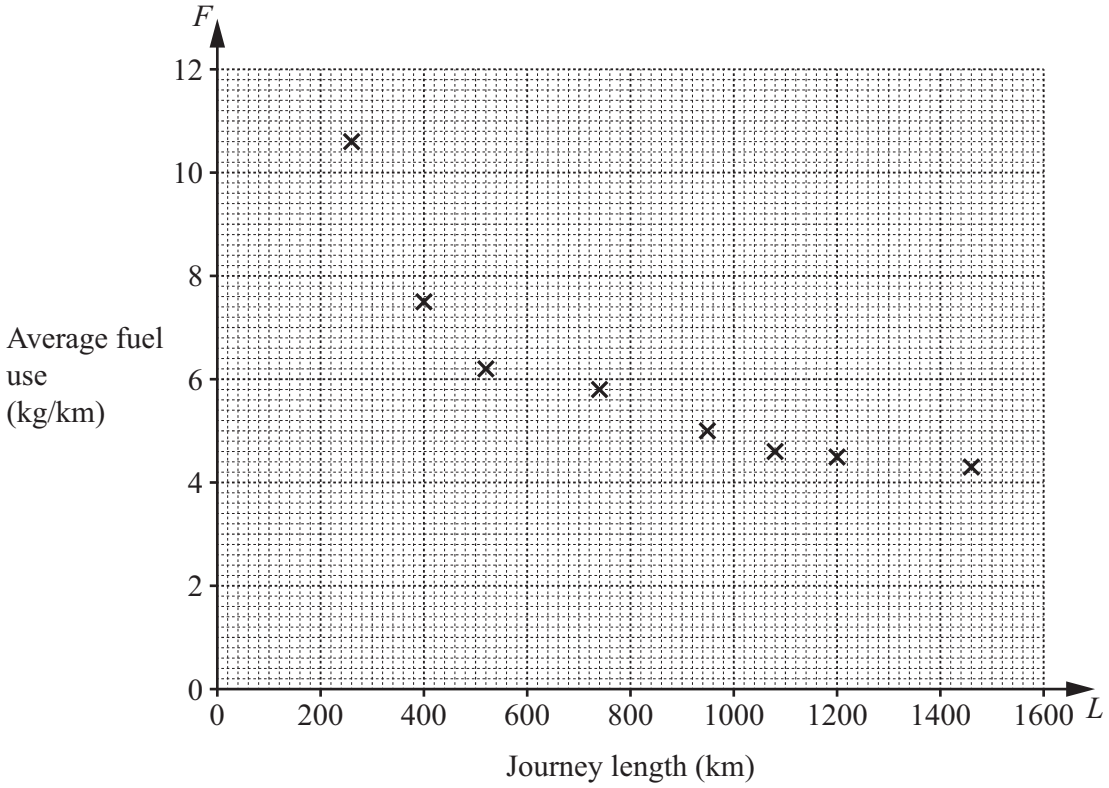
..... [2]

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8 Nikita finds this data for an aircraft making journeys of different lengths.

Journey length (L km)	260	400	520	740	950	1080	1200	1460
Average fuel use (F kg/km)	10.6	7.5	6.2	5.8	5.0	4.6	4.5	4.3

She plots the data on a grid.



(a) What happens to the average fuel use as the journey length increases?

..... [1]

(b) Nikita thinks a model for the data is $F = a + \frac{b}{L}$ where a and b are constants.

(i) Use the values of F from the table for $L = 400$ and $L = 1200$ to write down two equations in terms of a and b.

.....

..... [2]

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(ii) Solve your equations to find the value of a and the value of b and write down the model.

$a =$ .....

$b =$ .....

Model ..... [3]

9 The model gives these values of F, correct to one decimal place.

Journey length (L km)	200	400	600	800	1000	1200	1400	1600
Average fuel use (F kg/km)	12.0	7.5	6.0	5.3	4.8	4.5	4.3	4.1

(a) Plot these values on the grid in **Question 8** and draw the graph of the model.

[2]

(b) How well does the model fit Nikita's data?

..... [1]

(c) The aircraft makes a journey of 10 000 km.

Find the average fuel use that the model gives.

..... [2]

(d) Is your answer to **part (c)** valid?
Give a reason.

..... [1]

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10 The amount of carbon dioxide given out by an aircraft depends on the amount of fuel used.

A kilogram of aircraft fuel makes 3200 grams of carbon dioxide.

(a) The aircraft makes a journey of 800 km and uses 5.3 kg of fuel per km.

Work out the amount of carbon dioxide, in grams, that this aircraft gives out for each kilometre of the journey.

..... [2]

(b) The personal carbon footprint, C, for an aircraft passenger is the amount of carbon dioxide, in grams, that the aircraft gives out per kilometre divided by the number of passengers.

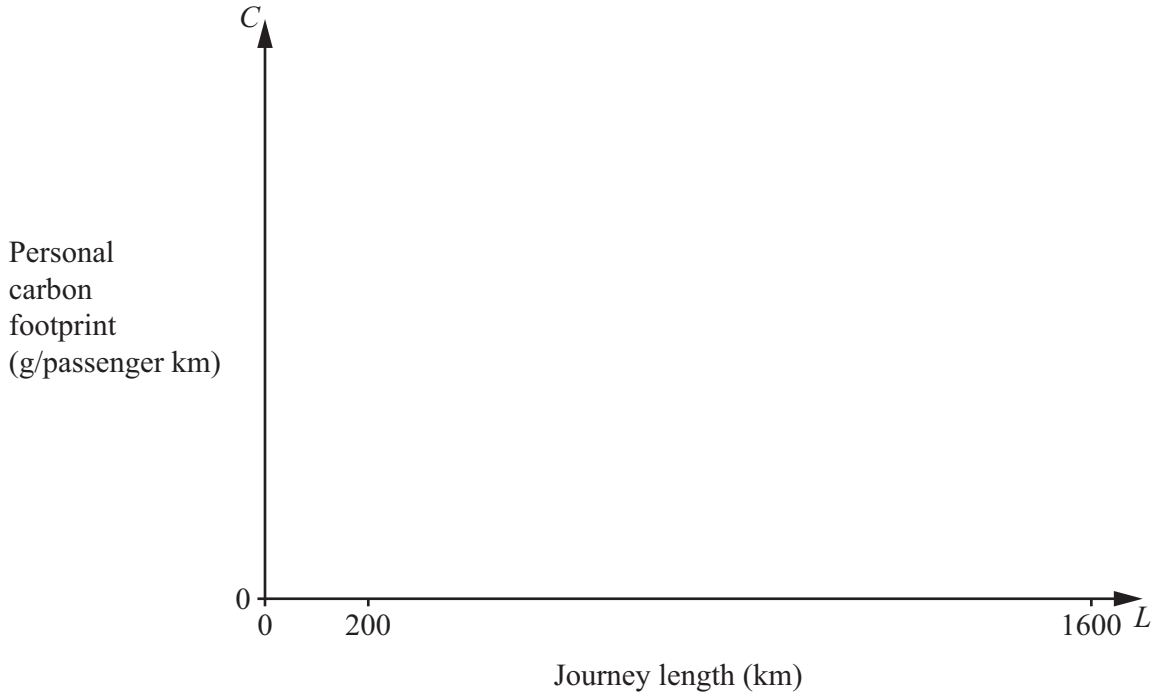
Nikita assumes that this aircraft has 200 passengers.

(i) Use your model for F in **Question 8(b)(ii)** to show that a model for C for a passenger on this aircraft with a journey length L is

$$C = 48 + \frac{28800}{L}.$$

[2]

(ii) Sketch the graph of this model for $200 \leqslant L \leqslant 1600$.



[3]

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(c) Nikita travels 900 km on this aircraft.

Use the model in **part (b)(i)** to calculate the amount of carbon dioxide, in kg, that the aircraft gives out for each passenger for the whole journey.

..... [3]

11 Nikita wants to compare her carbon footprint for air travel and train travel.

(a) She finds that the personal carbon footprint, C, for train travel is 40 g/km for each passenger for any length of journey.

Sketch the graph of the model for C for train travel on the axes in **Question 10(b)(ii)**. [2]

(b) One train company claims:

When you travel by train, your carbon footprint will never be more than 30% of your carbon footprint for the same journey by air.

For what lengths of journey is this claim correct?

..... [4]

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