Cambridge Assessment International Education

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2569757395

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 Investigation (Core) **May/June 2024**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

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Answer **all** the questions.

INVESTIGATION SUMS OF NUMBERS

This investigation looks at the sum of the positive integers, $1+2+3+\cdots$.

It also looks at the sum of their cubes, $1^3+2^3+3^3+\cdots$.

1 **(a)** Calculate $1+2+3+4+5+6+7+8$.

..... [1]

(b) The numbers can also be added using this method.

Complete this method for the sum of the numbers in **part (a)**.

Step 1	Write down the first half of the numbers in a row.	1 2 3 4
Step 2	Write down the second half of the numbers underneath the first half but in reverse order.	8 7 6 5
Step 3	Add each column of two numbers to make a third row.	9 9 9 9
Step 4	Find the total of the numbers in the third row by writing the calculation as a multiplication.	$4\times 9=$ .....

[1]

2 Use the method in **Question 1(b)** to calculate the sum of the first six positive integers, $1+2+3+4+5+6$.

[4]

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3 Complete the method to show that the sum of the first 30 positive integers is 465.

So $1+2+3+\cdots+28+29+30=465$.

1	2	3	⋯	15
.....	.....	.....	⋯	16
.....	.....	.....	⋯	.....
	..... ×	=	465	

[3]

4 Find the sum of the first 160 positive integers, $1+2+3+\cdots+160$.

..... [3]

5 Use **Questions 1**, **2**, **3** and **4** to help you complete the table.

	Number of positive integers starting at 1	Multiplication	Sum
	4	2×5	10
Question 2	6		
Question 1	8	4×9	
	10		
Question 3	30		465
	50	25×51	1275
	128	64×129	
Question 4	160		
	204		20910

[4]

6 $1 + 2 + 3 + \cdots + n$ has n positive integers and its sum is T.

Find a formula for T in terms of n.

..... [3]

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7 In the table in **Question 5** the number of integers is always even.

Does your formula in **Question 6** give the correct total when $n = 7$?

[3]

8 **(a)** Complete the table.

Sum of first n positive integers		Sum of first n cube numbers		
Calculation	Sum (T)		Calculation	Sum (S)
$1+2$	3	1^3+2^3	$1+8$	9
$1+2+3$		$1^3+2^3+3^3$		36
$1+2+3+4$			$1+8+27+64$	
	15	$1^3+2^3+3^3+4^3+5^3$		225

[4]

(b) **(i)** Write down the mathematical name for the positive integers in the Sum (S) column.

..... [1]

(ii) Write a formula for S in terms of T.

..... [1]

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(c) Use your answers to **Question 6** and **Question 8(b)** to find the sum of the cubes of the first 40 positive integers.

$$1^3 + 2^3 + 3^3 + \cdots + 40^3$$

..... [4]

(d) $1^3 + 2^3 + 3^3 + \cdots + n^3 = 396\,900$

Find how many cube numbers there are in this sum.

..... [4]

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