Cambridge Assessment International Education

Cambridge IGCSE™

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2062213110

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 Investigation (Core) **May/June 2023**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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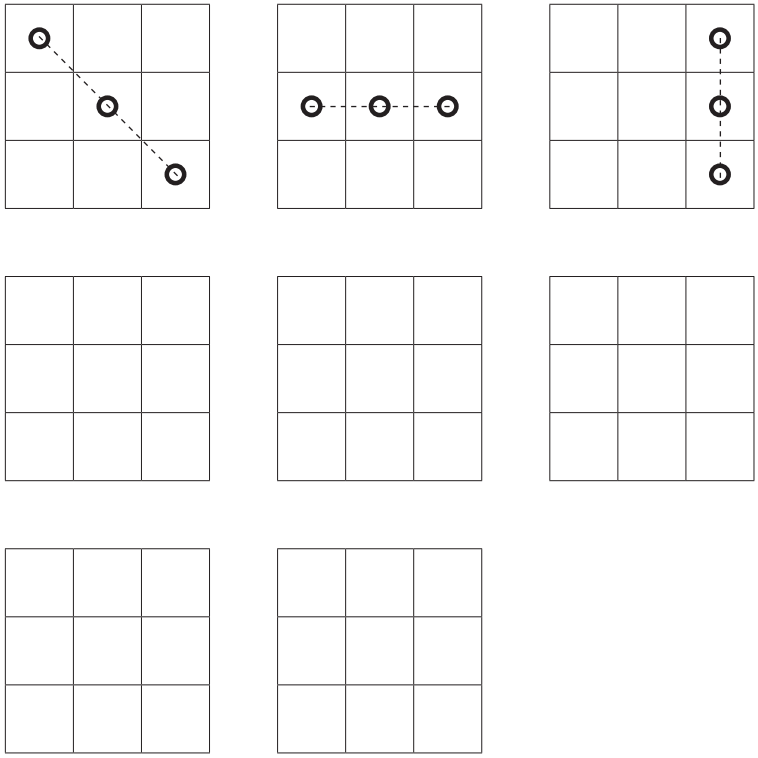
Answer **all** the questions.

INVESTIGATION **WINNING LINES**

This investigation looks at the number of winning lines on a grid.

1 In a game you win by making a straight line of three Os on a 3 by 3 grid.
There are 8 winning lines of three Os on a 3 by 3 grid.

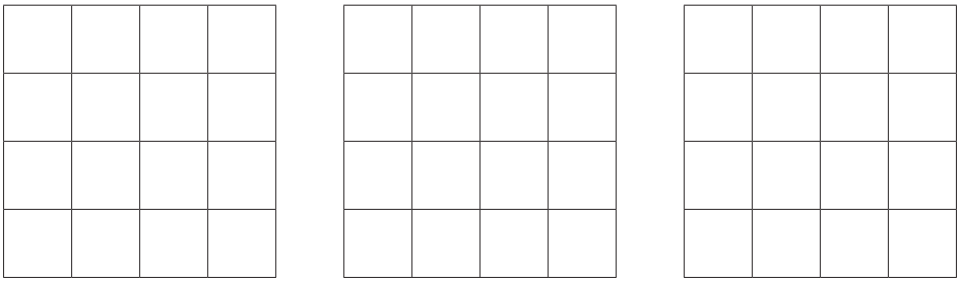
Show each winning line of three Os on the grids.
Three of the winning lines have been shown for you.



[2]

2 **(a)** Another grid is 4 by 4.
You now need four **O**s in a line to win.

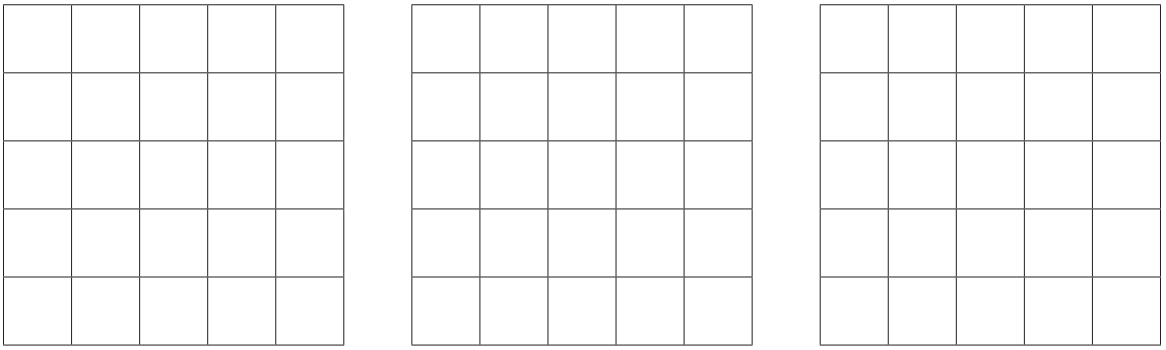
Find the number of winning lines on a 4 by 4 grid.
You may use the grids to help you.



..... [2]

(b) Another grid is 5 by 5.
You now need five **O**s in a line to win.

Find the number of winning lines on a 5 by 5 grid.
You may use the grids to help you.



..... [2]

(c) Another grid is 6 by 6.
You now need six **O**s in a line to win.

Write down the number of winning lines on a 6 by 6 grid.

..... [1]

3 **(a)** Complete this table using your answers to **Question 1** and **Question 2** and any patterns you notice.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
3 by 3				8
4 by 4				
5 by 5				
6 by 6				
7 by 7				
20 by 20				

[3]

(b) A grid is n by n.
You need n Os in a line to win.

Find an expression, in terms of n, for the number of winning lines.

..... [2]

(c) Jibreel draws a very large square grid.
He thinks there will be 583 winning lines of Os on his grid.

Give a reason why he is wrong.

..... [1]

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(d) Harriet draws a square grid with 324 squares.

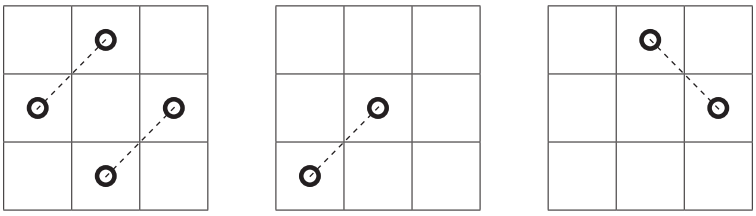
Find the number of winning lines of O s on this grid.

..... [3]

4 A grid is n by n.
In a different game a winning line is one O less than n.
To make a line, the Os must be in squares that are next to each other.

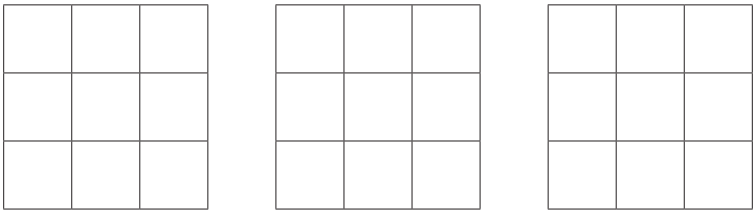
(a) In a 3 by 3 grid you need two Os in a line to win.

These diagrams show some of the diagonal winning lines.



Complete the table to find the number of winning lines with two Os.
You may use the grids below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
3 by 3				20

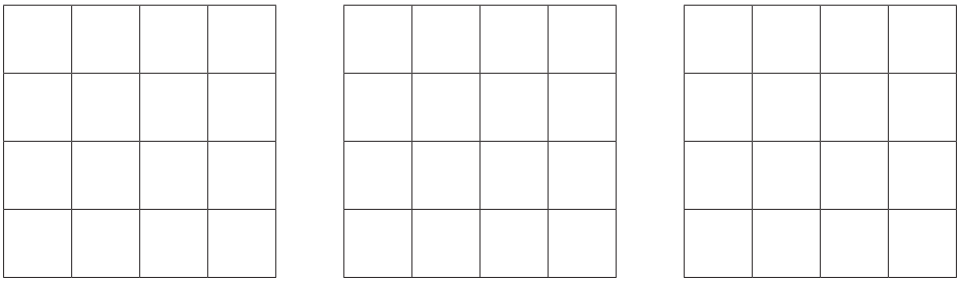


[2]

(b) In a 4 by 4 grid you need three Os in a line to win.

Complete the table to find the number of winning lines with three Os.
You may use the grids below the table to help you.

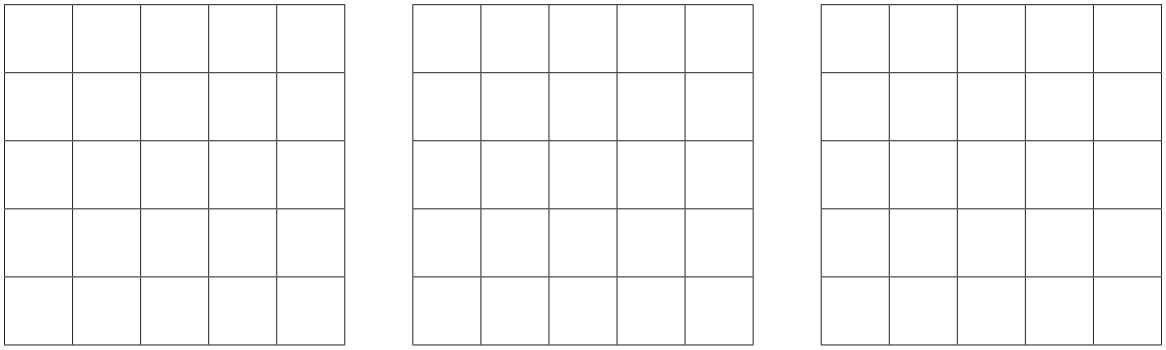
Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
4 by 4				



[2]

(c) Copy your results from **part (a)** and **part (b)** into this table.
Complete the table.
You may use the grids below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
3 by 3				
4 by 4				
5 by 5				
n by *n*				



[4]

(d) In an *n* by *n* grid you need $(n - 1)$ **O**s in a line to win.
n must be at least 3.

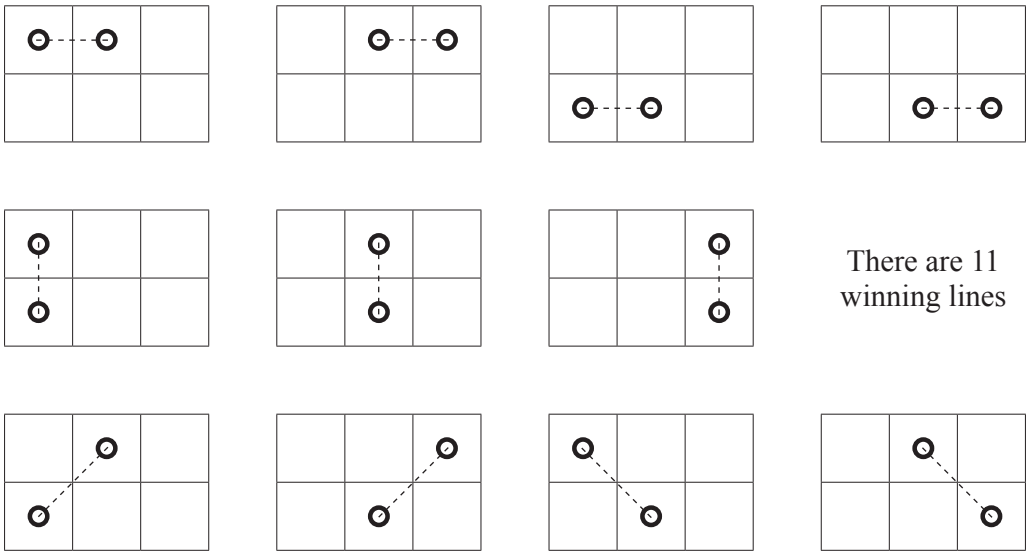
In one grid the total number of winning lines is a square number less than 50.

Find the grid size.

..... [4]

5 A rectangular grid has height 2 and width at least 2.
You need two Os in a line to win.

These diagrams show all the winning lines with two Os on a 2 by 3 grid.



(a) Complete the table for the number of winning lines with two Os.
You may use the grid below the table to help you.

Size of grid	Number of winning lines			
	Horizontal	Vertical	Diagonal	Total
2 by 2	2	2	2	6
2 by 3	4	3	4	11
2 by 4				
2 by 5				
2 by w				

[5]

(b) A 2 by w grid has 111 winning lines with two O s.

Find the width of the grid.

..... [3]

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