Cambridge Assessment International Education

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1882827971

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **May/June 2022**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 7) and part **B** (Questions 8 to 11).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 7)

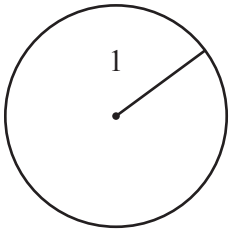
CIRCLES AND REGIONS (30 marks)

You are advised to spend no more than 50 minutes on this part.

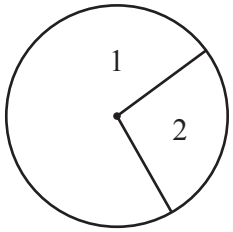
This is an investigation into the number of regions formed by drawing lines on a circle.

1 Radii

The diagrams show the number of regions inside a circle when 1 radius and 2 radii are drawn. The regions inside the circle are numbered.



1 radius



2 radii

(a) Complete the table.

Number of radii	Number of regions
1	1
2	2
3	
4	
5	
6	

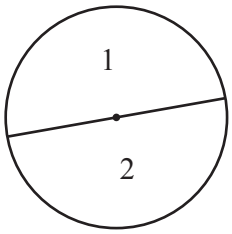
[1]

(b) Write a formula, in terms of n, for the number of regions, R, when there are n radii.

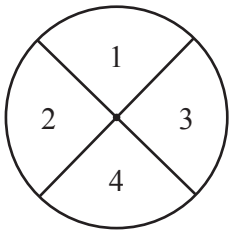
..... [1]

2 Diameters

The diagrams show the number of regions inside a circle when 1 diameter and 2 diameters are drawn.



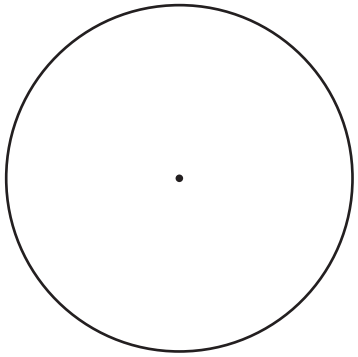
1 diameter



2 diameters

(a) Complete the table for 3, 4 and 5 diameters.
You may use the empty circle to help you.

Number of diameters	Number of regions
1	2
2	4
3	
4	
5	



[2]

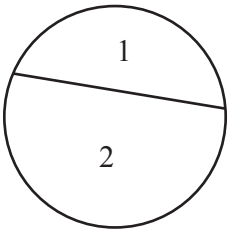
(b) Write a formula, in terms of n, for the number of regions, R, when there are n diameters.

..... [1]

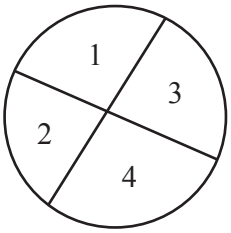
3 Chords

In this investigation:
- each chord must cut every other chord
- only two chords may intersect at any point.

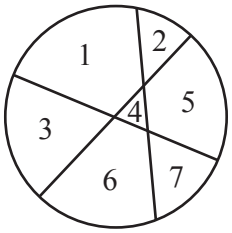
The diagrams show the number of regions inside a circle when 1 chord, 2 chords and 3 chords are drawn.



1 chord

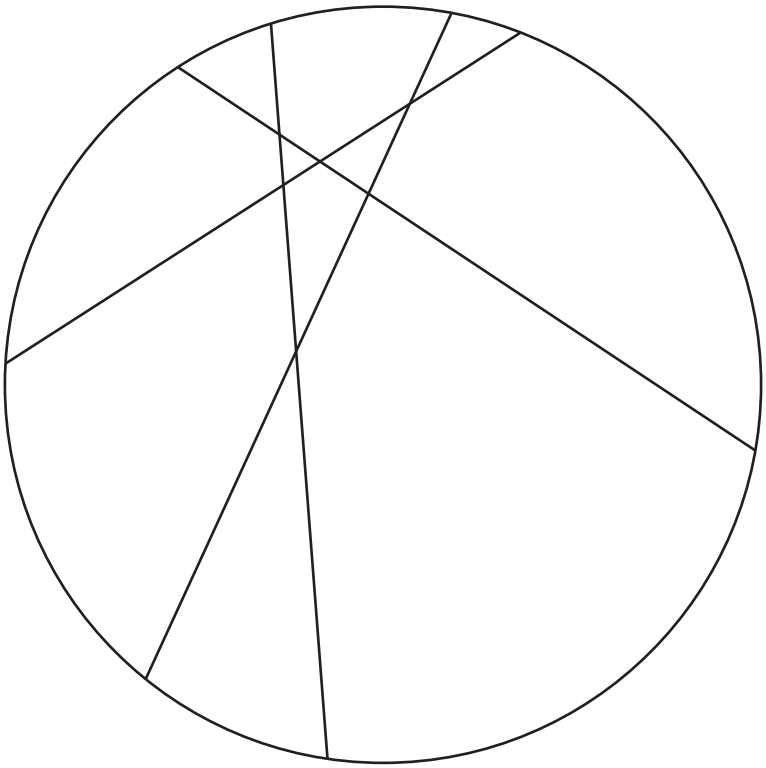


2 chords



3 chords

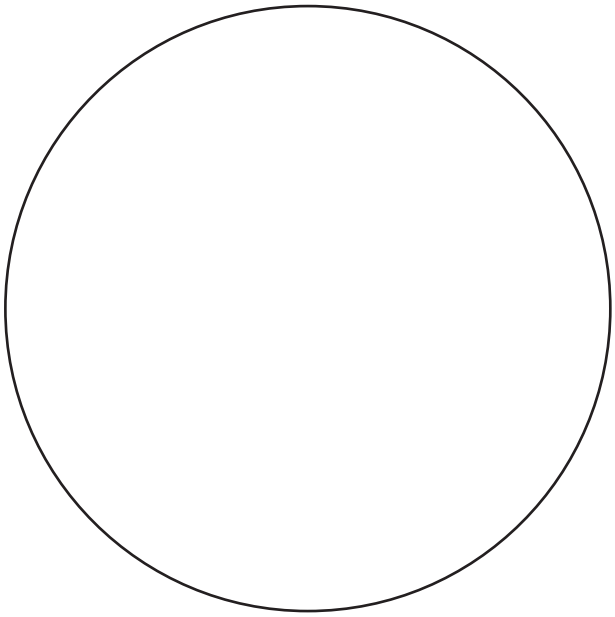
(a) Count the number of regions in the circle when 4 chords are drawn.



..... [1]

(b) Complete this table.
You may use the empty circle to help you.

Number of chords	Number of regions
1	2
2	4
3	7
4	
5	
6	22



[1]

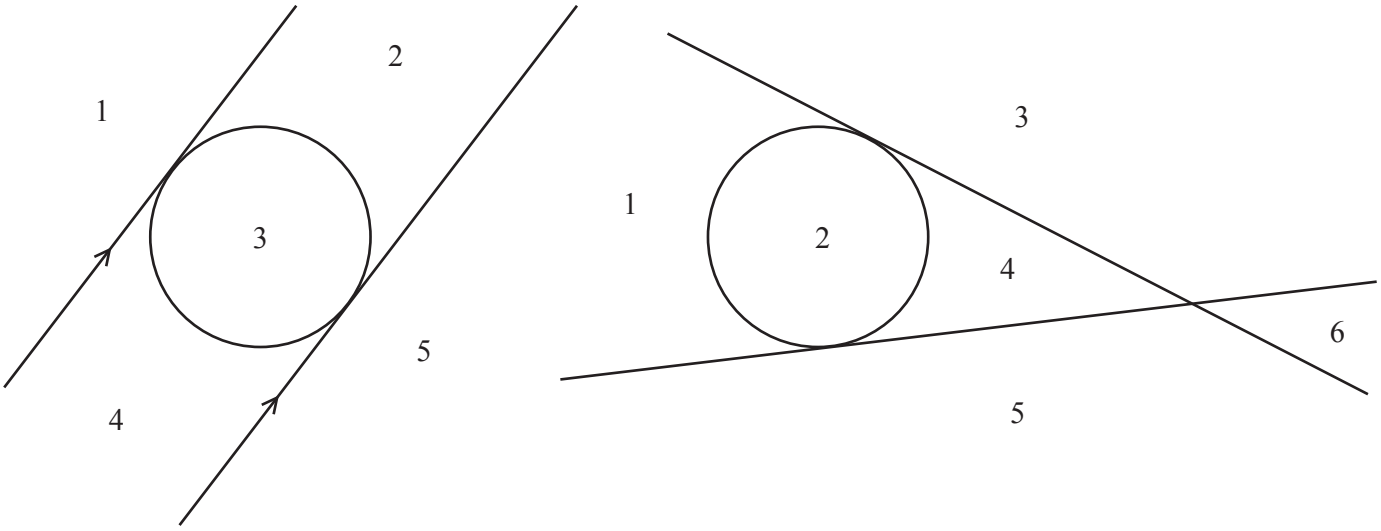
(c) Find a formula, in terms of n, for the number of regions, R, when there are n intersecting chords.

..... [4]

4 Tangents

A region can be inside or outside the circle when the lines are tangents.

These two diagrams both show a circle with 2 tangents and the regions numbered.
The maximum number of regions for a circle with 2 tangents is 6.

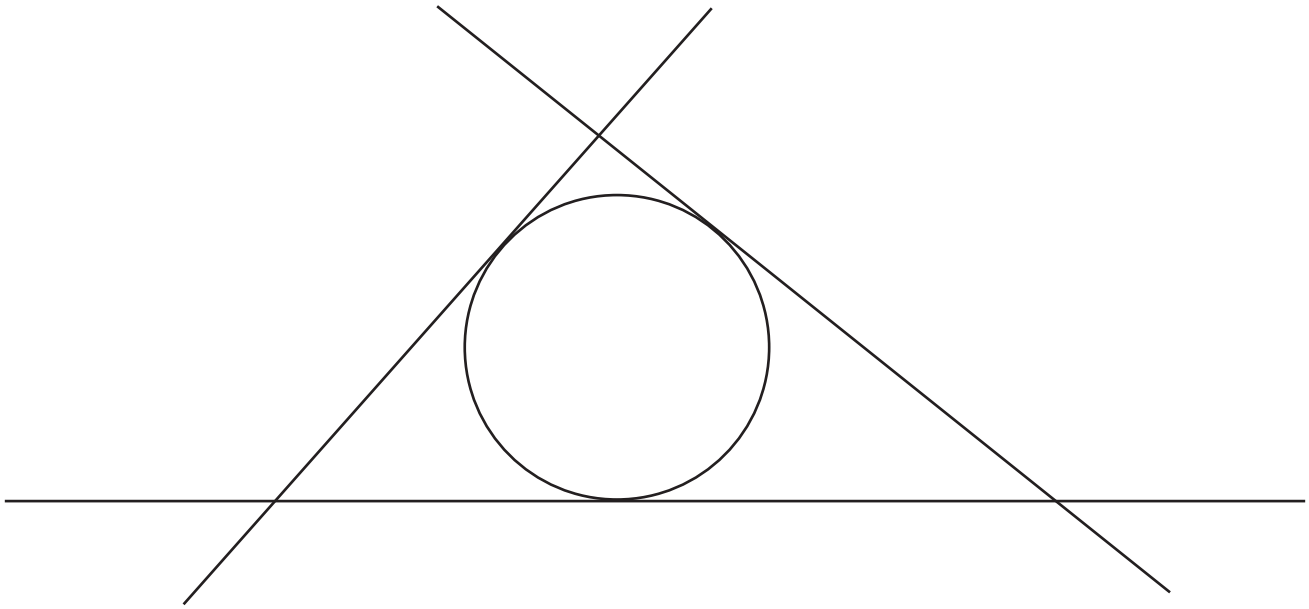


(a) Give a reason why the first diagram does not have the maximum number of regions with 2 tangents.

.....

..... [1]

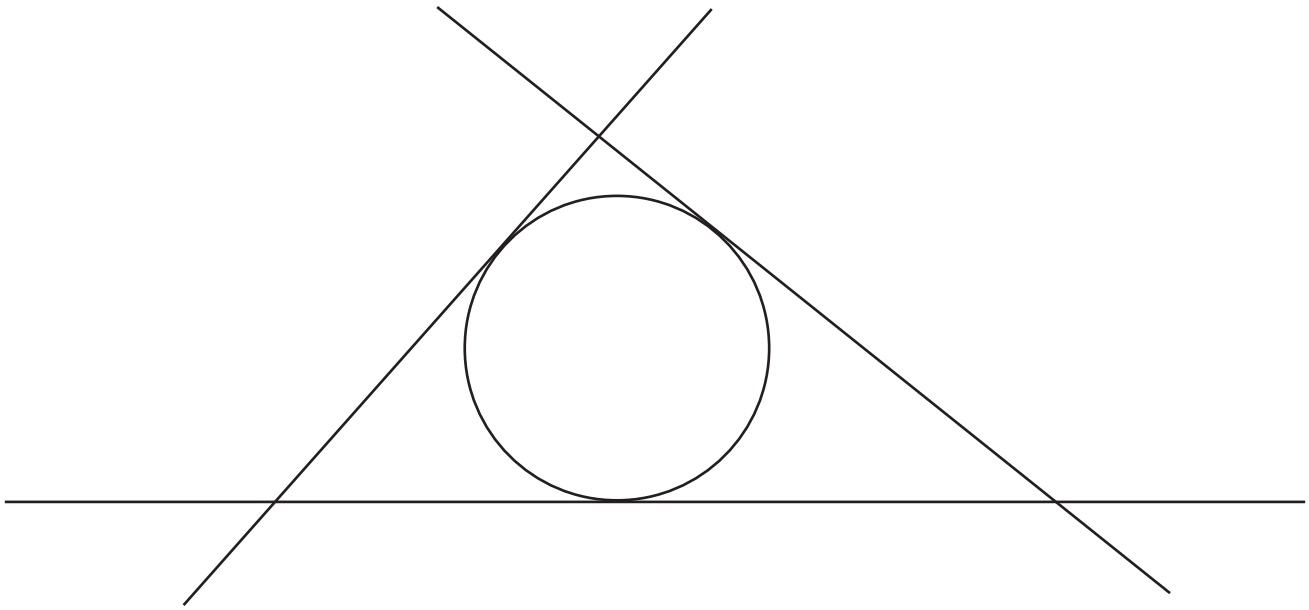
(b) Use this diagram to find the maximum number of regions when there are 3 tangents.



..... [1]

(c) Draw a fourth tangent on the diagram below to find the maximum number of regions.

Complete the table.



Number of tangents	Maximum number of regions
1	3
2	6
3	
4	
5	21

[2]

(d) This is a formula for the maximum number of regions, R, when there are n tangents.

$$R = \frac{1}{2}n^2 + bn + 1$$

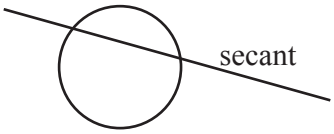
Find the value of b.

..... [2]

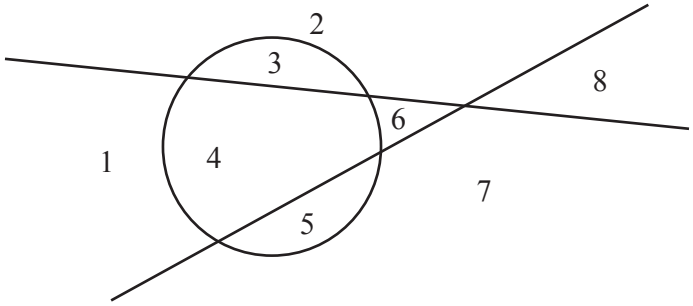
5 Secants

A secant is a straight line that intersects a circle at two points and extends outside the circle.

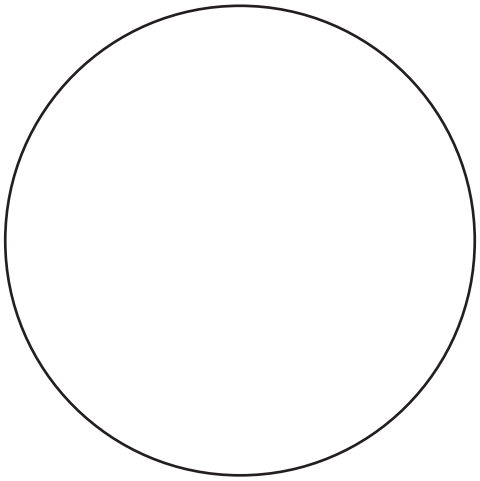
In this investigation:
- each secant must cut every other secant
- only 2 secants may intersect at any point
- secants must not intersect on the circumference of the circle.



The diagram shows the number of regions with 2 secants drawn on a circle.



(a) Find the number of regions when there are 3 secants.
Complete the table.



Number of secants	Number of regions
1	4
2	8
3	
4	19
5	26

[2]

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(b) This is a formula for the number of regions, R, when there are n secants.

$$R = \frac{1}{2}n^2 + bn + c$$

Find the value of b and the value of c.

$b =$ .....

$c =$ ..... [4]

6 Tangents are drawn on a circle to give the maximum number of regions.
There are 1225 regions.

Find the number of tangents.

..... [3]

7 There are two circles.
The first circle has **chords** drawn on it.
The second circle has **secants** drawn on it.

The number of chords on the first circle is the same as the number of secants on the second circle.

Each circle has the maximum number of regions.
One circle has 60 more regions than the other.

(a) Find the number of straight lines on each diagram.

..... [2]

(b) Find the larger number of regions.

..... [2]

B MODELLING (QUESTIONS 8 TO 11)

AIRPORT RUNWAY (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at the factors that affect the decision to build a second runway at an airport.

The factors are:
- the number of seconds a plane waits over the airport before it can start to land
- the number of seconds it then takes to land.

A plane cannot begin to land until the runway is free but must wait over the airport.
As soon as the runway is free the plane begins to land.

The number of seconds between one plane and the next plane arriving over the airport is called the *inter-arrival time*.
The number of seconds from when a plane begins to land and when it stops is called the *landing time*.

8 This table shows the data for the first 5 planes arriving at the airport for the 1080 seconds after 8 am on day 1.

For example: Plane B arrives 120 seconds after plane A. Plane A has not ended its landing. Plane B starts its landing 180 seconds after 8 am, as soon as plane A has ended its landing.

Plane	Inter-arrival time (seconds)	Arrival over airport (seconds after 8 am)	Start of landing (seconds after 8 am)	Landing time (seconds)	End of landing (seconds after 8 am)	Seconds waiting to land
A		30	30	150	180	0
B	120	150	180	110	290	30
C	360	510	510	100	610	0
D	25	535	610	280		75
E	60			44	934	

(a) Complete the table.

[5]

(b) A plane uses the runway for the whole of its landing time.

Calculate the total time that the runway was **not** used during these 1080 seconds.

..... [2]

9 To decide if a second runway should be built, more data is needed.
The table shows the data for 180 planes.
All values are given correct to the nearest integer.

(a) Complete the table.

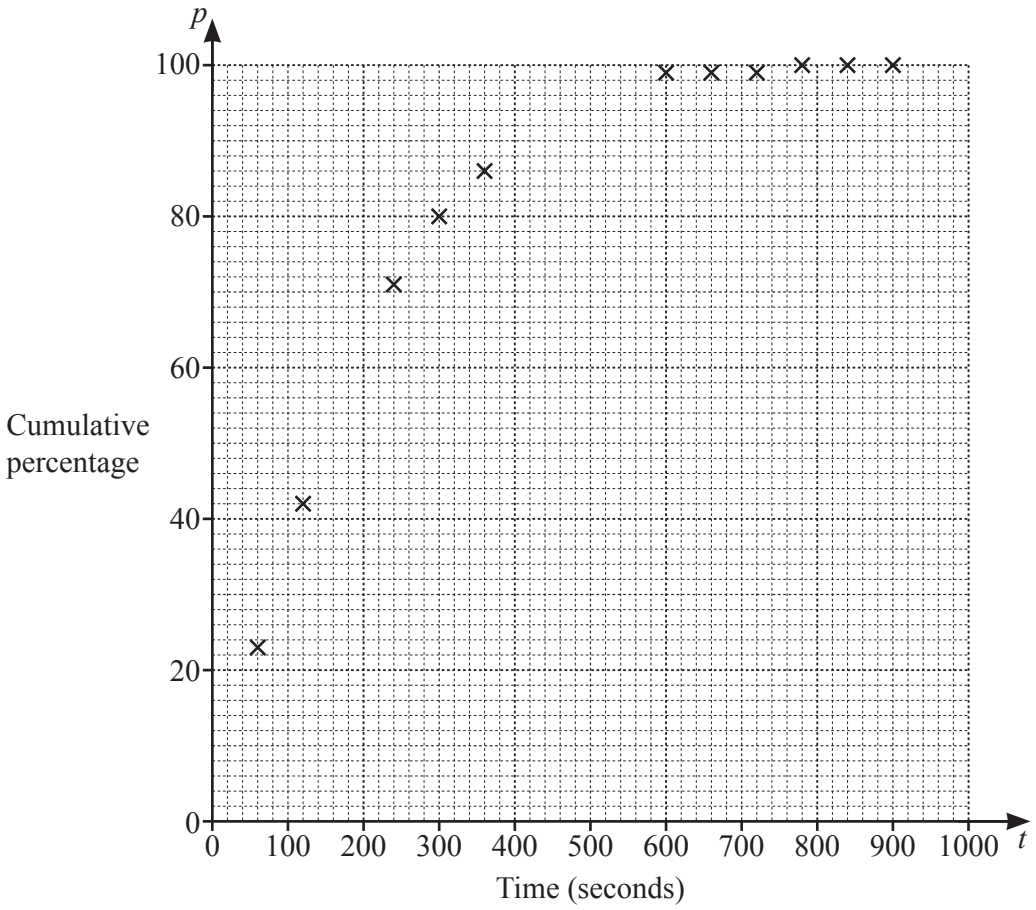
Inter-arrival time (t seconds)	Number of planes	Percentage of planes
$0 < t \leqslant 60$	42	23
$60 < t \leqslant 120$	34	19
$120 < t \leqslant 180$	29	
$180 < t \leqslant 240$	23	
$240 < t \leqslant 300$	16	
$300 < t \leqslant 360$	11	6
$360 < t \leqslant 420$	11	6
$420 < t \leqslant 480$	7	4
$480 < t \leqslant 540$	4	2
$540 < t \leqslant 600$	2	1
$600 < t \leqslant 660$	0	0
$660 < t \leqslant 720$	0	0
$720 < t \leqslant 780$	1	1
$780 < t \leqslant 840$	0	0
$840 < t \leqslant 900$	0	0

Inter-arrival time (t seconds)	Cumulative percentage of planes (p)
$t \leqslant 60$	23
$t \leqslant 120$	42
$t \leqslant 180$	
$t \leqslant 240$	71
$t \leqslant 300$	80
$t \leqslant 360$	86
$t \leqslant 420$	
$t \leqslant 480$	
$t \leqslant 540$	
$t \leqslant 600$	99
$t \leqslant 660$	99
$t \leqslant 720$	99
$t \leqslant 780$	100
$t \leqslant 840$	100
$t \leqslant 900$	100

[3]

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(b) On the grid below, complete the cumulative percentage curve.



[2]

(c) Use the graph to estimate the inter-arrival time for a cumulative percentage of 50.

..... [1]

10 This is a model for the cumulative percentage, p, in terms of the inter-arrival time, t.

$$p = k(t-a)^{\frac{1}{3}} \quad \text{where } a \text{ and } k \text{ are constants}$$

(a) Use the points (60, 23) and (360, 86) to write down two equations in terms of a and k.

.....

..... [1]

(b) Show that $52.3 = \dfrac{360-a}{60-a}$, where 52.3 is correct to 1 decimal place.

[2]

(c) Solve the equation in **part (b)** to show that $a = 54$, correct to the nearest integer.

[3]

(d) Find the value of k, correct to the nearest integer, and complete the model.

$p = \ldots\ldots\ldots\ldots (t - \ldots\ldots\ldots\ldots)^{\frac{1}{3}}$

[2]

(e) Use the model to find the inter-arrival time for a cumulative percentage of 50.

..... [3]

(f) Sketch the model on the axes in **Question 9(b)**. [2]

(g) Comment on the validity of this model.

.....

..... [1]

Question 11 is printed on the next page.

11 **(a)** Use the model to find the percentage of planes that arrived over the airport within 120 seconds of the previous plane.

..... [1]

(b) The table shows information about landing times for the 180 planes.
All values are given correct to the nearest integer.

Landing time (t seconds)	Number of planes	Percentage of planes
$0 < t \leqslant 60$	2	1
$60 < t \leqslant 120$	7	4
$120 < t \leqslant 180$	11	6
$180 < t \leqslant 240$	15	8
$240 < t \leqslant 300$	20	11
$300 < t \leqslant 360$	34	19
$360 < t \leqslant 420$	42	23
$420 < t \leqslant 480$	30	17
$480 < t \leqslant 540$	18	10
$540 < t \leqslant 600$	1	1

Landing time (t seconds)	Cumulative percentage of planes (p)
$t \leqslant 60$	1
$t \leqslant 120$	5
$t \leqslant 180$	11
$t \leqslant 240$	19
$t \leqslant 300$	30
$t \leqslant 360$	49
$t \leqslant 420$	72
$t \leqslant 480$	89
$t \leqslant 540$	99
$t \leqslant 600$	100

Find the percentage of planes where the landing time is more than 120 seconds.

..... [1]

(c) Based on your answers to **part (a)** and **part (b)**, should a second runway be built at the airport? Give a reason for your answer.

.....

..... [1]

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