Cambridge Assessment International Education

Cambridge IGCSE™

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7326343854

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **May/June 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 6)

ROLLING SQUARE (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at the path of a point on a square as it rolls along the *x*-axis.

A square of side 1 cm rolls along the *x*-axis.
One roll is a turn of 90° clockwise about its bottom right corner.

Diagram 1
Starting position

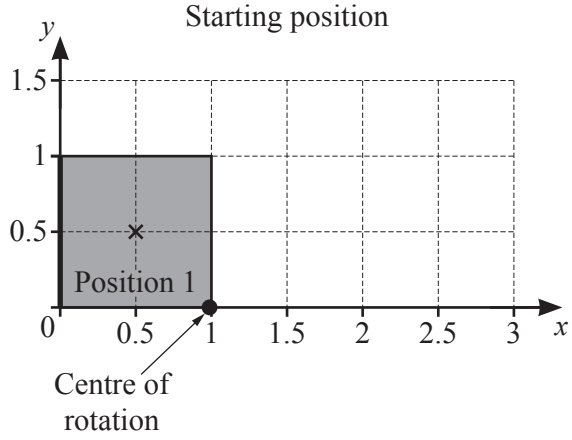


Diagram 2
After one roll

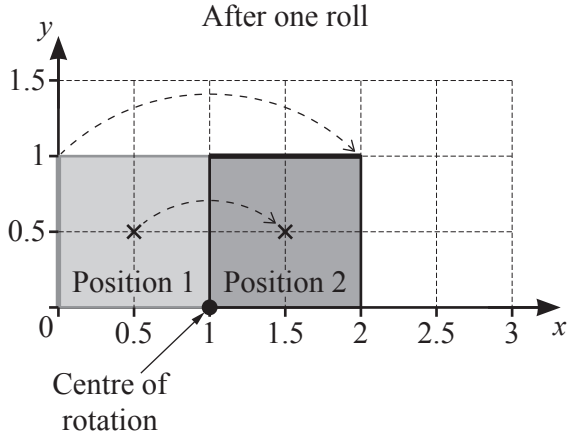
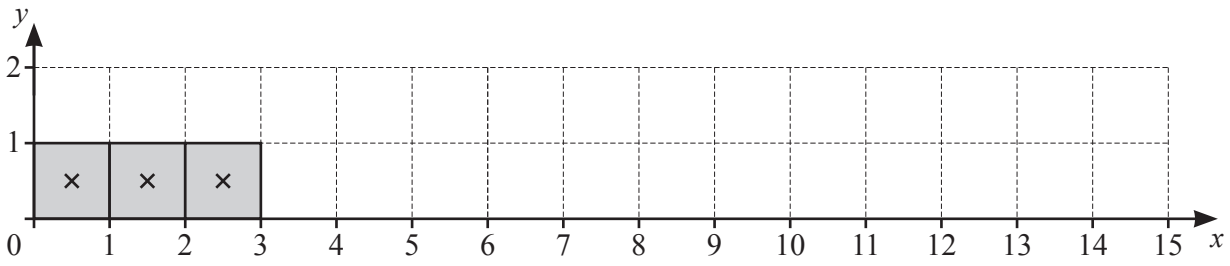


Diagram 1 shows the square in Position 1.
One side of the square is bold to help show the rotation.
The centre of the square is (0.5, 0.5).

Diagram 2 shows the square rolled 90° clockwise about (1, 0) to Position 2.

1 To get to Position 3 the square rolls 90° clockwise about (2, 0).
To get to Position 4 the square then rolls 90° clockwise about (3, 0).

(a) On the diagram below, draw the square in Position 4, Position 5 and Position 6.



[1]

(b) Complete this table to show the x-coordinate of the centre of the square in each position. You may use the diagram on page 2 to help you.

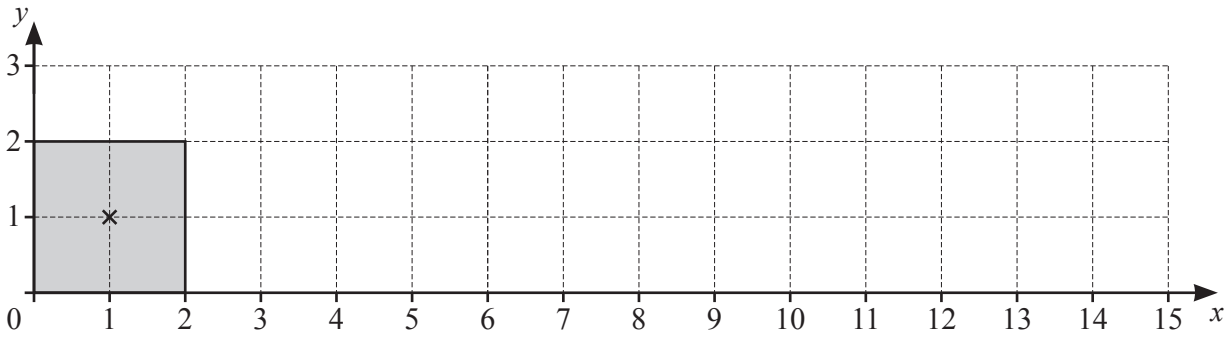
Position (n)	1	2	3	4	5	6		n
x-coordinate	0.5	1.5	2.5					

[2]

(c) Find the x-coordinate of the centre of the square in Position 92.

..... [2]

2 The side of the square is now 2 cm.



The square rolls along the x-axis in the same way as in **Question 1**.

(a) Complete the table of x-coordinates of the centre of the square in different positions.

Position (n)	1	2	3	4	5	6		n
x-coordinate	1	3						

[3]

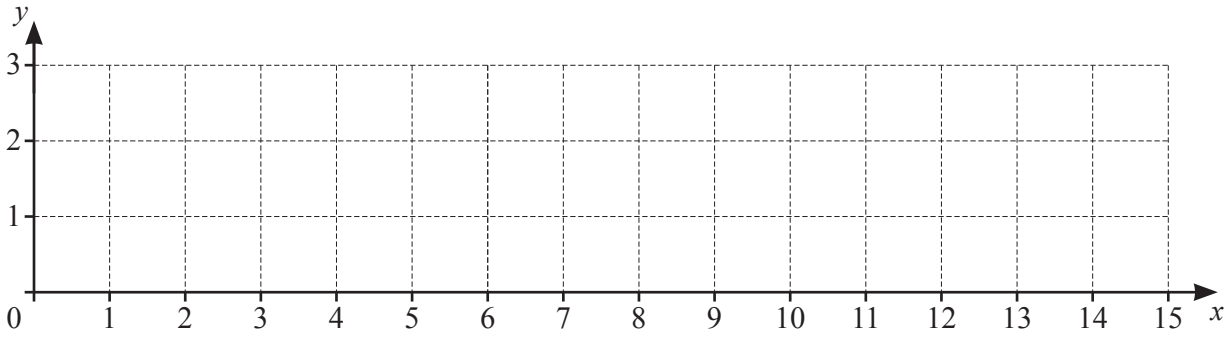
(b) Find the coordinates of the centre of the square in Position 35.

(..... , ...) [1]

3 The side of the square is now 3 cm.

Complete the table of x-coordinates of the centre of the square in different positions.
You may use the diagram below to help you.

Position (n)	1	2	3	4	5	6		n
x-coordinate	1.5							

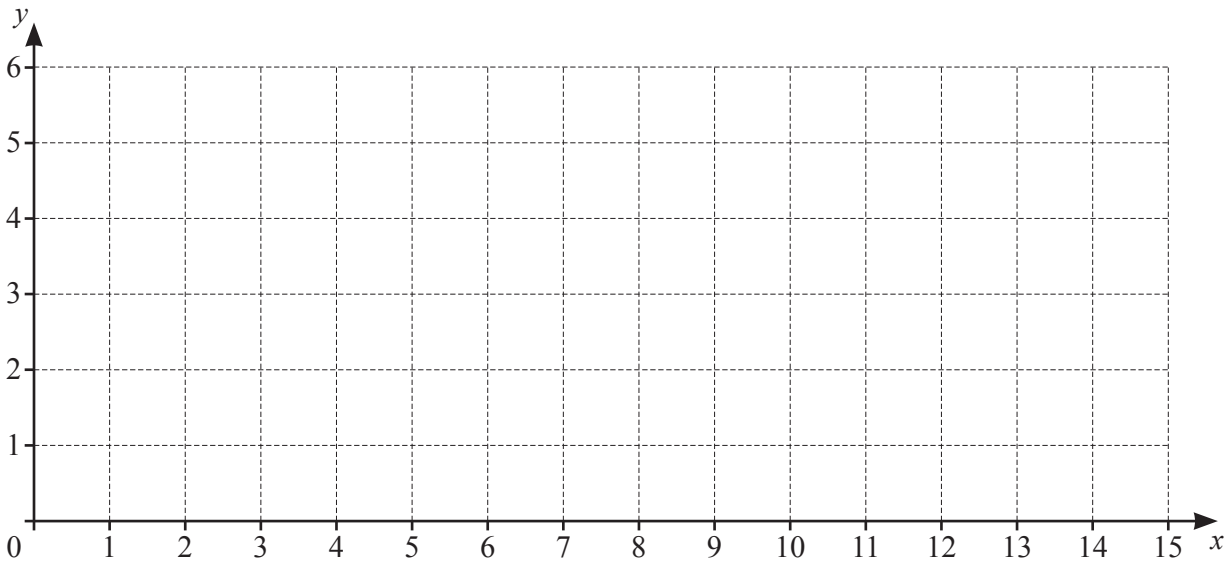


[3]

4 Write your expressions from **Questions 1(b)**, **2(a)** and **3** in the table below.
Complete the table using any patterns you notice.

You may use the grid on page 5 to help you.

Side of square (w cm)	x-coordinate in Position n
1	
2	
3	
4	
5	
w	



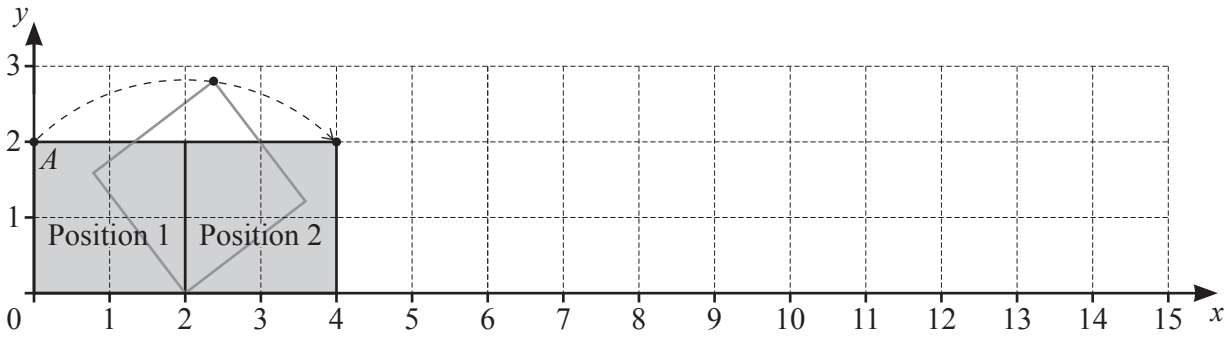
[5]

5 A square of side w cm rolls from Position 1 to Position 120.
At Position 120, the x-coordinate of the centre of the square is 2151.

Find the value of w.

..... [3]

6 A square of side 2 cm rolls along the x-axis.



(a) The table shows the x-coordinate of the point A for each position.
Complete the table.

Position	1	2	3	4	5	6	7	8	9	10	11	12	13
x-coordinate	0	4	6		8	12		14		20		22	24

[1]

(b) In the row of x-coordinates there are four sequences.

For Positions 4, 8, 12, ... the expression for the position is $4a$, where a is a positive integer.
Complete the table.

a	1	2	3	4	5	...	a
Position $(4a)$	4	8	12	16	20	...	$4a$
x-coordinate		14	22			...	$8a -$ .....
Position $(4a - 1)$	3	7	11	15	19	...	$4a - 1$
x-coordinate	6					...	.....
Position $(4a - 2)$	2	6	10	14	18	...	$4a - 2$
x-coordinate	4	12	20			...	.....
Position $(4a - 3)$	1					...	$4a - 3$
x-coordinate	0					...	.....

[5]

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(c) The 2 cm square rolls to Position 523.

Use **part (b)** to help you find the coordinates of point *A*.

(..... , ...) [4]

B MODELLING (QUESTIONS 7 TO 10)

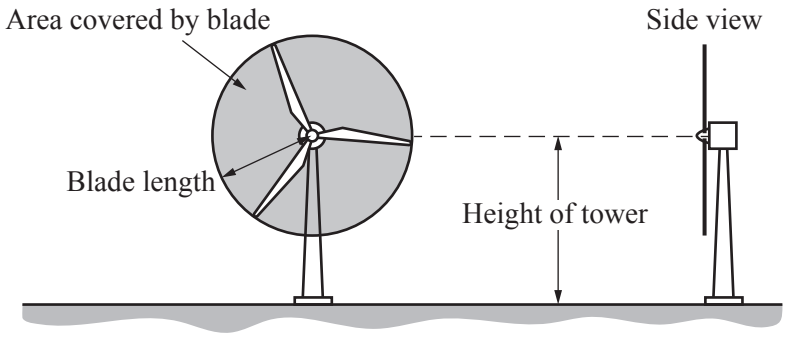
WIND TURBINES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at the use of wind turbines to generate electricity.

Area, A, of circle, radius r.	$A = \pi r^2$
Circumference, C, of circle, radius r.	$C = 2\pi r$

7 This is the front view and the side view of a wind turbine.



Wind turbines with longer blades generate more electrical power than wind turbines with shorter blades. Power is measured in kilowatts (kW).

A wind turbine has blades that are 27 m long and a tower of height 80 m.

(a) Find the greatest and least height above the ground for the tip of a blade as it turns.

Greatest height .....

Least height ..... [2]

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(b) An international soccer pitch is a rectangle measuring 70 m by 105 m.

(i) Find the area covered by the blades of this wind turbine.
Write your answer as a percentage of the area of the international soccer pitch.

..... [3]

(ii) New wind turbines have blades that are 107 m long.

Find the area covered by these blades as a percentage of the area of the international soccer pitch.

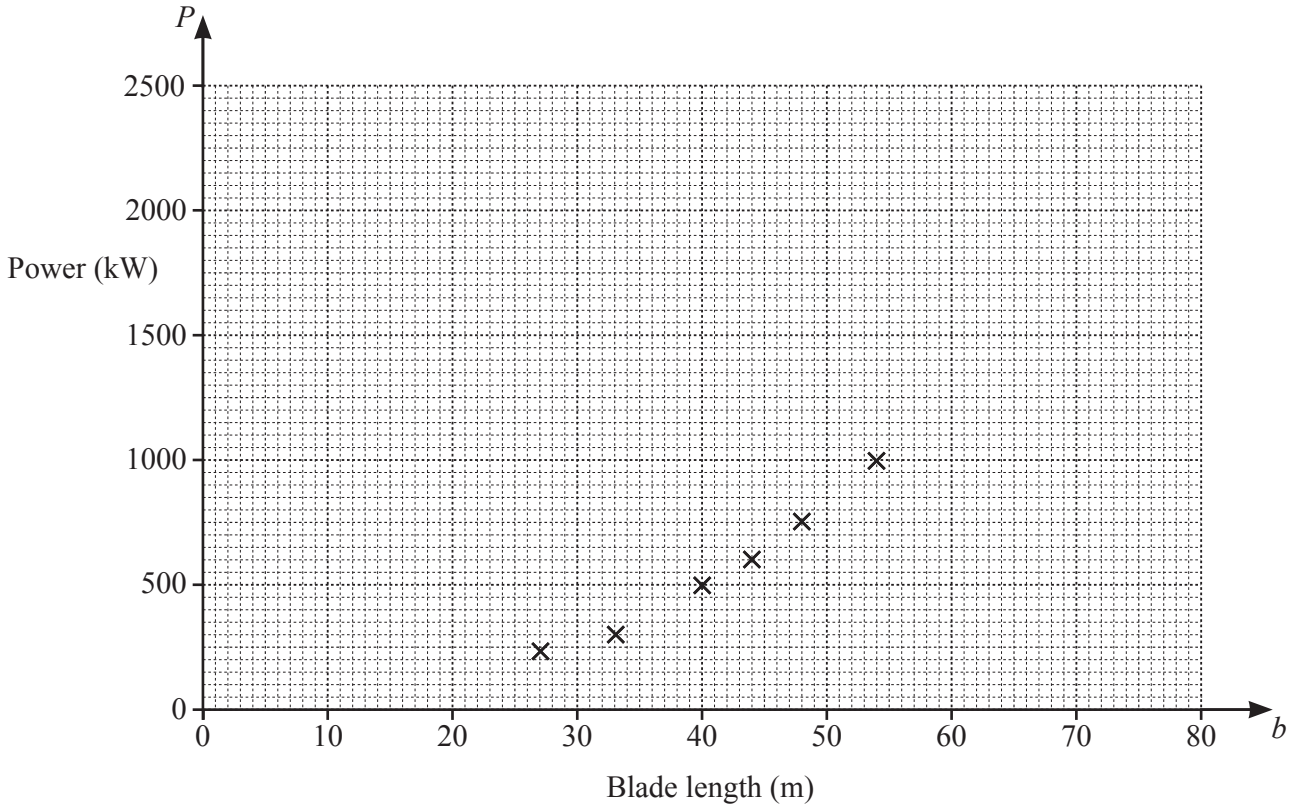
..... [2]

8 The amount of power generated depends on wind speed as well as the area covered.

This table shows the power (in kW) for blades of different lengths at a fixed wind speed.

Blade length (b metres)	27	33	40	44	48	54	64	72	80
Power (P kW)	225	300	500	600	750	1000	1500	2000	2500

(a) Plot the last three points on this graph.
The first six points have been plotted for you.



[1]

(b) A model for the power, P kW, is

$P = cb^2$, where b is the length of the blade in metres and c is a constant.

Use the information to find a value for c and write down the model.

..... [2]

(c) Another wind turbine generates 1200 kW.

Use your model to find the length of its blade.

..... [2]

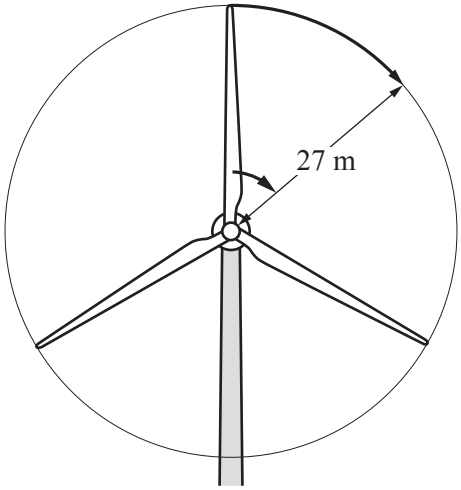
9 **(a)** A blade rotates through 30° every second.

(i) Find the time it takes to complete a full turn and the number of complete turns it makes in a minute.

Time = ...

Number of turns = ... [3]

(ii) Different parts of the blade travel through air at different speeds.



Show that the speed of the tip of this blade, with length 27 m, is 14.1 m/s, correct to 3 significant figures.

[3]

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(b) The blade with length 27 m now rotates through 40° every second.

Find the new speed of the blade tip in m/s.

..... [2]

(c) A blade turns through t degrees every second.
The length of the blade is L metres.

Write a model for the speed, S m/s, of the blade tip in terms of π, t and L.
Give your answer in its simplest form.

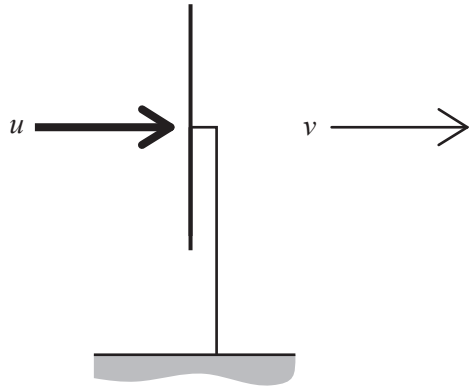
..... [2]

(d) The maximum speed for a blade tip is 72 m/s.

Find the maximum speed of rotation, in degrees per second, for a blade with length 107 m.

..... [3]

10 Wind enters a turbine at a speed of u m/s.
The wind leaves the turbine at a reduced speed of v m/s.



x is the fraction that v is of u, so $x = \frac{v}{u}$.

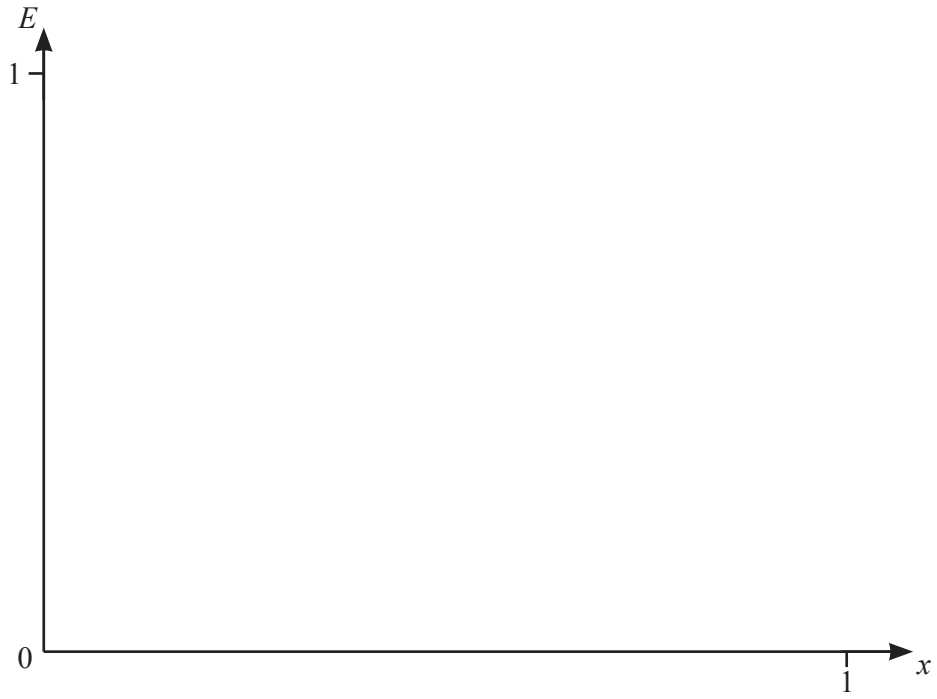
A model for the efficiency, E, of the wind turbine is $E = \frac{(1-x^2)(1+x)}{2}$.

(a) What can you say about the wind speeds v and u if the efficiency, E, is zero?

..... [1]

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(b) Sketch the graph of the model for E for $0 \leqslant x \leqslant 1$.



[2]

(c) Find the value of x that gives maximum efficiency.

..... [1]

(d) Find the greatest value for E.
Give your answer as a percentage.

..... [1]

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