Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

5499642845

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **May/June 2018**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 4) and **B** (Questions 5 to 9).
You must show all the relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 4)

ESTIMATING π (20 marks)

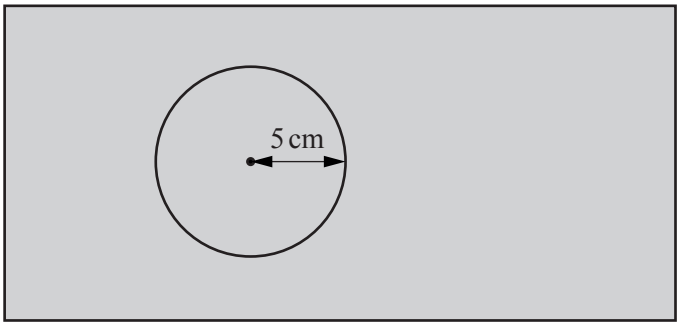
You are advised to spend no more than 45 minutes on this part.

This investigation is about using relative frequency to estimate the value of π.

Area, A, of circle radius r is $A = \pi r^2$.

1 Lee draws circles on rectangular pieces of paper.
He drops grains of rice at random onto the pieces of paper.
He counts the number of grains of rice inside each circle.

(a) Lee draws a circle of radius 5 cm on a rectangular piece of paper measuring 40 cm by 20 cm.



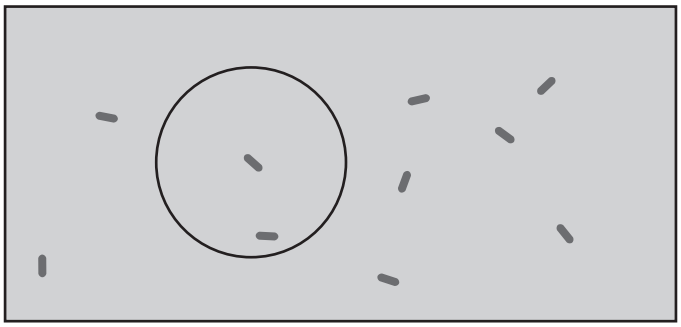
NOT TO SCALE

The probability, p, that a grain of rice lands **inside** the circle is

$$p = \frac{\text{area of circle}}{\text{area of paper}}.$$

(i) Show that p is approximately 0.098 for this piece of paper.

(ii) Lee drops 10 grains of rice at random onto the piece of paper.
The diagram shows the result.



NOT TO SCALE

Key:
– represents one grain of rice.

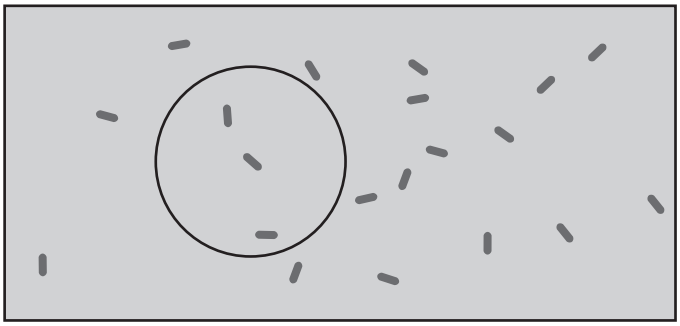
The relative frequency that a grain of rice is inside the circle

$$= \frac{\text{number of grains of rice inside the circle}}{\text{total number of grains of rice dropped}}.$$

Find the relative frequency that a grain of rice is inside the circle.

.....

(iii) Lee drops 10 more grains of rice at random onto the piece of paper.



NOT TO SCALE

Show that the relative frequency that a grain of rice is inside the circle is 0.15 .

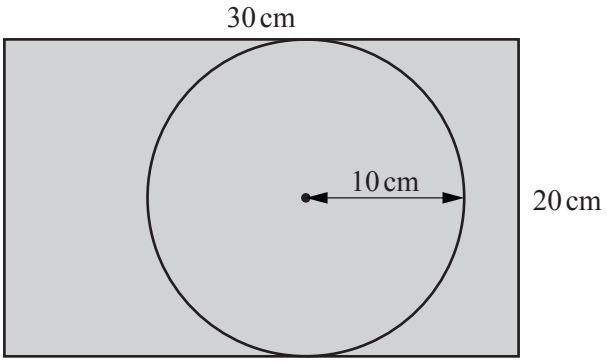
(iv) The relative frequency that a grain of rice is inside the circle gives an estimate for the probability, p.

The area of the circle is $25 \times \pi$.

Use $\frac{\text{area of circle}}{\text{area of paper}} = 0.15$ to show that an estimate for π is 4.8 .

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(b) Lee draws a circle of radius 10 cm on a rectangular piece of paper measuring 30 cm by 20 cm.



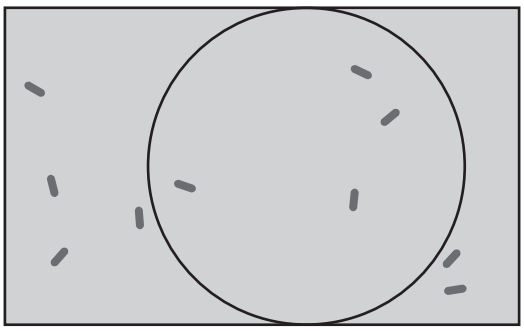
NOT TO SCALE

(i) Complete this statement with a number.

Area of circle = ... $\times \pi$

(ii) Lee drops 10 grains of rice at random onto the piece of paper.
Diagram A shows the result.

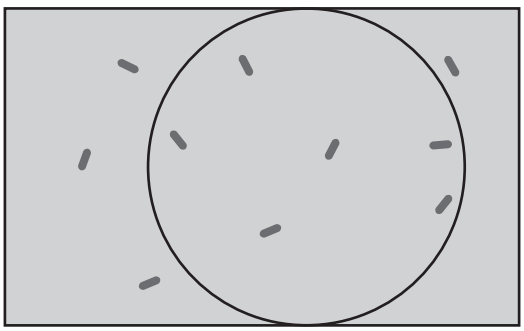
A



NOT TO SCALE

Lee removes the 10 grains of rice and drops another 10 grains of rice at random onto the piece of paper.
Diagram B shows the result.

B



NOT TO SCALE

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Complete the table.

	A	B	Combined results for all 20 grains of rice
Number of grains of rice inside circle			
Relative frequency			$\frac{10}{20}$

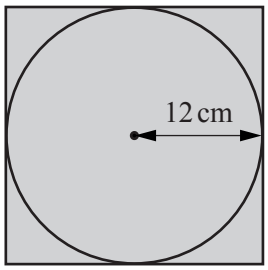
(iii) Use the formula

$$\frac{\text{area of circle}}{\text{area of paper}} = \text{relative frequency}$$

to estimate π using the combined results for all 20 grains of rice.

$\pi =$ .....

2 **(a)** Lee draws a circle of radius 12 cm on a different piece of paper.
The circle touches all four edges of the paper.

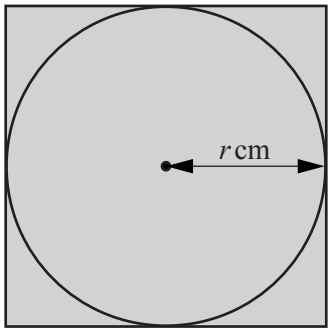


Lee drops 50 grains of rice at random onto the piece of paper.
He removes the 50 grains of rice and drops another 50 grains of rice at random onto the piece of paper.
The combined number of grains of rice inside the circle is 78.

Use the formula $\frac{\text{area of circle}}{\text{area of paper}} = \text{relative frequency}$ to estimate π.

$\pi =$ .....

(b) Lee draws a circle of radius r cm on a different piece of paper.
The circle touches all four edges of the paper.



Show that, for any value of r,

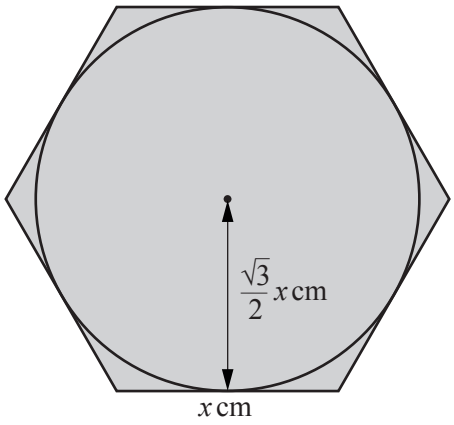
estimate for $\pi = k \times$ relative frequency, where k is an integer.

Find the value of k.

$k =$ .....

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3 Lee draws a circle on a piece of paper in the shape of a regular hexagon of side length x cm.
The circle touches all six edges of the paper.



NOT TO SCALE

Lee drops grains of rice at random onto the piece of paper.
He counts the number of grains of rice inside the circle and finds the relative frequency.

(a) Find an estimate for the value of π when $x = 30$ and the relative frequency that the grain of rice is inside the circle is 0.905 .

Use the formula $\dfrac{\text{area of circle}}{\text{area of paper}} = \text{relative frequency}$.

.....

(b) Show that, for any value of x,

$$\text{estimate for } \pi = k \times \text{relative frequency},$$

where k is an **exact** constant.

Find the value of k.

$k =$ .....

4 Lee draws a circle on a piece of paper in the shape of a regular polygon with n sides.
The circle touches all n edges of the paper.
He drops grains of rice at random onto the piece of paper.
He counts the number of grains of rice inside the circle and finds the relative frequency.

$$\text{Estimate for } \pi = k \times \text{relative frequency}$$

Explain clearly why the constant k gives an approximation for π, as the value of n increases.

B MODELLING (QUESTIONS 5 to 9)

SHOE BUSINESS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This task is about the profit made by making and selling shoes.

A company makes and sells two styles of shoe, X and Y.
The objective of the task is for the company to maximise the amount of profit it makes each day.
The number of shoes made each day is modelled using inequalities.

Each day the company
- makes x pairs of shoes of style X and y pairs of shoes of style Y
- makes at most 150 pairs of shoes.

5 **(a)** **(i)** Write down, in terms of x and y, an inequality to show how many pairs of shoes can be made each day.

.....

Two machines are used to make each pair of shoes.
Machine A is used for at least 3 hours each day.
Machine B is used for at least 4 hours each day.
The table shows some information about making each style of shoe.

	Style X	Style Y
Maximum number of pairs of shoes made each day	80	100
Time taken by Machine A for each pair of shoes	5 min	2 min
Time taken by Machine B for each pair of shoes	4 min	5 min
Cost of making one pair of shoes	\$20	\$15
Selling price of one pair of shoes	\$100	\$70

(ii) Explain why $0 \leqslant x \leqslant 80$ and $0 \leqslant y \leqslant 100$.

.....

.....

(iii) An inequality for the total time each day, in minutes, that Machine A is used is $5x + 2y \geqslant 180$.

Find, in terms of x and y, an inequality for the total time each day, in minutes, that Machine B is used.

.....

(b) The company sells all the pairs of shoes it makes each day.

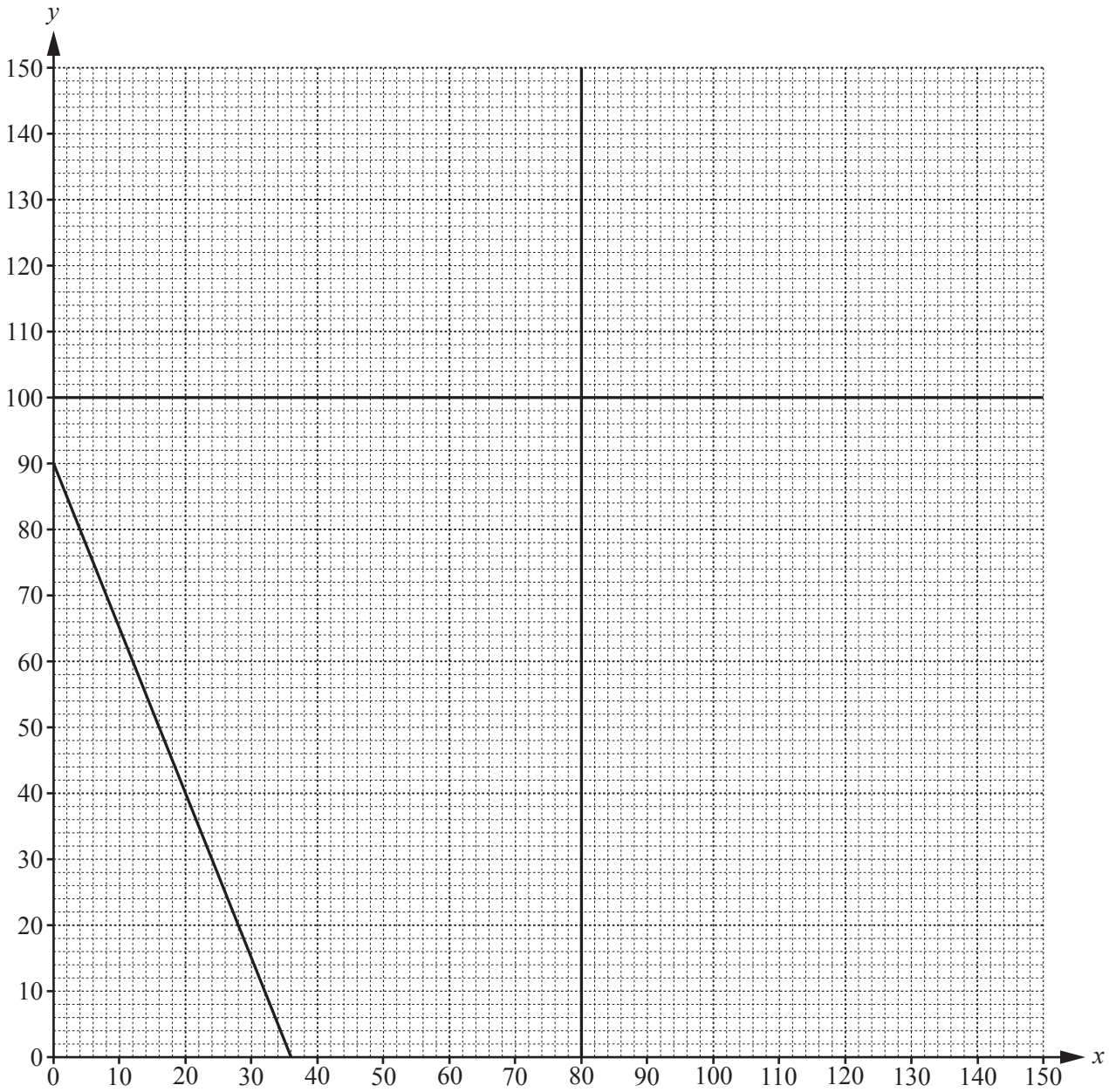
Find an **expression**, in terms of x and y, for the total **profit** made each day.

.....

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6 **(a)** Three lines $x = 80$, $y = 100$ and $5x + 2y = 180$ have been drawn on the grid.

Draw two more lines to find the region defined by the **five** inequalities from **question 5(a)**.
Shade the **unwanted** region.



(b) Show that the greatest profit each day is \$10 250.
Write down the number of pairs of shoes of each style to make this profit.

Style X ..... Style Y .....

7 The table shows the amount of material for each style.

	Style *X*	Style *Y*
Material for one pair of shoes	$0.5\,m^2$	$0.4\,m^2$

There is a shortage of material and $50\,m^2$ is used each day to make all the shoes.

(a) Draw a line on the grid in **question 6** to show this information.

(b) Work out the **decrease** in the greatest profit because of the shortage.

.....

8 The company is considering making bags.
Research shows that a model for the profit each day, \$$p$, from the sale of b bags is

$$p = b^2 - 30.5b - 65 .$$

(a) On the axes below, sketch the graph of $p = b^2 - 30.5b - 65$ for $0 \leqslant b \leqslant 35$.



(b) Find the minimum number of bags the company needs to sell each day in order to make a positive profit.

.....

(c) The greatest number of bags the company can make each day is 80.

Find the profit that the company will make each day when it sells all 80 bags.

.....

9 The company decides to make 80 bags each day.
For every 2 bags that it makes, it has to decrease the number of pairs of style *X* shoes by one.

The shortage of material for making shoes is the same as in **question 7**.
There is no shortage of material for making bags.

(a) Find the greatest number of pairs of shoes of style *Y* that the company can make.

.....

(b) Show that the company's profit each day has increased.

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