Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 (Extended) **May/June 2014**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **11** printed pages and **1** blank page.

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Answer **both** parts **A** and **B**.

A INVESTIGATION FRACTIONS WITHIN FRACTIONS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at sequences of fractions.

One way to form a sequence is by using fractions within fractions as shown below.

$\frac{1}{1}$ then $\frac{1}{1+\frac{1}{1}}$ then $\frac{1}{1+\frac{1}{1+\frac{1}{1}}}$ then $\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}$ and so on.

1 The first three terms of a sequence of fractions are $1, \frac{1}{2}, \frac{2}{3}$.

These terms are calculated in the following way.

$$\frac{1}{1} = 1$$

$$\frac{1}{1+\left(\frac{1}{1}\right)} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{1}{1+\left(\frac{1}{1+\frac{1}{1}}\right)} = \frac{1}{1+\left(\frac{1}{2}\right)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

(a) Fill in the box to complete the calculation of the 4th term.

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}} = \frac{1}{1+\frac{1}{1+\frac{1}{2}}} = \frac{1}{1+\frac{1}{\frac{3}{2}}}$$

$$= \boxed{} = \frac{1}{\frac{5}{3}}$$

$$= \frac{3}{5}$$

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(b) Show that the 5th term of this sequence of fractions is $\frac{5}{8}$.

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}} \quad =$$

(c) Complete the table to show the first eight terms of this sequence of fractions.

$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{8}$		$\frac{13}{21}$	

(d) Explain how you used a pattern to find the numerator and the denominator of the 8th term.

Numerator .....

Denominator .....

2 Here is a different sequence of fractions. The first three terms are 2, $\frac{2}{3}$, $\frac{6}{5}$.

(a) Calculate the 4th and 5th terms.
Give your answers as single fractions.

$\frac{2}{1}$ = $\frac{2}{1}$

$\frac{2}{1+\frac{2}{1}}$ = $\frac{2}{3}$

$\frac{2}{1+\frac{2}{1+2}}$ = $\frac{2}{1+\frac{2}{3}}$ = $\frac{6}{5}$

$\frac{2}{1+\frac{2}{1+\frac{2}{1+2}}}$ = $\frac{2}{1+\frac{2}{1+\frac{2}{3}}}$ = ...

$\frac{2}{1+\frac{2}{1+\frac{2}{1+\frac{2}{1+2}}}}$ = ...

(b) Explain how you can use a pattern to find the numerator and the denominator of the 5th term of this sequence.

Numerator .....

Denominator .....

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3

You may find this formula useful in this question.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

As more terms in these sequences of fractions are calculated the difference between the terms becomes smaller and smaller.
This means that the terms become nearly equal to the same number.
This number is called the limit of the sequence.

(a) In the sequence in **question 1** all the terms after the 7th term are the same when written correct to 3 decimal places.

If x is a fraction in the sequence in **question 1** then the next fraction is $\frac{1}{1+x}$.

In this case the sequence reaches its limit when $x = \frac{1}{1+x}$.

(i) Show that $x = \frac{1}{1+x}$ can be rearranged to give the quadratic equation $x^2 + x - 1 = 0$.

(ii) Solve $x^2 + x - 1 = 0$ and write down the positive decimal solution, correct to 3 decimal places.

.....

(iii) Complete the table for the sequence in **question 1**.

Fraction	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{8}$		$\frac{13}{21}$	
Decimal	1	0.5	0.667	0.6	0.625		0.619	

(b) (i) Complete the table for the sequence in **question 2**.

Fraction	$\frac{2}{1}$	$\frac{2}{3}$	$\frac{6}{5}$			$\frac{42}{43}$	$\frac{86}{85}$
Decimal	2	0.667	1.2			0.977	1.012

(ii) Solve $x = \frac{2}{1+x}$ for $x > 0$.

(iii) Explain the connection between the decimals in **part (b)(i)** and your answer to **part (b)(ii)**.

(c) The positive solution of $x = \frac{N}{1+x}$ gives the limit of these sequences of fractions.

(i) $x = \frac{N}{1+x}$ can be rearranged to give the quadratic equation $x^2 + x - N = 0$.

Solve $x^2 + x - N = 0$ and write down the positive value of x in terms of N.

.....

(ii) Find three integer values of N that make the limit a positive integer.

.....

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B MODELLING FITNESS TRAINING (20 marks)

You are advised to spend no more than 45 minutes on this part.

Viola starts her fitness training. She intends to walk, jog and run to increase her fitness.

1 **(a)** On day 1 she walks 1.5 km in 20 minutes.

Show that her average walking speed is 4.5 km/h.

(b) On day 2 she increases her average walking speed to 5 km/h.

How many minutes does it take her to walk the 1.5 km?

.....

(c) Viola wants to increase her average walking speed by 0.5 km/h each day.

If she does, on which day will she walk at an average of 6.5 km/h?

.....

2 She now begins to jog as well as walk.
On day 7 she jogs for 20 minutes at 8.1 km/h.

What distance does she jog?

.....

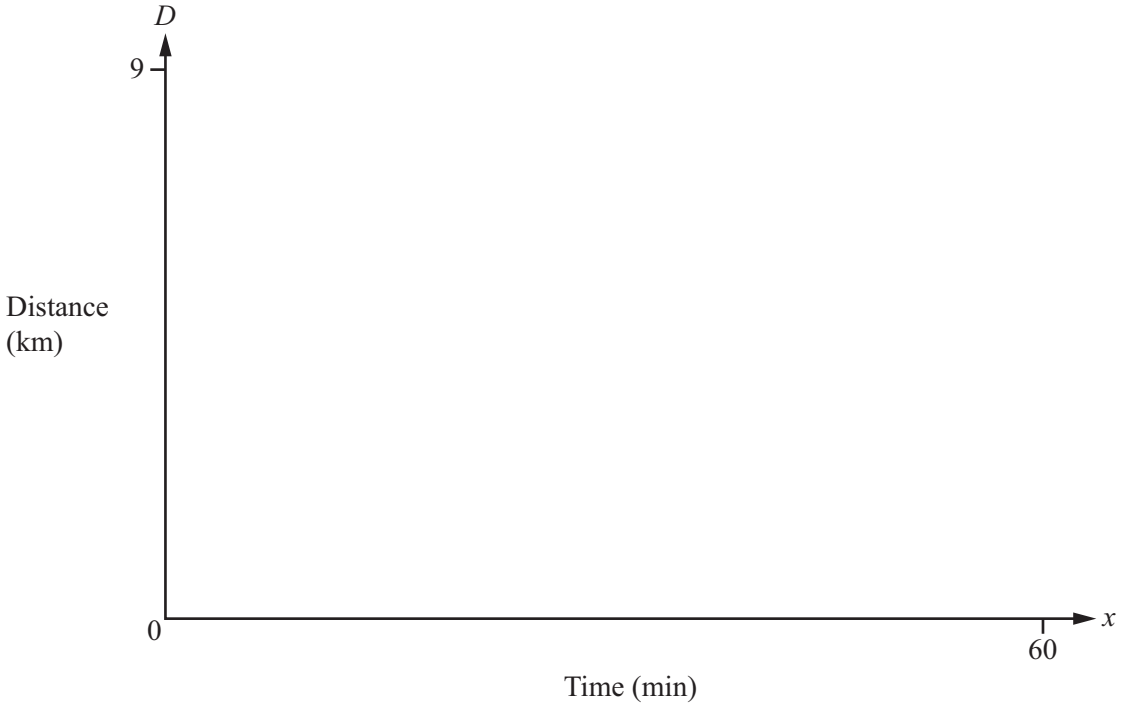
3 From day 10 she trains for one hour.
She walks at an average speed of 6.4 km/h and she jogs at an average speed of 8.1 km/h.
These speeds do not change.

(a) Construct a formula for the total distance, D km, when she walks for x minutes and jogs for the rest of the hour.

.....

(b) Show that your formula simplifies to the model $D = \frac{486 - 1.7x}{60}$.

(c) Sketch the graph of the model in **part (b)** on the axes below, for $0 \leqslant x \leqslant 60$.



(d) What distance will she travel in the hour when she walks and jogs for the same amount of time?

.....

(e) The next month Viola walks for 20 minutes, jogs for 20 minutes and runs for the rest of the hour. She travels a total distance of 9 km.

Work out her average speed when she is running.

.....

(f) The next month Viola walks for x minutes, jogs for y minutes and runs for the rest of the hour. Her average speed when she is running does not change from the value found in **part (e)**.

(i) Extend your model in **part (b)** for the total distance, D km, to include walking for x minutes, jogging for y minutes and running for the rest of the hour.

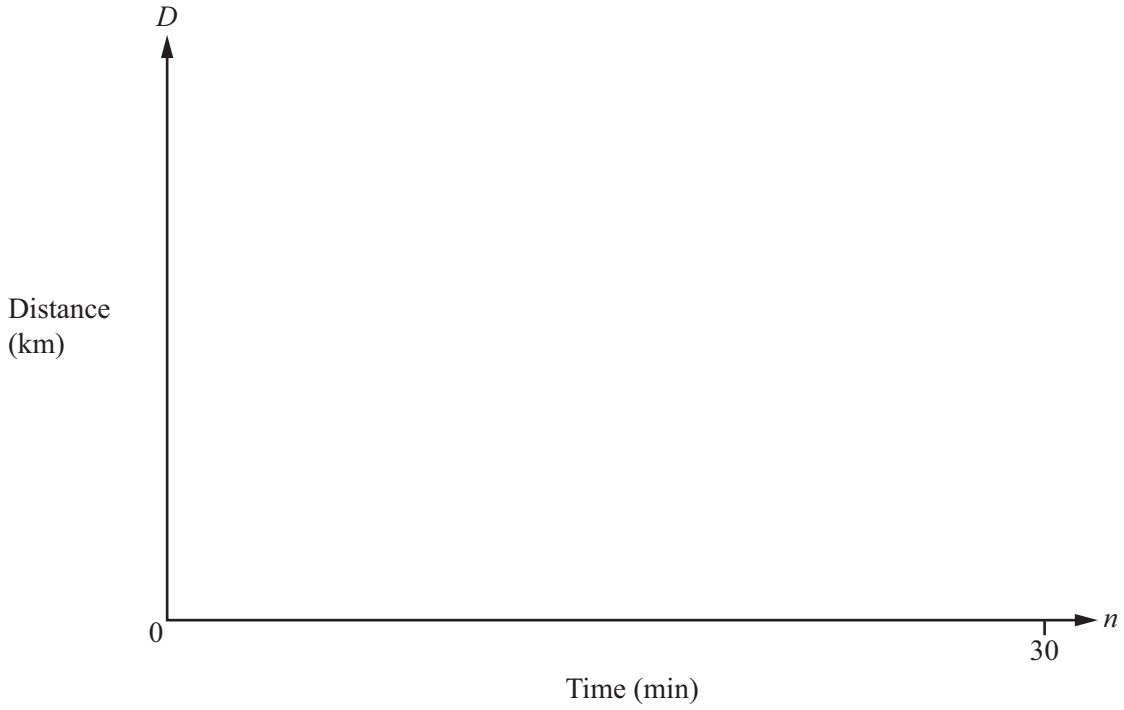
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(ii) Show that your model simplifies to $D = \frac{1}{60}(750 - 6.1x - 4.4y)$.

(g) (i) Rewrite the model in **part (f)** when Viola's walking and jogging times are both n minutes.

.....

(ii) Sketch the graph of the model in **part (g)(i)** on the axes below, for $0 \leqslant n \leqslant 30$.



(iii) What is Viola doing in her training when $n = 0$?

.....

(iv) What is Viola doing in her training when $n = 30$?

.....

(v) Modify the models in part **(b)** and part **(f)** when Viola spends H hours training.

Model in **part (b)** .....

Model in **part (f)** .....

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