Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	
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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 (Extended) **May/June 2014**

1 hour 30 minutes

Candidates answer on the Question Paper

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **8** printed pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION COUNTING FACTORS (20 marks)

You are advised to spend 45 minutes on part **A**.

This investigation looks for a method to find how many factors a number has.

1 **(a)** **(i)** Write down, in ascending order, the five factors of 16.
Two have been written down for you.

1 , ... , ... , ... , 16

(ii) Write down the five factors of 16 as powers of 2.
Two have been written down for you.

2^0 , ... , ... , ... , 2^4

(b) Write down, in ascending order, the four factors of 27 as powers of 3.

..... , ... , ... , ...

2 **(a)** p is a **prime** number.

(i) Write down the six factors of p^5 as powers of p.
One has been written down for you.

p^0 , ... , ... , ... , ... , ...

(ii) Write down, in terms of n, the number of factors of p^n.

.....

(b) $78\,125 = 5^n$ where 5 is a prime number.
Find the number of factors of 78 125. Do not write them out.

.....

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3 **(a)** 20 is **not** a prime number.
$20 = 2^2 \times 5^1$ where 2 and 5 are prime numbers.
Find all the factors of 20 by completing the table.

		Powers of 5	
		5^0	5^1
Powers of 2	2^0	$2^0 \times 5^0 = 1 \times 1 = 1$	$2^0 \times 5^1 =$ ..... $\times$ ..... $=$ .....
	2^1	$2^1 \times 5^0 = 2 \times 1 = 2$	$2^1 \times 5^1 =$ ..... $\times$ ..... $=$ .....
	2^2	$2^2 \times 5^0 =$ ..... $\times$ ..... $=$	$2^2 \times 5^1 = 4 \times 5 = 20$

(b) The table has 3 rows and 2 columns.
Describe how to find the number of factors of 20 from the number of rows and the number of columns.

(c) $4000 = 2^5 \times 5^3$.
Find the number of factors of 4000.

.....

4 **(a)** **(i)** $1\,000\,000 = 2^n \times 5^n$.
Find n.

.....

(ii) Find the number of factors of 1 000 000.

.....

(b) Find n such that 10^n has 900 factors.

.....

5 x has exactly two prime factors, 3 and 7.
So x can be written as $3^m \times 7^n$.

Find all the possible values of x that have exactly 12 factors.

.....

6 **(a)** Write 336 in the form $p^d \times q^e \times r^f$ where p, q and r are **prime** numbers.

..... × ..... × .....

(b) Find the number of factors of 336.

.....

7 A number has exactly three prime factors, 2, 3 and 5.
It has exactly 12 factors.

Find all possible numbers.

.....

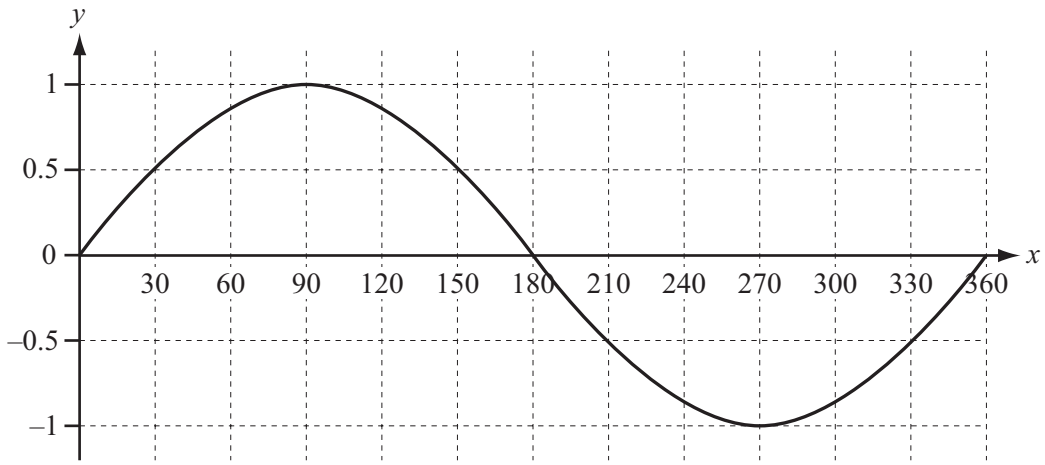
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B MODELLING TIDES (20 marks)

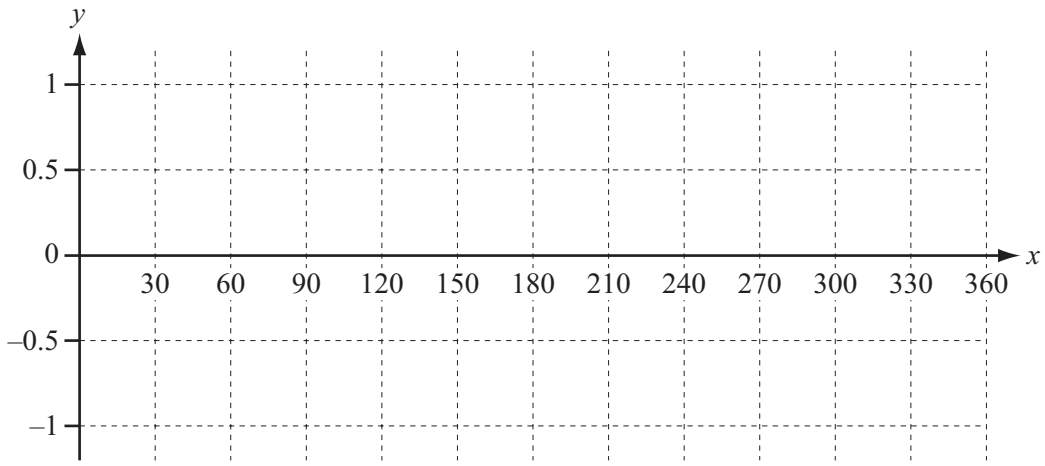
You are advised to spend 45 minutes on part **B**.

In this task you will model the height of a tide.

Here is the graph of $y = \sin x°$. Its period is 360.



1 (a) (i) On the grid below, sketch the graph of $y = \sin(3x)°$.



(ii) Write down the period of the graph of $y = \sin(3x)°$.

.....

(b) Write down the period of the graph of $y = \sin(10x)°$.

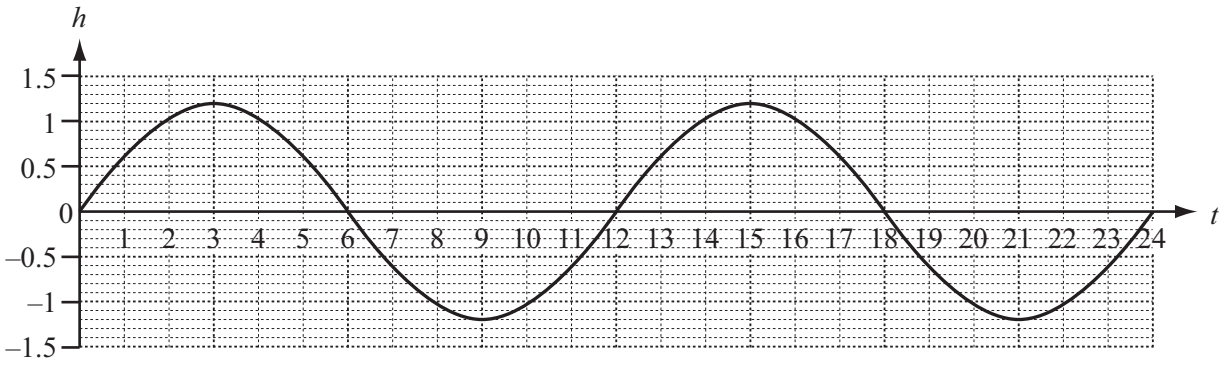
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(c) Write down an expression, in terms of b, for the period of the graph of $y = \sin(bx)°$.

.....

2 The graph below shows the approximate height of the tide at Auckland (New Zealand) during 25th April 2014.

t is the time of day starting at midnight.
h is the height in metres above a fixed height.

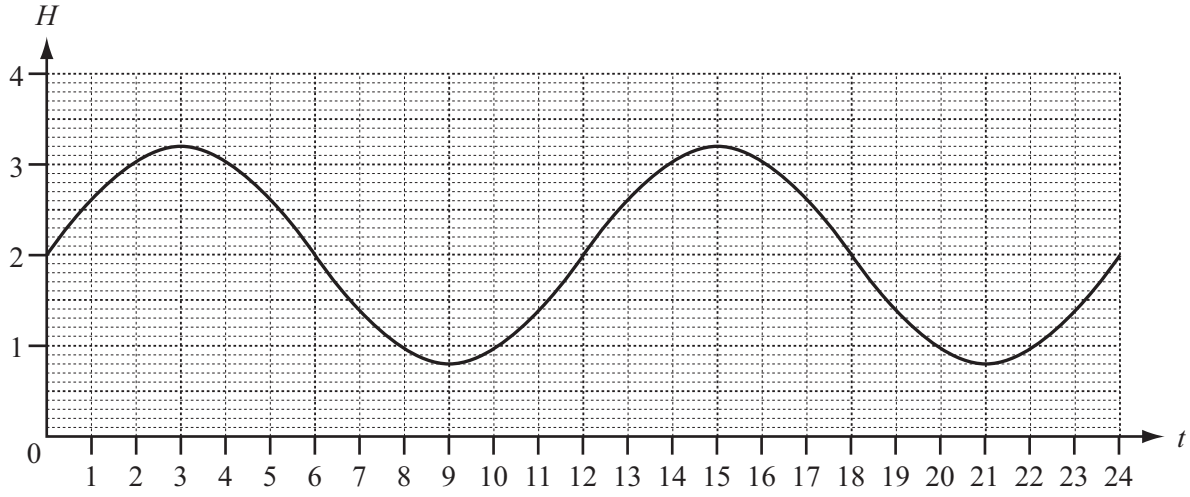


(a) Write down the period of the graph.

.....

(b) Explain why a model for the height of the tide on 25th April is $h = 1.2 \sin(30t)°$.

(c) Usually the height of a tide is measured as the height, H metres, above sea level.
The graph below shows this for the tide at Auckland on 25th April.



Modify the model in **part (b)**.

$H =$ .....

(d) The best fishing conditions are when the height of the tide is less than one metre above sea level.

(i) Use your calculator with your model for H to find the two times between which there were the best fishing conditions on the **morning** of 25th April.
Give your times correct to the nearest minute.

..... and .....

(ii) Using your answer to **part (i)**, write down the times between which there were the best fishing conditions on the **afternoon** of 25th April.

..... and .....

3 Here is a different model for the height of the tide, D metres, on the **morning** of 25th April.

$$D = 0.022t^3 - 0.403t^2 + 1.9t$$

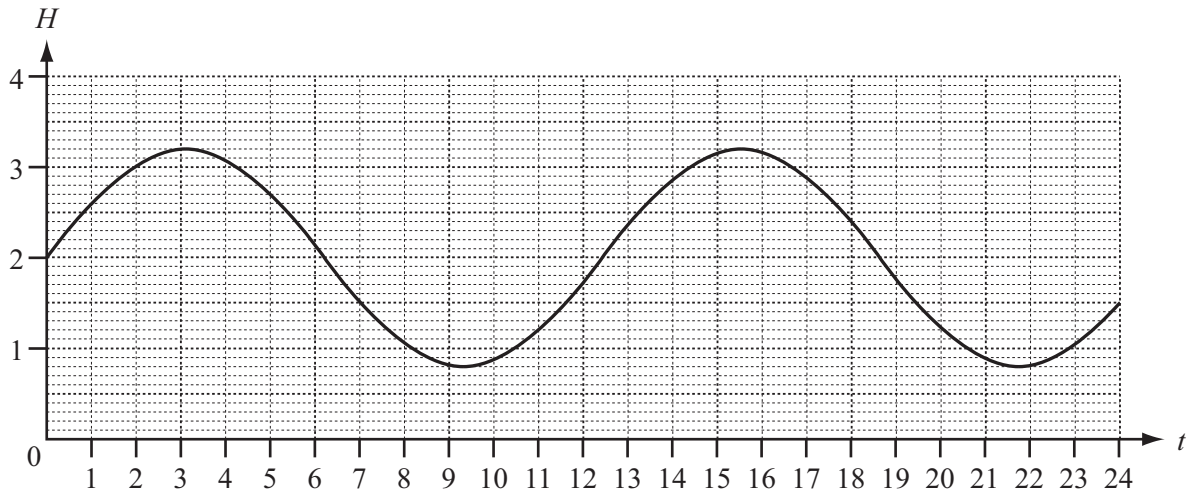
(a) Compare D with H from **question 2(c)** by drawing their graphs on your calculator for $\mathbf{0 < t < 12}$.
Comment on how the difference in height between D and H varies with time.

(b) Make **one** change to the model D that makes the graph of D very close to the graph of H for $3 < t < 9$.

$D =$ .....

Question 4 and 5 are printed on the next page.

4 The graph below shows the actual height of the tide above sea level on 25th April 2014 at Auckland.



The actual period for the height of a tide is 12 hours 25 minutes.
Use this fact to improve your model for the height of the tide above sea level on 25th April by changing one of the numbers in your answer to **question 2(c)**.

$H =$ .....

5 The graph for 26th April can be found by translating the graph for 25th April horizontally.
Modify your answer to **question 4** to give a model for the height of the tide above sea level at Auckland on **26th** April.

$H =$ .....

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