Cambridge Assessment International Education

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6407495703

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **February/March 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 5) and part **B** (Questions 6 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Answer **both** parts **A** and **B**.

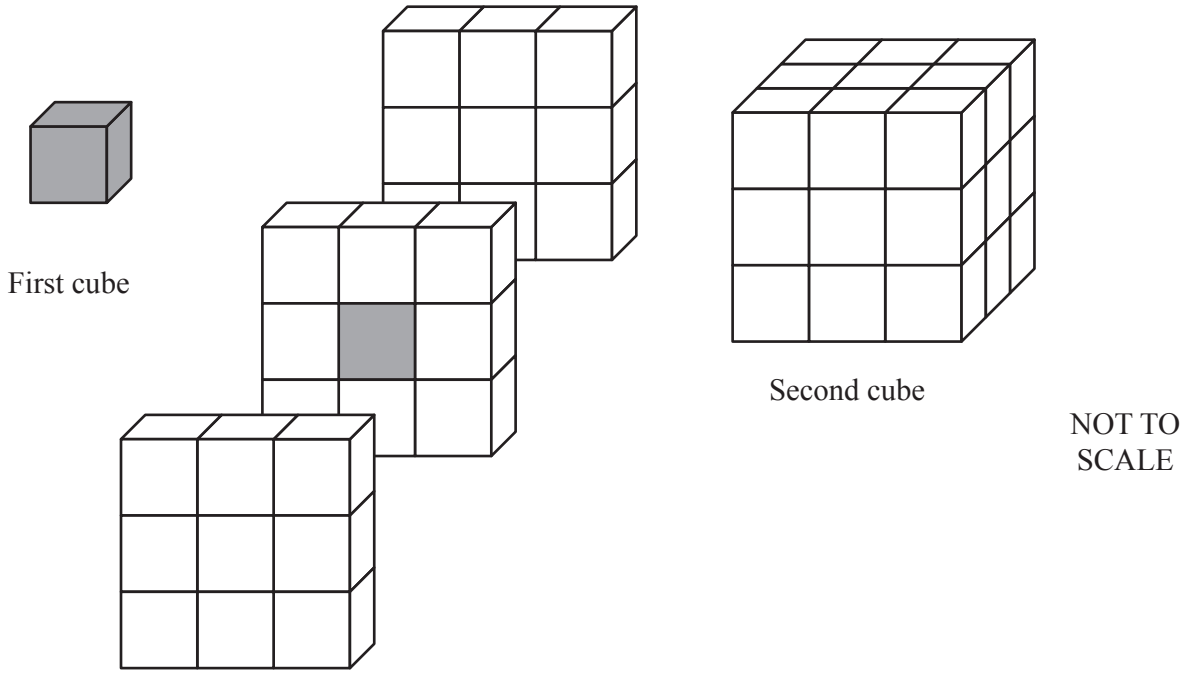
A INVESTIGATION (QUESTIONS 1 TO 5)

ENCLOSED CUBOIDS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task is about the number of white cubes needed to enclose a grey cuboid with integer sides.

1 The diagram shows:
- a grey cube with each edge 1 centimetre long (a 1 cm cube)
- a view showing how the grey cube is enclosed in a layer of 1 cm white cubes
- the resulting cube.



(a) The enclosing layer of white cubes is 1 cm thick.
This makes the second cube with each edge 3 cm long.

Find the number of white cubes.

..... [1]

(b) The second cube is enclosed in a layer of 1 cm white cubes to make the third cube.
There are now 2 layers enclosing the grey cube.
Each layer is 1 cm thick.

(i) Write down the length of one edge of the third cube.

..... [1]

(ii) Find the number of white cubes in the third cube.

..... [2]

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(c) For any cube made in this way, write down the relationship between the total number of 1 cm cubes and the number of white cubes.

..... [1]

(d) For any cube made in this way,

n = the number of enclosing layers
L = the length of the outer edge in centimetres
W = the number of white cubes.

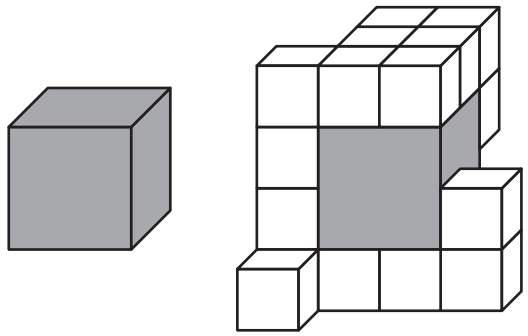
For the first cube, $n = 0$ and $L = 1$.
For the second cube, $n = 1$ and $L = 3$.

Complete this table for a grey cube of edge length 1 cm.

Number of enclosing layers	0	1	2	3	4		n
L	1	3					
W	0				728		

[5]

2 A grey cube has edge length 2 cm.
This cube will be enclosed in layers of 1 cm white cubes.
The diagram shows some of the white cubes in the first layer.



NOT TO SCALE

(a) This calculation gives the number of white cubes when there is one enclosing layer.

$4^3 - 2^3$

Explain why this calculation works.

.....

.....

..... [2]

(b) Complete this table for a grey cube of edge length 2 cm.

Number of enclosing layers	0	1	2	3	4	...	n
L	2				10	...	
W	0	56			992	...	

[4]

3 **(a)** Complete this table for a grey cube of edge length 3 cm.

Number of enclosing layers	0	1	2	3	4		*n*
L	3						
W	0	98	316	702	1304		

[2]

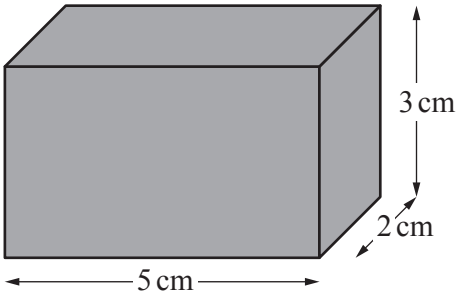
(b) A grey cube has edge length *e* cm.

Write your expressions for *W* from **questions 1(d)**, **2(b)** and **3(a)** in this table.
Complete the table.

Edge length of grey cube	*W*
1	
2	
3	
e	

[2]

4



NOT TO SCALE

This grey cuboid has edge lengths of 2 cm, 3 cm and 5 cm.

(a) Find the number of white cubes when there is one enclosing layer.

..... [3]

(b) A grey cuboid has edge lengths of a cm, b cm and c cm.
The cuboid has n enclosing layers.

Find the formula for the number of white cubes, W.

..... [2]

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5 A grey cuboid has edge lengths of k cm, k cm and $2k$ cm.
The grey cuboid has 8 enclosing layers of 1 cm white cubes.

The largest **face** of the resulting cuboid has 546 white cubes.

Find the value of k.

..... [5]

B MODELLING (QUESTIONS 6 to 10)

ENCLOSURES FOR CATS (30 marks)

You are advised to spend no more than 50 minutes on this part.

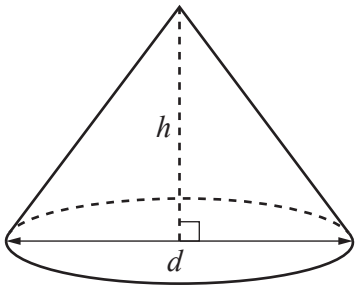
Volume, V, of cylinder of radius r, height h.	$V = \pi r^2 h$
Volume, V, of cone of radius r, height h.	$V = \frac{1}{3}\pi r^2 h$
Volume, V, of sphere of radius r.	$V = \frac{4}{3}\pi r^3$

This task looks at modelling the floor area and volume of enclosures for cats.

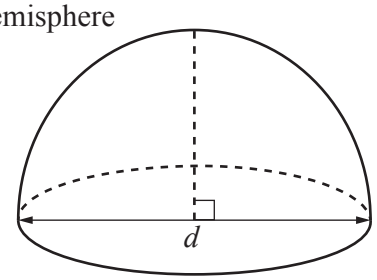
Dorothy has two cats and is going to build an enclosure for them.
She considers three designs.

6 Design 1 is a cone and design 2 is a hemisphere.
The cone and the hemisphere have the same diameter, d.

Cone



Hemisphere



(a) (i) Show that the volume of the cone is $\frac{\pi d^2 h}{12}$.

[1]

(ii) Find the volume of the hemisphere.

Write your answer in its simplest form, in terms of π and d.

..... [2]

(b) Write h in terms of d when the volume of the cone is equal to the volume of the hemisphere.

..... [1]

(c) (i) Find the volume of the cone when $h = \frac{d}{2}$.

Write your answer in its simplest form, in terms of π and d.

..... [2]

(ii) Which is smaller, the volume of the cone in **part (c)(i)** or the volume of the hemisphere?

Show how you decide.

..... [1]

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[Turn over

Information about cat enclosures	
Minimum floor sleep area per cat	$0.5\,m^2$
Minimum floor exercise area per cat	$2.8\,m^2$
Minimum extra floor area per cat	$1\,m^2$
Highest point above floor must be 2 m or more	

7 **(a)** Show that the total minimum floor area for two cats is $8.6\,m^2$.

[1]

(b) Calculate the diameter that gives this minimum floor area for both the cone and the hemisphere.

..... [2]

(c) Explain why a hemisphere with this minimum floor area is not a suitable enclosure.

.....

..... [1]

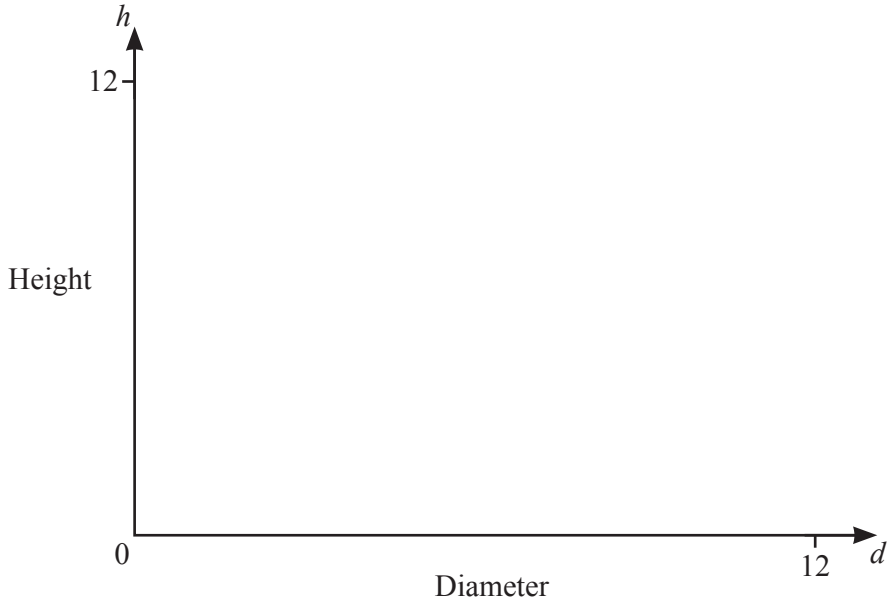
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8 Dorothy decides on a volume of $10\,m^3$ for the enclosure for the two cats.

(a) Use **question 6(a)(i)** to find a model for h, in terms of π and d, for the cone enclosure.

..... [2]

(b) Sketch your model for h for $0 < d \leqslant 12$.



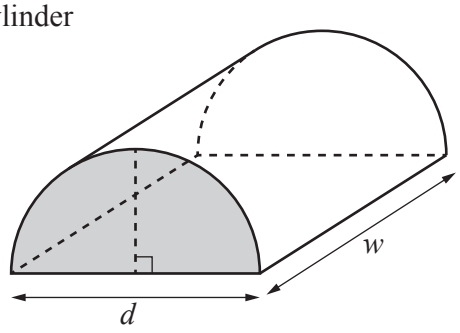
[2]

(c) Find the total floor area of the enclosure when the height of the cone is 2 m.

..... [3]

9 Design 3 is a half-cylinder.
It has diameter d and length w.

Half-cylinder



(a) Use the information on page 10 to write down the smallest value of d.

..... [1]

(b) Show that the volume of the half-cylinder is $\frac{\pi d^2 w}{8}$.

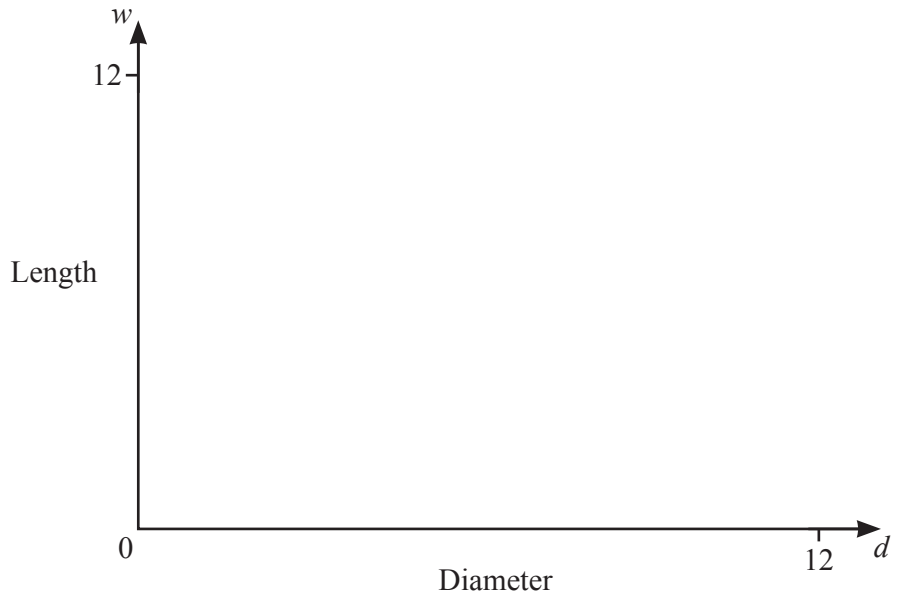
[2]

(c) **(i)** Show that a model for w for a half-cylinder enclosure that has a volume of $10\,m^3$ is

$$w = \frac{80}{\pi d^2}\ .$$

[1]

(ii) Sketch the model for w for $0 < d \leqslant 12$.



[1]

(d) (i) The total floor area is $8.6\,m^2$.

Write down a model for w in terms of d.

..... [1]

(ii) On the axes above sketch your model from **part (d)(i)**. [2]

(e) Find the height of the half-cylinder enclosure that fits both models.

..... [3]

10 Write down which enclosure of volume $10\,m^3$, cone or half-cylinder, you think Dorothy should choose for her two cats. Explain your choice.

.....

..... [1]

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