Level 2 Certificate
FURTHER MATHEMATICS
8365/2

Paper 2 Calculator

Mark scheme

June 2021

Version: 1.0 Final

2 1 6 G 8 3 6 5 / 2 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Copyright information

AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2021 AQA and its licensors. All rights reserved.

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	$10x - 5$ or $44 - 4x$	M1	may be seen in a grid
	$10x - 5 + 44 - 4x$ or $6x + 39$ or $3 \times (2x + 13)$	M1dep	oe all terms correct in a single expression
	$3(2x + 13)$	A1	
	Additional Guidance		

Q	Answer	Mark	Comments
2(a)	$5m \times (1 - 0.4)$ or $5m \times 0.6$ or $3m$	M1	oe eg $5m - 0.4 \times 5m$ or $5m - 2m$ may be seen in an equation eg $3m = m + 1$
	$\frac{1}{2}$ or 0.5	A1	
	Additional Guidance		
	$2m$ only		M0
	$2m = 1$		M1

Q	Answer	Mark	Comments
2(b)	$2w - 10 = 18^3$ or $2w - 10 = 5832$ or $\frac{18^3 + 10}{2}$	M1	oe eg $2w = 5842$
	2921	A1	
	Additional Guidance		

Q	Answer	Mark	Comments
3	$12de \times 2d$ or $24d^2e$ or $\frac{1}{2} \times 8e^2 \times 9d$ or $36de^2$	M1	oe eg $24ed^2$ or $36e^2d$
	$24d = 36e$	M1dep	oe equation with squared terms eliminated eg $2d = 3e$ or $\frac{2d}{e} = 3$ or $\frac{e}{d} = \frac{12}{18}$
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	
	Additional Guidance		
	Equivalent fraction to $\frac{3}{2}$ or $1\frac{1}{2}$ with no incorrect working		M2A0

Q	Answer	Mark	Comments
4	$\pi \times 100$ (÷ 4) or 100π (÷ 4) or 25π or $\pi \times 36$ (÷ 4) or 36π (÷ 4) or 9π	M1	oe
	$\pi \times 100 \div 4 - \pi \times 36 \div 4$	M1dep	oe eg $\frac{100\pi - 36\pi}{4}$ or $\frac{64\pi}{4}$
	16π	A1	SC2 2176π
	Additional Guidance		
	Use of circumference instead of area throughout		M0M0A0
	Allow substitution of π = [3.14, 3.142] for M marks		
	16π in working with eg 50.3 on answer line		M2A0
	SC2 is for using radii of 100 and 36		
	Omission of π in working must be recovered		

Q	Answer	Mark	Comments
5	$k - 1 = 15 - 6$ or $\tan 45 = \frac{k-1}{15-6}$	M1	oe equation eg $k - 1 = 9$ or $\frac{k-1}{9} = 1$
	$(k =)\ 15 - 6 + 1$ or $(k =)\ (15 - 6)\tan 45 + 1$ or $(k =)\ 10$	M1dep	oe eg $(k =)\ 9 + 1$ may be seen on diagram
	(5.5, 8)	A1	oe eg $\left(5\frac{1}{2},\ 8\right)$ or $\left(\frac{11}{2},\ 8\right)$ SC1 answer (…., 8)
	Additional Guidance		
	First M1 can be scored using Pythagoras' theorem and cosine rule eg $(PR^2 =)\ 9^2 + (15 - 6)^2$ or 162 and $(PQ =)\ 6 - 1$ or 5 and (angle QPR =)180 – 45 or 135 and $(k - 1)^2 + (15 - 1)^2$ = their 162 + (their 5)^2 $-2 \times \sqrt{\text{their } 162} \times$ their 5 × cos their 135	M1	
	10 but not seen or implied to be k	M0M0	

Q	Answer	Mark	Comments	
6	**Alternative method 1**			
	$y^2 = \frac{x+2w}{3}$	M1		
	$3y^2 - x = 2w$ or $\frac{3y^2 - x}{2}$ or $\frac{3y^2}{2} - \frac{x}{2}$	M1dep		
	$w = \frac{3y^2 - x}{2}$ or $w = \frac{3y^2}{2} - \frac{x}{2}$	A1		
	Alternative method 2			
	$y^2 = \frac{x}{3} + \frac{2w}{3}$	M1		
	$y^2 - \frac{x}{3} = \frac{2w}{3}$ or $\frac{3}{2}\left(y^2 - \frac{x}{3}\right)$ or $\frac{3y^2}{2} - \frac{3x}{6}$	M1dep		
	$w = \frac{3}{2}\left(y^2 - \frac{x}{3}\right)$ or $w = \frac{3y^2}{2} - \frac{3x}{6}$	A1		
	Additional Guidance			
	Condone eg $w = \frac{3y^2 - x}{2}$ seen in working with $\frac{3y^2 - x}{2}$ on answer line			M2A1
	$w = \frac{3}{2}y^2 - \frac{1}{2}x$ etc			M2A1

Q	Answer	Mark	Comments
7(a)	a^{4m} or a^{10m} or $4m = 10m$	M1	oe eg $a^{4\times m}$
	0	A1	
	Additional Guidance		
	Allow a to be replaced by any value greater than 1		

Q	Answer	Mark	Comments
7(b)	$w^{13}x^7 \div w^3x^2$ or $w^{10}x^5$ or $x^2y^5 = w^{10}x^7$ or $y^5 = \frac{w^{10}x^7}{x^2}$ or $w^3y^5 = w^{13}x^5$ or $y^5 = \frac{w^{13}x^5}{w^3}$	M1	oe eg $\frac{w^{13}x^7}{w^3x^2}$ may be embedded eg $\sqrt[5]{w^{10}x^5}$
	$w^2x^{(1)}$	A1	oe eg xw^2
	Additional Guidance		
	$y = w^{10}x^5$		M1A0

Q	Answer	Mark	Comments
8	-3 2 6 14 with no other solutions	B4	B3 three correct with at most one incorrect B2 two correct with at most two incorrect B1 one correct with at most three incorrect SC2 -3 2 6 14 with no other values seen SC1 Two or three of -3 2 6 14 with no other values seen
	Additional Guidance		
	Solutions may be in any order eg1 -3 14 6 2 eg2 14 -3		B4 B2
	$x < -3$ $2 < x < 6$ $x > 14$		SC2
	$2 \leqslant x \leqslant 6$		SC1
	-3 2 6 14 seen in working with no other values and answer line $-3 \leqslant x \leqslant 14$		SC2

Q	Answer	Mark	Comments
9(a)	$\frac{b+a}{ab}$	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
9(b)	$\frac{c^3}{6c+1}$	B3	B2 $c^3(6c-1)$ and $(6c+1)(6c-1)$ B1 $c^3(6c-1)$ or $(6c+1)(6c-1)$
	Additional Guidance		
	$\frac{c^3}{6c+1}$ followed by incorrect further work		B2

Q	Answer	Mark	Comments
10	**Alternative method 1**		
	$\frac{4}{3} \times \pi \times \frac{27k^3}{8}$ or $\frac{9\pi k^3}{2}$	M1	oe
	$k^3 = 972 \times 2 \div 9$ or $k^3 = 216$ or $\sqrt[3]{216}$	M1dep	oe eg $k^3 = 972\pi \times 2 \div 9\pi$ must have k^3 seen or implied
	6	A1	
	Alternative method 2		
	$972\pi \div \frac{4}{3}\pi$ or 729 or $972\pi \div \frac{4}{3}\pi$ or 729 or $\sqrt[3]{729}$ or 9	M1	oe eg (r^3 =) $972 \div \frac{4}{3}$
	$\frac{3k}{2} = \sqrt[3]{\text{their } 729}$ or $\frac{3k}{2} = 9$ or $9 \times 2 \div 3$	M1dep	oe must have $\frac{3k}{2}$ seen or implied
	6	A1	
	Additional Guidance		
	Alt 1 $\frac{4}{3} \times \frac{27k^3}{8}$ is M0 unless recovered		
	Alt 2 $972\pi \div \frac{4}{3}$ is M0 unless recovered		
	Alt 1 $\frac{4}{3} \times \pi \times \left(\frac{3k}{2}\right)^3$ not subsequently simplified correctly		M0

Q	Answer	Mark	Comments
11	3 terms from $20x^2 \quad -5xy^2 \quad (+)12xy^2 \quad -3y^4$	M1	may be seen in a grid
	$20x^2 - 5xy^2 + 12xy^2 - 3y^4$	A1	four correct terms in any order may be seen in a grid implied by correct answer
	$20x^2 + 7xy^2 - 3y^4$	A1	terms may be in any order
	Additional Guidance		
	Terms seen in a grid must have the correct signs		
	Terms must be fully processed eg do not allow $4x3y^2$ unless recovered		
	xy^2 may be y^2x throughout		
	$20x^2 + 7xy^2 - 3y^4$ followed by incorrect further work		M1A1A0

Q	Answer	Mark	Comments
12	10	B1	y-coordinate of C may be seen on the graph
	$(-)\frac{\text{their } 10}{5}$ or $(-)2$	M1	$\pm$ their gradient of L
	$(y =) -\frac{\text{their } 10}{5}x + \text{their } 10$	M1dep	oe eg $y - 0 = -\frac{\text{their } 10}{5}(x - 5)$ or $y - \text{their } 10 = -\frac{\text{their } 10}{5}(x - 0)$ must use a negative gradient
	$-\frac{\text{their } 10}{5}x + \text{their } 10 = 3x + 2$ or $5x = 8$	M1dep	oe
	1.6	A1ft	oe eg $\frac{8}{5}$ ft B0M3
	Additional Guidance		
	A1ft values must be exact or rounded to 1 decimal place or better		
	Ignore any y-coordinate of B calculated after working out the x-coordinate		
	Assuming the lines are perpendicular can score a maximum of B1		
	y-coordinate of $C = 8$	B0	
	gradient L $= -\frac{8}{5}$	M1	
	$y = -\frac{8}{5}x + 8$	M1	
	$-\frac{8}{5}x + 8 = 3x + 2$	M1	
	1.3 $\left(\text{Note that the exact value is } \frac{30}{23}\right)$	A1ft	

Q	Answer	Mark	Comments
13	$3ax^2$ or $20x$	M1	oe eg $3 \times ax^{3-1}$ or $2 \times 10x^{2-1}$
	$3a \times 2^2 + 20 \times 2$ or $12a + 40$	M1	ft substitution of $x = 2$ into their derivative must have attempted differentiation and have two terms with one involving a may be seen in a denominator
	their $(12a + 40) = -1 \div -\frac{1}{4}$ or their $(12a + 40) = 4$	M1dep	oe eg $-\frac{1}{\text{their } (12a+40)} = -\frac{1}{4}$ dep on 2nd M1
	-3	A1	
	Additional Guidance		
	Only substituting $x = 2$ into y		Zero
	$ax^2 + 10x$ $4a + 20 = 4$		M0 M1M1
	$3x^2 + 20x$ $12 + 20$		M1 M0M0

Q	Answer	Mark	Comments
14(a)	Reflection in the x-axis or reflection in $y = 0$	B1	
	Additional Guidance		
	Reflect(ed) in the x-axis		B1
	Do not allow if there is additional incorrect information eg1 Reflection in the x-axis about the origin eg2 Reflection in the x-axis and rotated		 B0 B0
	Reflection		B0

Q	Answer	Mark	Comments
14(b)	Rotation through 180° centre the origin or enlargement scale factor –1 centre the origin	B2	B1 $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ or enlargement scale factor -1 or rotation through 180° or indication that **B** represents rotation through 270° (anticlockwise centre the origin) or indication that **B** represents rotation through 90° clockwise (centre the origin)

Additional Guidance	
For B2 ignore any reference to clockwise or anticlockwise rotation	
Condone omission of degrees symbol throughout eg B is rotation through 270	 B1
Mark intention eg1 Rotate(d) 180 about O eg2 Enlarge(d) sf –1	 B2 B1
Allow rotation through 540 centre the origin	B2
Do not allow if there is additional information that is incorrect eg1 Rotation through 180° and a reflection eg2 Enlargement sf –1 rotated through 90° eg3 Rotation through 180° centre the origin so the shape turns	 B0 B0 B2
Rotation	B0
Enlargement	B0
Do not allow turn for rotation	
Do not allow eg half turn for 180°	
Do not allow negative enlargement	

Q	Answer	Mark	Comments
15	**Alternative method 1**		
	angle $ABO = x$	M1	may be seen on diagram implied by angle $AOB = 180 - 2x$
	angle $ACB = 180 - w$	M1	oe eg angle $ACB + w = 180$ may be seen on diagram
	angle $AOB = 2 \times (180 - w)$ or angle $AOB = 360 - 2w$	M1dep	may be seen on diagram dep on 2nd M1 angle AOB may be seen as $180 - 2x$
	$x + x + 2 \times (180 - w) = 180$	M1dep	oe eg $2(180 - w) = 180 - 2x$ dep on M3
	$w = x + 90$ with M4 and all reasons given	A1	eg of reasons isosceles triangle and angles on a straight line and angle at centre and angle sum of triangle
	Alternative method 2		
	angle $ABO = x$	M1	may be seen on diagram implied by angle $AOB = 180 - 2x$
	angle $AOB = 180 - x - x$ or angle $AOB = 180 - 2x$	M1dep	oe eg $2x$ + angle $AOB = 180$ may be seen on diagram
	angle $ACB = \frac{1}{2} \times (180 - x - x)$ or angle $ACB = 90 - x$	M1dep	oe eg angle $ACB = \frac{1}{2} \times (180 - 2x)$ may be seen on diagram angle ACB may be seen as $180 - w$
	$\frac{1}{2} \times (180 - x - x) + w = 180$	M1dep	oe eg $w = 180 - (90 - x)$
	$w = x + 90$ with M4 and all reasons given	A1	eg of reasons isosceles triangle and angle sum of triangle and angle at centre and angles on a straight line

Mark scheme and Additional Guidance continues on the next two pages

Q	Answer	Mark	Comments
15 cont	**Alternative method 3** Draws tangent (eg PQ) at A		
	angle $QAB = 90 - x$	M1	oe eg x + angle $QAB = 90$ may be seen on diagram
	angle $ACB = 180 - w$	M1	oe eg angle $ACB + w = 180$ may be seen on diagram
	angle QAB = angle ACB	M1	may be seen on diagram eg both angles labelled y
	$90 - x = 180 - w$	M1dep	oe eg $90 - x + w = 180$ dep on M3
	$w = x + 90$ with M4 and all reasons given	A1	eg of reasons radius perpendicular to tangent and angles on a straight line and alternate segment

Additional Guidance is on the next page

	Additional Guidance	
15 cont	Allow angle *BCD* for w throughout	
	3rd M1 and 4th M1 may be seen in one line of working eg1 Alt 1 angle $ABO = x$ angle $ACB = 180 - w$ $180 - 2x = 2 \times (180 - w)$ eg2 Alt 2 angle $ABO = x$ angle $AOB = 180 - 2x$ $180 - w = \frac{1}{2} \times (180 - 2x)$	 M1 M1 M1M1 M1 M1 M1M1
	Condone slips in notation only if angles are marked in correct position on the diagram eg1 Do not allow angle $C = 180 - w$ unless marked in correct position on the diagram eg2 Allow *ACB* for angle *ACB*	
	For reasons, allow if the intention is clear eg1 Allow isos triangle for isosceles triangle eg2 Allow angles in a triangle for angle sum of a triangle eg3 Allow angles on a line for angles on a straight line	
	For reasons do not allow incorrect statements eg do not allow angles in a triangle add to 360	

Q	Answer	Mark	Comments
16	15×2^4 or 15×16 or 240	M1	oe eg $\binom{6}{4}2^4$ or $2^6 \times \frac{6\times5}{2} \times \left(\frac{1}{2}\right)^2$ may include a^2 and/or x^4 allow embedded eg ${}^6C_4\, a^2(2x)^4$
	$240a^2 = 1500$ or $a^2 = \frac{1500}{240}$ or $(\pm)\sqrt{\frac{1500}{240}}$ or $\frac{5}{2}$ or $-\frac{5}{2}$	M1dep	must evaluate $\binom{6}{4}$ oe eg $15 \times 2^4a^2 = 1500$ or $(\pm)\sqrt{\frac{1500}{15\times2^4}}$ may include x^4 on both sides of an equation
	$\frac{5}{2}$ and $-\frac{5}{2}$ with no other values	A1	oe eg 2.5 and –2.5 SC2 [2.236, 2.24] and [–2.24, –2.236] SC1 [2.236, 2.24] or [–2.24, –2.236]
	Additional Guidance		
	The relevant term must be selected from a full expansion but the other terms can be ignored		
	Allow $\binom{6}{4}$ to be $\binom{6}{2}$		
	$240a^2x^4 = 1500x^4$		M1M1
	$240a^2x^4 = 1500$ recovered to $(\pm)\sqrt{\frac{1500}{240}}$ oe		M1M1
	$240a^2x^4 = 1500$ not recovered to $(\pm)\sqrt{\frac{1500}{240}}$ oe		M1M0

Q	Answer	Mark	Comments
17	**Alternative method 1** Works out *MD* and *BD* and uses $\tan MBD$		
	$\tan 28 = \frac{GN}{32}$ or $32\tan 28$ or [17, 17.015]	M1	oe eg $\frac{32}{\tan(90-28)}$ working out *GN* or *HM*
	$32 - 32\tan 28$ or [14.985, 15]	M1dep	oe working out *NC* or *MD*
	$\sqrt{32^2+32^2}$ or $\sqrt{2048}$ or [45.2, 45.3]	M1	oe eg $32\sqrt{2}$ working out *BD*
	$\tan MBD = \frac{\text{their [14.985, 15]}}{\text{their [45.2, 45.3]}}$	M1dep	oe eg $\tan^{-1}\frac{\text{their [14.985, 15]}}{\text{their [45.2, 45.3]}}$ dep on M3
	[18.3, 18,4]	A1	
	Alternative method 2 Works out *BD* and *MB* and uses $\cos MBD$		
	$\tan 28 = \frac{GN}{32}$ or $32\tan 28$ or [17, 17.015]	M1	oe eg $\frac{32}{\tan(90-28)}$ working out *GN* or *HM*
	$32 - 32\tan 28$ or [14.985, 15]	M1dep	oe working out *NC* or *MD*
	$\sqrt{32^2+32^2}$ or $\sqrt{2048}$ or [45.2, 45.3] or $\sqrt{32^2+32^2+\text{their [14.985,15]}^2}$ or [47.67, 47.7]	M1	oe eg $32\sqrt{2}$ working out *BD* or *MB* if awarding this mark for working out *MB* it is dependent on M2
	$\cos MBD =$ $\frac{\sqrt{32^2+32^2}}{\sqrt{32^2+32^2+\text{their [14.985, 15]}^2}}$	M1dep	oe eg $\cos^{-1}\frac{\sqrt{32^2+32^2}}{\sqrt{32^2+32^2+\text{their [14.985, 15]}^2}}$ dep on M3
	[18.3, 18,4]	A1	

Mark scheme and Additional Guidance continue on the next page

17 cont	**Alternative method 3** Works out *MD* and *MB* and uses $\sin MBD$		
	$\tan 28 = \frac{GN}{32}$ or $32\tan 28$ or [17, 17.015]	M1	oe eg $\frac{32}{\tan(90-28)}$ working out *GN* or *HM*
	$32 - 32\tan 28$ or [14.985, 15]	M1dep	oe working out *NC* or *MD*
	$\sqrt{32^2 + 32^2 + \text{their [14.985,15]}^2}$ or [47.67, 47.7]	M1dep	oe working out *MB*
	$\sin MBD = \frac{\text{their [14.985,15]}}{\sqrt{32^2 + 32^2 + \text{their [14.985, 15]}^2}}$	M1dep	oe eg $\sin^{-1}\frac{\text{their [14.985,15]}}{\sqrt{32^2 + 32^2 + \text{their [14.985, 15]}^2}}$
	[18.3, 18,4]	A1	
	Additional Guidance		
	1st M1 *GN* may be seen as a letter, eg x, but do not award if subsequently used as the length of an incorrect side (eg *MN*)		
	4th M1 *MBD* may be seen as a letter, eg y, but do not award if subsequently used as the size of an incorrect angle (eg *DMB*)		
	Alt 1 or Alt 2 $32\sqrt{1^2 + 1^2}$		3rd M1
	Alt 1 $\tan MBD = \frac{32(1-\tan 28)}{32\sqrt{2}}$ or $\tan MBD = \frac{(1-\tan 28)}{\sqrt{2}}$		M4

Q	Answer	Mark	Comments
18	**Alternative method 1**		
	12 or $-3x^{-2}$	M1	oe eg $12x^0$ or $3 \times -1x^{-1-1}$ or $-\frac{3}{x^2}$
	12 and $-3x^{-2}$	M1dep	oe eg $12 - \frac{3}{x^2}$ or $12x^0$ and $3 \times -1x^{-1-1}$
	$12 - 3x^{-2} = 0$ and $x = 0.5$ or $12 - 3 \times 0.5^{-2} = 0$	M1dep	oe = 0 must be seen condone inclusion of $x = -0.5$
	$6x^{-3}$	M1	oe eg $-2 \times -3x^{-2-1}$ ft differentiation of their first derivative if it involves a negative power of x
	M4 and 6×0.5^{-3} (= 48) which is positive (so minimum)	A1	oe do not allow if $\frac{6}{0.5^3}$ is evaluated incorrectly

Mark scheme and Additional Guidance continue on the next page

18 cont	**Alternative method 2**		
	12 or $-3x^{-2}$	M1	oe eg $12x^0$ or $3 \times -1x^{-1-1}$
	12 and $-3x^{-2}$	M1dep	oe eg $12 - \frac{3}{x^2}$ or $12x^0$ and $3 \times -1x^{-1-1}$
	$12 - 3x^{-2} = 0$ and $x = 0.5$ or $12 - 3 \times 0.5^{-2} = 0$	M1dep	oe = 0 must be seen condone inclusion of $x = -0.5$
	Substitutes one x value in range (0, 0.5) into $12 - 3x^{-2}$ and substitutes one x value > 0.5 into $12 - 3x^{-2}$	M1	eg $12 - 3 \times 0.25^{-2}$ and $12 - 3 \times 1^{-2}$ ft substitution into their first derivative if it involves a negative power of x
	M4 and two correct evaluations (so minimum) or M4 and two correct signs shown with no incorrect evaluations (so minimum)	A1	eg M4 and $12 - 3 \times 0.25^{-2} = -36$ and $12 - 3 \times 1^{-2} = 9$ (so minimum) or M4 and $12 - 3 \times 0.25^{-2}$ is negative and $12 - 3 \times 1^{-2}$ is positive (so minimum)
	Additional Guidance		
	Alt 1 $12 + 3x^{-2} = 0$ $-6x^{-3}$		M1M0M0 M1A0
	Alt 2 $12 - 3x^{-2}$ $6x^{-3}$ $12 - 3 \times 0.25^{-2} = -36$ $12 - 3 \times 1^{-2} = 9$ so minimum (A1 only possible after awarding M4)		M1M1M0 M1 A0
	Ignore any testing of the stationary point at $x = -0.5$		

Q	Answer	Mark	Comments
19(a)	x^4	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
19(b)	$2x^2 + 10$ or $2(x^2 + 5)$	B2	B1 $k(x) = 2x$ or $(k(x))^2 = 4x^2$ or $h(2x) = 4x^2 + 5$ or $(2x)^2 + 5$
	Additional Guidance		
	$2(x^2 + 5)$ in working with answer $2x^2 + 5$		B1

Q	Answer	Mark	Comments
20	**Alternative method 1** Uses a common denominator of $\sin x$		
	$\dfrac{2\sin x+\cos x}{\dfrac{\sin x}{\cos x}}-\dfrac{1}{\sin x}$ or $\dfrac{\cos x(2\sin x+\cos x)}{\sin x}-\dfrac{1}{\sin x}$	M1	implied by 2nd M1 condone omission of $-\dfrac{1}{\sin x}$
	$\dfrac{2\sin x\cos x+\cos^2 x}{\sin x}-\dfrac{1}{\sin x}$ or $2\cos x+\dfrac{\cos^2 x}{\sin x}-\dfrac{1}{\sin x}$	M1dep	condone omission of $-\dfrac{1}{\sin x}$
	$\dfrac{2\sin x\cos x+\cos^2 x-\cos^2 x-\sin^2 x}{\sin x}$ or $\dfrac{2\sin x\cos x+1-\sin^2 x-1}{\sin x}$ or $\dfrac{2\sin x\cos x-\sin^2 x}{\sin x}$ or $2\cos x-\dfrac{\sin^2 x}{\sin x}$	M1dep	
	$2\cos x-\sin x$ with M3	A1	

Mark scheme and Additional Guidance continues on the next two pages

20 cont	**Alternative method 2** Uses a common denominator of $\tan x \sin x$		
	$\dfrac{\sin x(2\sin x + \cos x) - \tan x}{\tan x \sin x}$ or $\dfrac{2\sin^2 x + \cos x \sin x - \tan x}{\tan x \sin x}$	M1	$\tan x$ may be seen as $\dfrac{\sin x}{\cos x}$
	$\dfrac{2\sin^2 x \cos x + \cos^2 x \sin x - \sin x}{\sin^2 x}$	M1dep	
	$\dfrac{2\sin x \cos x + \cos^2 x - \cos^2 x - \sin^2 x}{\sin x}$ or $\dfrac{2\sin x \cos x + 1 - \sin^2 x - 1}{\sin x}$ or $\dfrac{2\sin x \cos x - \sin^2 x}{\sin x}$	M1dep	allow the fractions with denominator $\sin^2 x$ eg $\dfrac{2\sin^2 x \cos x + \sin x - \sin^3 x - \sin x}{\sin^2 x}$
	$2\cos x - \sin x$ with M3	A1	

Additional Guidance is on the next page

	Additional Guidance	
	$2\cos x - \sin x$ with no method	Zero
	Allow eg $1\sin x$ for $\sin x$	
	Condone x missing for M marks and award A mark if recovered	
	Fractions may be seen as separate fractions/terms eg1 Alt 1 $\frac{2\sin x}{\frac{\sin x}{\cos x}} + \frac{\cos x}{\frac{\sin x}{\cos x}} - \frac{1}{\sin x}$	M1
	$2\cos x + \frac{\cos^2 x}{\sin x} - \frac{1}{\sin x}$	M1
	$2\cos x + \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} - \frac{1}{\sin x}$	M1
	eg2 Alt 2 $\frac{2\sin^2 x + \cos x \sin x}{\tan x \sin x} - \frac{\tan x}{\tan x \sin x}$	M1
	$\frac{2\sin^2 x \cos x + \cos^2 x \sin x}{\sin^2 x} - \frac{\sin x}{\sin^2 x}$	M1
	$\frac{2\sin x \cos x + \cos^2 x}{\sin x} - \frac{\cos^2 x + \sin^2 x}{\sin x}$	M1

Q	Answer	Mark	Comments
21	**Alternative method 1**		
	$3(x^2 + ax + ax + a^2)$ … or $3(x^2 + 2ax + a^2)$ … or $3\left(x + \frac{b}{3}\right)^2$ … or $2b = 6a$ or $8a = 3a^2 + b + 2$	M1	oe eg $3x^2 + 6ax + 3a^2$ … or $\frac{b}{3} = a$ or $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
	$2b = 6a$ and $8a = 3a^2 + b + 2$	M1dep	oe equations eg $\frac{b}{3} = a$ and $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
	$3a^2 + 3a - 8a + 2$ (= 0) or $3a^2 - 5a + 2$ (= 0)	M1dep	oe quadratic equation in a
	$(3a - 2)(a - 1)$ or $\frac{--5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 2}}{2 \times 3}$	M1	oe eg $\frac{5}{6} \pm \sqrt{\frac{25}{36} - \frac{2}{3}}$ ft their 3-term quadratic
	$a = \frac{2}{3}$ and $a = 1$ or $a = \frac{2}{3}$ and $b = 2$ or $a = 1$ and $b = 3$	A1	
	$a = \frac{2}{3}$ and $b = 2$ and $a = 1$ and $b = 3$	A1	

Mark scheme and Additional Guidance continue on the next page

21 cont	**Alternative method 2**		
	$3(x^2 + ax + ax + a^2)$ … or $3(x^2 + 2ax + a^2)$ … or $3\left(x + \frac{b}{3}\right)^2$ … or $2b = 6a$ or $8a = 3a^2 + b + 2$	M1	oe eg $3x^2 + 6ax + 3a^2$ … or $\frac{b}{3} = a$ or $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
	$2b = 6a$ and $8a = 3a^2 + b + 2$	M1dep	oe equations eg $\frac{b}{3} = a$ and $b + 2 = -3\left(\frac{b}{3}\right)^2 + 8a$
	$\frac{8b}{3} = 3\left(\frac{b}{3}\right)^2 + b + 2$ or $b^2 - 5b + 6$ (= 0)	M1dep	oe quadratic equation in b
	$(b - 2)(b - 3)$ or $\frac{--5 \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$	M1	oe eg $\frac{5}{2} \pm \sqrt{\frac{25}{4} - 6}$ ft their 3-term quadratic
	$b = 2$ and $b = 3$ or $a = \frac{2}{3}$ and $b = 2$ or $a = 1$ and $b = 3$	A1	
	$a = \frac{2}{3}$ and $b = 2$ and $a = 1$ and $b = 3$	A1	
	Additional Guidance		
	Allow $0.\dot{6}$ for $\frac{2}{3}$		
	Allow 0.67 for $\frac{2}{3}$ for first A1		
	In quadratic formula allow 5^2 for $(-5)^2$ but use of -5^2 must be recovered		