CAMBRIDGE
International Education

# Cambridge International A Level

**MATHEMATICS** **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2025**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.



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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| Annotation | Meaning |
|---|---|
|  | More information required |
|  | Accuracy mark awarded zero |
|  | Accuracy mark awarded one |
|  | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one |
|  | Independent accuracy mark awarded two |
|  | Benefit of the doubt |
|  | Blank Page |
|  | Incorrect |
| Dep | Used to indicate DM0 or DM1 |

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| Annotation | Meaning |
|---|---|
| DM1 | Dependent on the previous M1 mark(s) |
| FT | Follow through |
|  | Indicate working that is right or wrong |
| Highlighter | Highlight a key point in the working |
| ISW | Ignore subsequent work |
| J | Judgement |
| JU | Judgement |
| M0 | Method mark awarded zero |
| M1 | Method mark awarded one |
| M2 | Method mark awarded two |
| MR | Misread |
| O | Omission or Other solution |
| Off-page comment | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment | Allows comments to be entered in speech bubbles on the candidate response. |
| PE | Judgment made by the PE |
| Pre | Premature approximation |
| SC | Special case |
| SEEN | Indicates that work/page has been seen |

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| Annotation | Meaning |
|---|---|
| SF | Error in number of significant figures |
|  | Correct |
| TE | Transcription error |
| XP | Correct answer from incorrect working |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | State or imply non-modular inequality $(3x+2)^2 < 3^2(2x-1)^2$, or pair of linear equations $(3x + 2) = \pm 3(2x - 1)$ | **B1** | Allow '=', or any inequality sign. |
| | Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for $x$ | **M1** | E.g. $27x^2 - 48x + 5 = 0$, $(9x - 1)(3x - 5) = 0$, $x = \ldots$<br>Allow even if quadratic not given in a 3-term form.<br>See guidelines document for solving a quadratic. |
| | Obtain critical values $x = \frac{1}{9}$ and $x = \frac{5}{3}$ | **A1** | Allow '=', or any inequality sign. |
| | State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only | **A1** | Allow 'OR' but not 'AND'.<br>Allow '$\cup$' but not '$\cap$'.<br>No marks can be scored if no working is seen. |
| | **Alternative Method for Question 1** | | |
| | Obtain critical value $x = \frac{5}{3}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality | **B1** | Allow '=', or any inequality sign. |
| | Obtain critical value $x = \frac{1}{9}$ similarly | **B2** | Allow '=', or any inequality sign. |
| | State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only | **B1** | Allow 'OR' but not 'AND'.<br>Allow '$\cup$' but not '$\cap$'.<br>No marks can be scored if no working is seen. |
| | | **4** | |

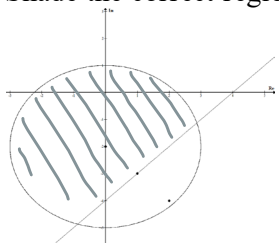
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Commence division and reach partial quotient of the form $3x^3 \pm 3x^2$ or $3x^4 - 2x^2 \equiv (x+1)(Ax^3 + Bx^2 + Cx + D) + Ex + F$, and reach $A = 3$ and $B = \pm 3$ | **M1** | May be seen in synthetic division. |
| | Obtain quotient $3x^3 - 3x^2 + x - 1$ Do not ISW | **A1** | Don't need to state which is the quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct. |
| | Obtain remainder of 1 | **A1** | Do not ISW.<br>Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$. |
| | **Alternative Method for Question 2** | | |
| | f (–1) = 3 – 2 = 1 = remainder | **B1** | Do not ISW. |
| | Use division or inspection or compare coefficients | **M1** | $3x^4 - 2x^2 - 1 \equiv (x+1)(3x^3 - 3x^2 + x - 1)$ |
| | Obtain quotient $3x^3 - 3x^2 + x - 1$ | **A1** | Do not ISW.<br>Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use log quotient law, e.g. $\ln \frac{3}{5^x} = \ln 3 - \ln 5^x$<br>or log product law, e.g. $\ln 5^x + \ln(2^{3x-4})$ | **M1** | May work in log to any base for first 3 marks. |
| | Use log power law, e.g. $\ln 5^x = x \ln 5$ or $\ln(2^{3x-4}) = (3x-4)\ln 2$ | **M1** | Condone missing brackets if recovered at some point. |
| | Obtain correct expression for $x$ in any exact form, e.g. $\frac{\ln 3 + 4\ln 2}{3\ln 2 + \ln 5}$ | **A1** | |
| | Obtain final answer $\frac{\ln 48}{\ln 40}$ | **A1** | Do not ISW.<br>Final answer of $\log_{40} 48$ scores 3 marks only.<br>No working award 0 marks. |
| | **Alternative Method for Question 3** | | |
| | Rearrange to obtain $(2^3 \times 5)^x = 3 \times 2^4$ | **B1** | OE |
| | Use log power law on an equation of the form $a^{bx} = c$<br>e.g. $x \ln(2^3 \times 5) = \ln(3 \times 2^4)$ | **M1** | May work in log to any base for first 3 marks. |
| | Obtain correct expression for $x$ in any exact form, e.g. $x = \frac{\ln(3 \times 2^4)}{\ln(2^3 \times 5)}$ | **A1** | |
| | Obtain final answer $\frac{\ln 48}{\ln 40}$ | **A1** | Do not ISW.<br>Final answer of $\log_{40} 48$ scores 3 marks only.<br>No working award 0 marks. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Show a circle centre (0, –2) | **B1** | For all marks: Accept a scale or dashes representing a scale or points labelled. If scale only on one axis, allow this to imply the same scale on the other axis. Condone dashed circle and dashed perpendicular bisector for all marks |
| | Show a circle with radius 3 | **B1FT** | FT centre not at the origin.<br>Allow circle with 3 radii correct out of 4 'compass directions'.<br>If no indication of scale, allow **SC B1FT** only for circle. |
| | Show the point representing (2, –4) or the midpoint (1, –3) | **B1** | May be implied by a correct perpendicular bisector.<br>Condone if (2, –4) on the circle. |
| | Show the perpendicular bisector of the line joining (2, –4) and (0, –2)<br>or the perpendicular bisector of the line joining (2, –4) and centre of their circle<br>or the perpendicular bisector going through (1, –3) | **B1FT** | FT is on the positions of (2, –4) and (0, –2) or on the position of (2, –4) and centre of their circle or on the position of (1, –3).<br>Cuts (or would cut) $x$-axis between 3 and 5 and $y$-axis between –5 and –3 if everything else correct.<br>If no indication of scale, allow **SC B1FT** only for perpendicular bisector. |
| | Shade the correct region<br> | **B1** | Dependent on all previous marks.<br>Allow **SC B1** for correct shading if the perpendicular bisector looks correct and the only error is that it is slightly out when crossing the axes.<br>If no indication of scale, allow **SC B1** for shading if everything is relatively correct. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Use correct double angle formula to express $\cos 4x$ in terms of $\sin^2 2x$ | **B1** | Need to see $1 - 2\sin^2 2x$ or $(1 - \sin^2 2x) - \sin^2 2x$. May be implied by further work. |
| | Use correct double angle formula to express $\cos 4x$ in terms of single angles | **M1** | Allow with $\cos x$ but not $\cos 2x$. E.g. $1 - 2(2\sin x \cos x)^2$ or $2(1 - 2\sin^2 x)^2 - 1$. |
| | Obtain a correct expression in $\sin x$ in any form | **A1** | E.g. $1 - 2[4\sin^2 x(1 - \sin^2 x)] + 2\sin^2 x - 1$, or $2(1 - 2\sin^2 x)^2 - 1 + 2\sin^2 x - 1$. |
| | Obtain the given answer $8\sin^4 x - 6\sin^2 x$ | **A1** | AG<br>Must show at least one intermediate line of working including $\sin^4 x$ between first A1 and AG.<br>Allow, e.g., $A$ for $x$ except in the final answer.<br>Allow recovery on the next line after a slip.<br>Allow recovery on the next line after missing $x$. |
| | **Alternative Method for Question 5(a)** | | |
| | Use correct double angle formula to express whole expression in terms of $\cos 2x$ | **M1** | E.g. $(1 - \cos 2x)(2 - 2\cos 2x - 3)$. |
| | Use correct double angle formula to express $2\cos^2 2x - 1$ as $\cos 4x$ | **B1** | |
| | Use correct double angle formula to express whole expression as $\cos 4x - \cos 2x$ | **A1** | |
| | Obtain the given answer $\cos 4x + 2\sin^2 x - 1$ | **A1** | AG<br>Allow, e.g., $A$ for $x$ except in the final answer.<br>Allow recovery on the next line after a slip.<br>Allow recovery on the next line after missing $x$. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | Obtain answers 0°, 180° and –180° | **B1** | |
| | Carry out a correct method to find a value of $x$ in the given interval for $8\sin^2 x - 6 = 0$ | **M1** | Condone a wrong value of $x$ if $\sin x = \sqrt{\frac{3}{4}}$ OE seen.<br>Allow M1A1A1 if dividing by $\sin^2 x$, but B1 is not scored. |
| | Obtain answer, e.g. 60° | **A1** | In radians, would be $\frac{1}{3}\pi$. |
| | Obtain remaining answers, e.g. –60°, 120° and –120° and no other in the interval | **A1** | Ignore answers outside the given interval. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | Commence integration by parts and reach $Ax^2\cos 2x - \int Bx\cos 2x\,dx$ | ***M1** | OE<br>Condone sign error in formula. |
| | Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x\,dx$ | **A1** | OE<br>Unsimplified. |
| | Complete integration by parts and reach $Ax^2\cos 2x + Bx\sin 2x + C\cos 2x$ | ***M1** | OE<br>$\int Bx\cos 2x\,dx = Bx\sin 2x + C\cos 2x$ may be written separately to $Ax^2\cos 2x$ for M1, but not for A1. |
| | Obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ | **A1** | OE<br>Unsimplified. |
| | Substitute limits correctly in an expression of the form $Ax^2\cos 2x + Bx\sin 2x + C\cos 2x$ and evaluate the trig expressions | **DM1** | $A\left(\frac{\pi}{6}\right)^2\left(\frac{1}{2}\right) + B\left(\frac{\pi}{6}\right)\left(\frac{\sqrt{3}}{2}\right) + C\left(\frac{1}{2}\right) - C$<br>Allow one slip, including omitting the '– $C$'.<br>Do not allow only decimals |
| | Obtain answer $-\frac{1}{144}\pi^2 + \frac{\sqrt{3}}{24}\pi - \frac{1}{8}$ or exact three-term equivalent | **A1** | ISW<br>Allow equivalent fractions.<br>Allow 0.125 for $\frac{1}{8}$. |
| | | **6** | |

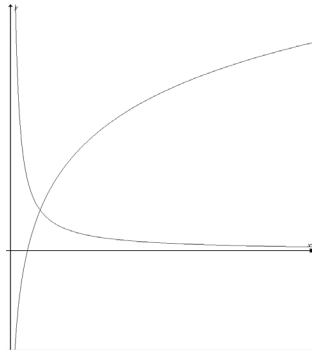
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7 | Attempt to remove fraction by multiplying throughout by $2 - i$ or multiply fraction by $\frac{2+i}{2+i}$ | ***M1** | Allow if still includes brackets and/or $i^2$.<br>E.g. $5z - zz^*(2 - i) + (20 + 8i)(2 - i) = 0$ or better.<br>E.g. $\frac{5z}{2-i} \times \frac{2+i}{2+i} - zz^* + 20 + 8i = 0$ or better. |
| | Substitute $z = x + iy$ and $z^* = x - iy$ throughout the equation | **B1** | May see $5x + 5iy - 2(x^2 + y^2) + i(x^2 + y^2) + 48 - 4i = 0$, or $2x + ix + 2iy - y - x^2 - y^2 + 20 + 8i = 0$. |
| | Use $i^2 = -1$ correctly at least once and equate real and imaginary parts to zero | ***DM1** | OE, e.g. $5x - 2x^2 - 2y^2 + 48 = 0$ and $5y + x^2 + y^2 - 4 = 0$, or $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$.<br>For *their* horizontal equation. |
| | Obtain two correct equations e.g. $5x - 2(x^2 + y^2) + 48 = 0$ and $5y + (x^2 + y^2) - 4 = 0$ | **A1** | E.g. $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$.<br>Allow $5iy + i(x^2 + y^2) - 4i = 0$ or $ix + 2iy + 8i = 0$. |
| | Solve a quadratic and a linear equation for $x$ or for $y$ | **DM1** | |
| | Obtain answers $2 - 5i$ and $-\frac{16}{5} - \frac{12}{5}i$ only | **A1** | Accept $x = 2$ $y = -5$ and $x = -\frac{16}{5}$ $y = -\frac{12}{5}$ only<br>OE, or $(2, -5)$ and $\left(-\frac{16}{5}, -\frac{12}{5}\right)$ only OE.<br>Allow decimals.<br>Do not ISW. |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Use the correct product or quotient rule, e.g. $e^{-5x}\ \frac{d}{dx}(\ln 5x) + \ln 5x\ \frac{d}{dx}(e^{-5x})$ | **M1** | M0 if $y = e^{-5p} \ln 5p$ seen prior to differentiation. Accept if only seen in actual derivative = 0. |
| | Obtain the correct derivative in any form e.g. $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x$ | **A1** | |
| | Obtain the given answer $\ln 5p = \frac{1}{5p}$ after full and correct working | **A1** | AG<br>May go from $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x = 0$, or<br>$\frac{1}{x}e^{-5x} = 5e^{-5x} \ln 5x$ to the given answer without intermediate working. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(b) | Sketch an acceptable graph, e.g. $y = \ln 5x$ or $y = \frac{1}{5x}$<br> | **M1** | For both marks:<br>Note: Allow without scale on either axis, but if $y = \ln 5x = 0$ identified to be not $x = 0.2$, then 0 marks for $y = \ln 5x$.<br>Allow graphs not labelled, or labelled with $p$ instead of $x$.<br>Allow $\ln 5x$ starting at the $x$-axis.<br>If either graph shown in other quadrants, must be correct.<br>For $y = \frac{1}{5x}$, asymptotic behaviour needed for at least one axis. Must not touch axes. |
| | Sketch a second acceptable graph, e.g. $y = \frac{1}{5x}$ or $\ln 5x$, and justify the given statement by dot, cross or statement only one intersection. | **A1** | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(c) | Calculate the values of a relevant expression or pair of expressions at $p = 0.2$ and $p = 0.6$<br><br>Note can use, e.g., $p = 0.3$ and $p = 0.5$, or any smaller interval which works | **M1** | $f(p) = \ln 5p - \frac{1}{5p}$<br>$f(0.2) = -1 < 0$, $f(0.6) = 0.765 > 0$<br>At least one correct value to at least 2sf.<br>Or comparing $\ln 5p$ and $\frac{1}{5p}$.<br>At least 3 correct values to at least 2sf. |
| | Complete the argument correctly with correct calculated values | **A1** | |
| | | **2** | |
| 8(d) | Use the iterative formula correctly at least twice | **M1** | M0 for 0.3526, 0.3526, 0.3526… |
| | Obtain final answer $p = 0.35$, Answer = 0.35, or just 0.35 stated | **A1** | Allow, e.g., $a_1$, $a_2$, $a_3$ … or $x_1$, $x_2$, $x_3$ … or answer$_1$ , answer$_2$, answer$_3$ … for M1 and second A1.<br>For first A1, must be $p = 0.35$ or answer = 0.35 unless just 0.35 is stated, e.g. not $x = 0.35$, $p_7 = \ldots$, $p_\infty = \ldots$ etc. |
| | Show sufficient iterations to 4 dp to justify 0.35 to 2 dp or show there is a sign change in the interval (0.345, 0.355)<br><br>0.2, 0.5437, 0.2889, 0.3996, 0.3299, 0.3667, 0.3451, 0.3571, 0.3502, 0.3541<br>0.25, 0.4451, 0.3135, 0.3786, 0.3392, 0.3607, 0.3482, 0.3552, 0.3512, 0.3535<br>0.3, 0.3895, 0.3342,0.3639, 0.3465, 0.3562, 0.3507, 0.3538 0.3520, 0.3530<br>0.45, 0.3119, 0.3797, 0.3387, 0.3610, 0.3480, 0.3553, 0.3512, 0.3535<br>0.5, 0.2984,0.3910, 0.3336, 0.3643, 0.3463, 0.3563, 0.3506, 0.3538<br>0.55, 0.2877, 0.4008, 0.3294, 0.3670, 0.3449, 0.3572, 0.3501, 0.3541<br>0.6, 0.2791, 0.4095, 0.3260, 0.3694, 0.3437, 0.3579, 0.3497, 0.3543 | **A1** | E.g. 0.4, 0.3297, 0.3668, 0.3450, 0.3571, 0.3502, 0.3541.<br>Allow M1(A1 or A0) A1 if more values are to at least 4dp than to 3dp.<br>**SC B1** for starting from either 0.3526 or 0.3527 and using iterative formula correctly at least twice if the sequence shows a correct change in the 4th decimal place (and **SC DB1** for getting $p = 0.35$), but 0 marks otherwise. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | Obtain $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ | **B1** | OE<br>Must have **r** = …, but penalise missing **r** = only once in **(a)**.<br>Do not allow column vectors with **i**, **j** and **k** included. |
| | Carry out a correct method for finding a direction vector for $l_2$, $(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})\cdot(y\mathbf{j} + z\mathbf{k}) = 0$ | **M1** | E.g. $-2y + z = 0$ SOI. |
| | Obtain $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} + \mu(\mathbf{j} + 2\mathbf{k})$ | **A1** | OE<br>Must have **r** = …, but penalise missing **r** = only once in **(a)**.<br>Do not allow column vectors with **i**, **j** and **k** included. |
| | | **3** | |
| 9(b) | Carry out correct process for evaluating the scalar product of the direction vectors of $l_1$ and $l_2$ | ***M1** | Using *their* direction vectors from **(a)**.<br>Ignore symbol if state e.g. '×' in place of '·'.<br>Allow the same parameter for both lines here. |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and state $\cos\theta$ = the result | **DM1** | Using *their* direction vectors from **(a)**. |
| | Obtain answer 28.6° or 0.498$^c$ | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(c) | Equate components of general points on *their* $l_1$ and *their* $l_2$, provided these are both equations of lines and solve for $\lambda$ or for $\mu$ | **M1** | $(3 + 2\lambda, 1 + \lambda, -6 + 4\lambda) = (-1, 3 + \mu, -6 + 2\mu)$<br>Do not allow the same parameter for both lines here if solving using two linear equations.<br>Allow M1 for $3 + 2\lambda = -1 \Rightarrow \lambda = \ldots$ even if the other equations are incorrect or not stated. |
| | Obtain correct answer for $\lambda$ or $\mu$, e.g. $\lambda = -2$, $\mu = -4$ | **A1** | Allow M1A1 for $3 + 2\lambda = -1$ leading to $\lambda = -2$, even if the other equations are incorrect or not stated. |
| | Obtain position vector of point of intersection is $-\mathbf{i} - \mathbf{j} - 14\mathbf{k}$ | **A1** | OE<br>Do not accept coordinates.<br>Allow even if **j** and **k** equations are incorrect or not stated. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | Carry out a relevant method to find $A$ and $B$ such that $\frac{2}{1-9y^2} = \frac{A}{1+3y} + \frac{B}{1-3y}$ o | **M1** | OE<br>Allow M1 for finding $A$ and $B$ for $\frac{A}{1+3y} + \frac{B}{3y-1}$<br>and A1 for $A = 1$, $B = -1$ if $-2 = A(3y - 1) + B(1 + 3y)$. But, A0 for $A = -1$, $B = 1$ if $2 = A(3y - 1) + B(1 + 3y)$. |
| | Obtain $A = 1$ and $B = 1$ | **A1** | If work with $x$ and never see $y$, award M1A0, but allow M1A1 if $y$ is seen anywhere on right hand side. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | Separate variables correctly and attempt integration of at least one side | **M1** | Integrate to obtain at least one log term of the form $p\ln(a+by)$ OE, $q\ln\frac{a+by}{a-by}$ on one side, or a tan term on the other, and disregard the 2 if it appears. |
| | Integrate $\frac{2}{1-9y^2}$ to obtain $\frac{1}{3}\ln(1+3y)-\frac{1}{3}\ln(1-3y)$ | **A1FT** | OE, e.g. $\frac{1}{3}\ln\left(\frac{1+3y}{1-3y}\right)$.<br>FT $\frac{A}{3}\ln(1+3y)-\frac{B}{3}\ln(1-3y)$<br>or $\frac{A}{3}\ln(1+3y)+\frac{B}{3}\ln(3y-1)$ if *their* partial fractions used.<br>The '2' must have been dealt with correctly for this mark (check right hand side for 2 appearing here) |
| | Obtain $r\tan 3x$ | **B1** | |
| | Obtain term $\frac{1}{3}\tan 3x$ | **B1** | Allow $\frac{1}{6}\tan 3x$ if '2' not dealt with correctly earlier. |
| | Use $y = 0$ when $x = \frac{1}{12}\pi$ to evaluate a constant or as limits in a solution of the form $p\ln(1+3y)+q\ln(1-3y)+r\tan 3x$ where $p,q,r \neq 0$ | **M1** | $0 + 0 + 1/3 + C = 0$<br>No errors in substitution. |
| | $\frac{1}{3}\ln(1+3y)-\frac{1}{3}\ln(1-3y)=\frac{1}{3}\tan 3x-\frac{1}{3}$<br>Obtain answer $y=\frac{1}{3}-\frac{2}{3\left(1+\mathrm{e}^{\tan 3x-1}\right)}$ or $y=\frac{\mathrm{e}^{\tan 3x-1}-1}{3\left(\mathrm{e}^{\tan 3x-1}+1\right)}$ | **A1** | OE<br>ISW |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | Differentiate $\sqrt{3+2\tan x}$ to obtain $\sec^2 x\left(3+2\tan x\right)^{-\frac{1}{2}}$ | **B1** | OE (can be unsimplified).<br>Not dependent on product rule, but must be convincing or seen in isolation, not as a derivative of the whole expression.<br>B0B0 for e.g.<br>$2\sec x\sec x\tan x\sec^2 x\left(3+2\tan x\right)^{-\frac{1}{2}}$. |
| | Differentiate $\sec^2 x$ to obtain $2\sec x\sec x\tan x$ | **B1** | OE<br>Not dependent on product rule but must be convincing or seen in isolation, not as a derivative of the whole expression.<br>B0B0 for e.g.<br>$2\sec x\sec x\tan x\sec^2 x\left(3+2\tan x\right)^{-\frac{1}{2}}$ OE, e.g. $\dfrac{2\sin x}{\cos^3 x}$. |
| | Use correct product (or quotient) rule | **M1** | $\sec^2 x\ \dfrac{\mathrm{d}}{\mathrm{d}x}(\sqrt{\ldots\ldots}) + \sqrt{\ldots\ldots}\ \dfrac{\mathrm{d}}{\mathrm{d}x}\ (\sec^2 x)$ |
| | [Obtain derivative, if correct<br>$2\sec^2 x\tan x\left(3+2\tan x\right)^{\frac{1}{2}}+\sec^4 x\left(3+2\tan x\right)^{-\frac{1}{2}}$ ]<br>and equate derivative to zero and obtain an equation in one trig function | **M1** | Must include $\left(\ldots\right)^{\frac{1}{2}}$ and $\left(\ldots\right)^{-\frac{1}{2}}$ here.<br>Arithmetical errors only for this M1.<br>May work in terms of sin $x$ and cos $x$. |
| | Obtain $5\tan^2 x + 6\tan x + 1 = 0$ | **A1** | OE, e.g. $5\tan^4 x + 6\tan^3 x + 6\tan^2 x + 6\tan x + 1 = 0$<br>$52\sin^4 x - 28\sin^2 x + 1 = 0$<br>$52\cos^4 x - 76\cos^2 x + 25 = 0$<br>$\cot^2 x + 6\cot x + 5 = 0$ |
| | Obtain AWRT $x = -0.197$ only | **A1** | ISW<br>May be more accurate. |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(b) | $\frac{\mathrm{d}u}{\mathrm{d}x} = 2\sec^2 x$ | **B1** | SOI |
| | Reach an integral of the form $\int A\sqrt{u}\ \mathrm{d}u$ | ***M1** | OE |
| | Obtain $\int \frac{1}{2}u^{\frac{1}{2}}\ \mathrm{d}u$ | **A1** | OE |
| | Obtain $\frac{1}{3}u^{\frac{3}{2}}$ | **A1FT** | OE<br>FT *their* coefficient. |
| | Substitute correct limits correctly in an expression of the form $Bu^{\frac{3}{2}}$<br>or $B(3+2\tan x)^{\frac{3}{2}}$<br>and obtain $c\left(5^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$ | **DM1** | OE<br>$u = 1$ and $u = 5$<br>$x = -\frac{1}{4}\pi$ and $x = \frac{1}{4}\pi$<br>Do not allow only decimals. |
| | Obtain answer $\frac{5\sqrt{5}}{3} - \frac{1}{3}$ | **A1** | Or exact equivalent, e.g. $\frac{\sqrt{125}-1}{3}$.<br>ISW |
| | | **6** | |

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