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Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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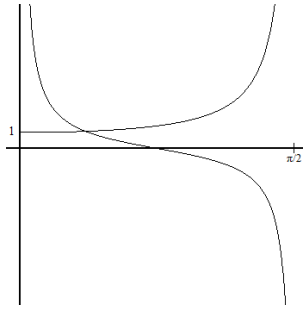
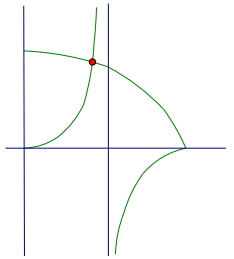
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(9-3x)^{\frac{1}{2}}$ or $\left(1-\frac{1}{3}x\right)^{\frac{1}{2}}$	**M1**	E.g. $-\frac{1}{2}\times\frac{1}{3}x$ or $-\frac{\frac{1}{2}\times\frac{1}{2}}{2}\times\frac{1}{9}x^2$ or $\frac{1}{2}9^{\frac{-1}{2}}(-3x)^1$ or $\frac{\frac{1}{2}\times\frac{-1}{2}}{2}9^{\frac{-3}{2}}(-3x)^2$. Not for symbolic coefficients in the form ${}^{n}\mathrm{C}_{r}$.
	State correct first term 3	**B1**	
	Obtain the next two terms $-\frac{1}{2}x-\frac{1}{24}x^2$	**A1 A1**	A1 for each term correct. Do not ISW.
			SC M1A1 for $1-\frac{1}{6}x-\frac{1}{72}x^2$ seen on its own or as a factor.
		4	

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Question	Answer	Marks	Guidance
2(a)	Sketch a relevant graph, e.g. $y = \cot 2x$  	**B1**	Alt: use $\tan 2x$ and $\cos x$. And only one root in range. (also cross at $\frac{\pi}{2}$) 
	Sketch a second relevant graph on the same axes, e.g. $y = \sec x$ and justify the given statement	**B1**	Need to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
2(b)	State $x = \frac{1}{2}\tan^{-1}(\cos x)$ and rearrange to the **given equation** cot 2x = sec x Note: If using the alternative approach in **(a)**, can stop at $\tan 2x = \cos x$	**B1**	Should see $\tan 2x = \cos x$ before the given conclusion. Or rearrange cot 2x = sec x to $x = \frac{1}{2}\tan^{-1}(\cos x)$ and state iterative formula $x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$.
		1	

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Question	Answer	Marks	Guidance
3	Square $x + iy$ and equate real and imaginary parts to 6 and −8 respectively	***M1**	Condone +8 in place of -8 and/or $i^2 = 1$.
	Obtain equations $x^2 – y^2 = 6$ and $2xy = –8$	**A1**	OE
	Eliminate one variable and find an equation in the other (from 2 equations each in 2 unknowns)	**DM1**	Condone a slip but not seriously incorrect algebra, e.g. use of $x = –4y$ is M0.
	Obtain $x^4 – 6x^2 – 16 = 0$ or $y^4 + 6y^2 – 16 = 0$	**A1**	Accept 3-term equivalents e.g. $x^4 = 6x^2 + 16$. Condone missing '= 0' if implied by subsequent work.
	Obtain answers $\pm\left(2\sqrt{2} – \sqrt{2}i\right)$ or exact equivalents	**A1**	Allow if values of x and y stated separately but the pairing is clear. Ignore additional correct solutions for x and y not real, but A0 if any additional incorrect answers.
		5	

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Question	Answer	Marks	Guidance
4	Use laws of indices correctly and solve for 5^x	***M1**	E.g. obtain $5^x = \frac{5}{12}$ OE. Allow for $y =$ … if they have previously stated $y = 5^x$. Could be implied if they have a correct simplified equation in 5^x, e.g. $12 \times 5^x = 5$.
	Use a correct method for solving an equation of the form $5^x = a$, where $a > 0$	**DM1**	Allow $x \ln 5 = \ln\left(\frac{10}{24}\right)$.
	Obtain answer –0.544	**A1**	CWO. If no working shown, 0/3. Note: 3 dp required.
		3	
5(a)	$\frac{4\pi}{7}$ and/or $-\frac{\pi}{7}$	**M1**	SOI Allow $\frac{4}{7}\pi - \left(-\frac{1}{7}\pi\right)$ or $\frac{4}{7}\pi - \frac{1}{7}\pi$ Note: Many multiply top and bottom by the conjugate, which is fine, but to score the M1 they need to state or imply the argument of a complex number.
	Obtain arg $u = \frac{5}{7}\pi$	**A1**	Do not accept degrees.
		2	

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Question	Answer	Marks	Guidance
5(b)	Reflection (in the) real axis	**B1**	Correct non-contradictory statement. Condone *x*–axis or horizontal axis. Need 'reflection'. Not 'mirror', 'flip'.
	arg $u^* = -\frac{5}{7}\pi$	**B1FT**	FT *their* exact **(a).** Accept $2\pi -$ *their* exact **(a)**. Accept an 'exact' expression in place of an exact value. Need to see a value or an expression. Do not accept $\arg u^* = -\arg u$ without a value seen.
		2	
6	Form a pair of equations in *a* and *b*	***M1**	Condone sign slips but must be using the given coordinates correctly. e.g. $\begin{cases}\ln a + 8.27 = 3.4\ln b \\ \ln a + 2.24 = 0.5\ln b\end{cases}$ or $\begin{cases}ae^{2.24} = b^{0.5} \\ ae^{8.27} = b^{3.4}\end{cases}$
	Carry out a correct method for finding ln*a* or ln*b* or *a* or *b*	**DM1**	Condone sign slip.
	Obtain value $a = 0.3$	**A1**	(0.30109…)
	Obtain value $b = 8$	**A1**	(7.99895…) Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.

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Question	Answer	Marks	Guidance
6	**Alternative Method for Question 6:**		
	Carry out a correct method for finding ln$b$ or b (Need to link the gradient to ln$b$ at some point)	***M1**	Condone sign slips but must be using the given coordinates correctly. $\ln b = \frac{8.27 - 2.24}{3.4 - 0.5} (= 2.079....)$
	Obtain value $b = 8$	**A1**	
	Correct method to find $\pm \ln a$ or a	**DM1**	Condone sign slip $(\ln a = -1.200...)$.
	Obtain value $a = 0.3$	**A1**	Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.
		4	

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Question	Answer	Marks	Guidance
7(a)	Use correct double angle formula to obtain an equation in tan x	**M1**	e.g. $\tan^3 x + \frac{2\times 2\tan x}{1-\tan^2 x} - \tan x (=0)$. Allow if the correct formula is quoted but then they lose the 2 from the numerator when they use the formula.
	Obtain a correct equation in tan x in any form without fractions	**A1**	E.g. $\tan^3 x - \tan^5 x + 4\tan x - \tan x + \tan^3 x (=0)$. Condone if '= 0' is missing here.
	Reduce to the **given answer** of $\tan^4 x - 2\tan^2 x - 3 = 0$ correctly	**A1**	Obtain given answer from correct working but condone if never mention $\tan x \neq 0$. Condone the right terms in a different order 'Show that' so each line must be correct.
		3	
7(b)	A complete correct method to solve the equation to obtain a value for θ	**M1**	$\left(\tan 2\theta = \pm\sqrt{3}\right)$ Allow if they make a slip in copying the equation but do have a complete method to obtain a value of θ. M0 if they get a value for 2θ but never halve it.
	Obtain two of (θ =) $\frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$	**A1**	
	Obtain the other two of (θ =) $\frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$ and no others in the interval	**A1**	Exact, ignore any answers outside interval Accept $\frac{2}{6}\pi$ for $\frac{1}{3}\pi$ and $\frac{4}{6}\pi$ for $\frac{2}{3}\pi$. Do not need to see $\theta = \frac{\pi}{2}$ (from $\tan 2\theta = 0$).
		3	

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Question	Answer	Marks	Guidance
8(a)	Use correct product rule or chain rule to find derivative of x with respect to t	**M1**	Obtain $k\tan 2t\sec^2 2t$.
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = 4\tan 2t\sec^2 2t$ oe	**A1**	
	$\frac{\mathrm{d}y}{\mathrm{d}t} = -2\sin 2t$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$ to obtain **given answer** $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}\cos^3 2t$	**B1**	Condone if $\frac{\mathrm{d}y}{\mathrm{d}x}$ missing.
		4	
8(b)	Obtain $x = 1$ and $y = \frac{\sqrt{2}}{2}$	**B1**	Accept $y = 0.707\ldots$
	State or imply gradient of tangent is $\frac{-\sqrt{2}}{8}$ or gradient of normal is $4\sqrt{2}$	**B1**	Any equivalent form, e.g. $2^{\frac{5}{2}}$. Accept – 0.177 or 5.66.
	Use correct method to find equation of **normal** using *their* values	**M1**	Need a fully substituted equation for the normal (in any form) or to get at least as far as finding value for m and expression for c.
	Obtain equation of normal is $y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$ or equivalent 3 term equation	**A1**	E.g., $y = 5.66x - 4.95$. Must be $y = \ldots$
		4	

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Question	Answer	Marks	Guidance
9(a)	Use a correct method to find $\overrightarrow{OD}$	**M1**	E.g. $\overrightarrow{OC}+3\left(\overrightarrow{OA}-\overrightarrow{OB}\right)=$ $(-3\mathbf{i}-2\mathbf{j}+2\mathbf{k})+3((2\mathbf{i}+\mathbf{j}-3\mathbf{k})-(4\mathbf{j}+\mathbf{k}))$ $\left(\overrightarrow{AB}=-2\mathbf{i}+3\mathbf{j}+4\mathbf{k}\right)$ Accept column vectors throughout.
	Obtain position vector of D is $3\mathbf{i}-11\mathbf{j}-10\mathbf{k}$	**A1**	Accept coordinates.
		2	
9(b)	Carry out correct method for finding a vector equation for $\overrightarrow{AC}$ or $\overrightarrow{BD}$	***M1**	E.g. $2\mathbf{i}+\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+3\mathbf{j}-5\mathbf{k})$ or $4\mathbf{j}+\mathbf{k}+\mu(3\mathbf{i}-15\mathbf{j}-11\mathbf{k})$. Condone missing $\mathbf{r}=\ldots$
	Both diagonal equations correct.	**A1ft**	Seen or implied. Follow their D. Condone missing $\mathbf{r}=\ldots$
	Equate at least two pairs of corresponding components and solve for λ or for μ	**DM1**	Dependent on using relevant lines and two different parameters.
	Obtain $\lambda=-\frac{1}{4}$ or $\mu=\frac{1}{4}$	**A1**	The values will depend on the directions of their lines
	Obtain position vector of P is $\frac{3}{4}\mathbf{i}+\frac{1}{4}\mathbf{j}-\frac{7}{4}\mathbf{k}$	**A1**	OE Accept coordinates. Do not ISW.

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Question	Answer	Marks	Guidance
9(b)	**Alternative Method for Question 9(b):**		
	State or imply $\overrightarrow{AC} = 5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$	**B1 FT**	Or $\overrightarrow{BD} = 3\mathbf{i} - 15\mathbf{j} - 11\mathbf{k}$ Follow *their D* if used.
	Identify similar triangles with ratio 1 : 3	**M1**	
	Use similar triangles to obtain $\overrightarrow{OP}$, e.g. $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AC}$	**M1**	Must be correct fraction.
	Obtain position vector of *P* is $\frac{3}{4}\mathbf{i} + \frac{1}{4}\mathbf{j} - \frac{7}{4}\mathbf{k}$	**A2**	OE Allow A1A0 if any two values are correct.
		5	
9(c)	Find direction vector $\overrightarrow{BA} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ and $\overrightarrow{BC} = -3\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ or equivalent	**B1FT**	Or $\overrightarrow{AB}$ and $\overrightarrow{CB}$. FT if using an incorrect $\overrightarrow{AB}$ from earlier work.
	Carry out correct process for evaluating the scalar product of two relevant vectors	**M1**	Allow if one is going in the negative direction, e.g. $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of their moduli and evaluate the inverse cosine of the result to obtain an angle	**M1**	Independent of the first M1. For their two vectors $\theta = \cos^{-1}\frac{8}{\sqrt{29}\sqrt{46}} = ...$
	Obtain answer 77.3° (or 1.35 radians)	**A1**	77.347… Correctly rounded to more than 3 sf or AWRT 77.3.
		4	

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Question	Answer	Marks	Guidance
10(a)	Obtain $\frac{\mathrm{d}V}{\mathrm{d}t} = 40\pi - 0.8\pi r$ or equivalent	**B1**	Need a complete correct statement seen or implied.
	Obtain $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ or equivalent e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$	**B1**	Need a complete correct statement seen or implied.
	Use the chain rule to obtain **given answer (including the derivative)**	**B1**	Allow if $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{50-r}{5r^2}$ follows $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{40-0.8r}{4r^2}$ without further explanation (π already cancelled) and no incorrect statements seen.
		3	
10(b)	Commence division and reach quotient of the form $-5r \pm 250$ or $5r^2 = (50 - r)(Ar + B) + C$ and reach $A = -5$ and $B = \pm 250$	**M1**	Allow M1 if divide by $r - 50$ to obtain $5r \pm 250$.
	Obtain quotient $-5r - 250$	**A1**	Do not need to state which is quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder 12 500	**A1**	Note: 12 500 following division by $r - 50$ is correct and scores this A1 ISW.
			SC B1 only for correct use of remainder theorem to obtain correct remainder.
		3	

Question	Answer	Marks	Guidance
10(c)	Prepare to integrate e.g. separate variables correctly Or express in the form $\frac{\mathrm{d}t}{\mathrm{d}r} = \frac{5r^2}{50-r}\left(= -(5r+250) + \frac{12500}{50-r}\right)$	**B1FT**	$\int \frac{5r^2}{50-r}\mathrm{d}r = \int 1\mathrm{d}t$ Condone missing dr, dt or missing integral signs, but not both. Follow their division in **(b)** if substitute before separating.
	Obtain term t	**DB1**	
	Obtain terms $\frac{A}{2}r^2 + Br - C\ln(50-r)$	**M1**	From *their* $Ar + B + \frac{C}{50-r}$ in **(b)** where $ABC \neq 0$. Allow a single slip in the coefficients.
	Obtain terms $-\frac{5}{2}r^2 - 250r - 12500\ln(50-r)$	**A1FT**	FT *their* **(b)**, provided of the correct form.
	Use $t = 0$, $r = 0$ to evaluate a constant or as limits in a solution containing terms of the form r^2, r, $\ln(50-r)$ and t	**M1**	
	Obtain final answer $t = -\frac{5}{2}r^2 - 250r - 12500\ln(50-r) + 12500\ln 50$	**A1**	OE Must be $t =$ ….. Allow with $12500\ln 50 = 48900$ or better.
		6	
10(d)	Obtain $t = 70.5$	**B1**	May be more accurate (70.4605…).
		1	

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Question	Answer	Marks	Guidance
11(a)	Use correct quotient rule NB the question asks for $f'(x)$ so need complete form	**M1**	Or correct product rule.
	Obtain correct derivative in any form, e.g. $\dfrac{4e^{2x}(e^{2x}-3e^{x}+2)-2e^{2x}(2e^{2x}-3e^{x})}{\left(e^{2x}-3e^{x}+2\right)^{2}}$	**A1**	
	Equate *their* derivative to zero	***M1**	Can be implied by numerator equated to zero for quotient rule. $\left(8=6e^{x}\right)$
	Solve for x to obtain $x = \ln a$	**DM1**	a positive.
	Obtain $x=\ln\dfrac{4}{3}$ and $y = -16$	**A1**	No errors seen. Accept equivalent exact forms, e.g. $x=\ln\dfrac{8}{6}$.

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Question	Answer	Marks	Guidance
11(a)	**Alternative Method for Question 11(a)**		
	Complete method to express $\mathrm{f}(x)$ in partial fractions	**M1**	As far as $p+\frac{q}{\mathrm{e}^x-2}+\frac{r}{\mathrm{e}^x-1}$ with values for *p*, *q* and *r* $\left(2+\frac{8}{\mathrm{e}^x-2}-\frac{2}{\mathrm{e}^x-1}\right)$. Allow in $u\left(u=\mathrm{e}^x\right)$.
	Differentiate to obtain $\mathrm{f}'(x)=\frac{s\mathrm{e}^x}{\left(\mathrm{e}^x-2\right)^2}+\frac{t\mathrm{e}^x}{\left(\mathrm{e}^x-1\right)^2}$	***M1**	Note: the question requires f'(*x*) so if they have substituted for e^x, they will also need chain rule.
	Obtain $\mathrm{f}'(x)=\frac{-8\mathrm{e}^x}{\left(\mathrm{e}^x-2\right)^2}+\frac{2\mathrm{e}^x}{\left(\mathrm{e}^x-1\right)^2}$	**A1**	From correct work.
	Equate derivative to zero and solve for *x* to obtain $x=\ln a$	**DM1**	Must follow correctly to give a positive value of *a*.
	Obtain $x=\ln\frac{4}{3}$ and $y=-16$	**A1**	No errors seen. Accept $x=\ln\frac{8}{6}$, or equivalent.
		5	

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Question	Answer	Marks	Guidance
11(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$	**B1**	
	Obtain $\int \frac{2u}{u^2-3u+2}\,\mathrm{d}u$ or equivalent Or$_1$ $\int \left(\frac{2}{u}+\right)\frac{8}{u(u-2)} - \frac{2}{u(u-1)}\mathrm{d}u$	**B1**	Correct expression in u. Condone missing du or missing integral but not both. Allow FT if using their partial fractions from **(a).**
	State or imply partial fractions of the form $\frac{A}{u-1}+\frac{B}{u-2}$ Or$_1$ $\frac{C}{u}+\frac{D}{u-2}+\frac{E}{u-1}$ Or$_2$ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u^2-3u+2}=\frac{2u-3}{u^2-3u+2}+\frac{F}{u-2}+\frac{G}{u-1}$	**B1 FT**	Complete reduction to partial fractions. Correct form for *their* integrand.
	Use a correct method for finding a constant	**M1**	Available if they have incorrect form.
	Obtain correct $\frac{-2}{u-1}+\frac{4}{u-2}$ Or$_2$ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u-2}-\frac{3}{u-1}$	**A1**	
	Integrate to obtain $a\ln(u-1)+b\ln(u-2)$ or equivalent	***M1**	M0 if they have additional terms that do not cancel out.
	Obtain correct $-2\ln(u-1)+4\ln(u-2)$ or equivalent	**A1FT**	FT values of *their* partial fraction coefficients.
	Correctly use limits $u = 5$ and 3 in an expression of the form $a\ln(u-1)+b\ln(u-2)$ or $x = \ln 5$ and $\ln 3$ in an expression of the form $a\ln(\mathrm{e}^x-1)+b\ln(\mathrm{e}^x-2)$	**DM1**	
	Obtain $\ln\frac{81}{4}$	**A1**	Accept $\ln 20.25$.
		9	

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