Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5370356203

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A random variable X has the distribution N(410, 400).

Find the probability that the mean of a random sample of 36 values of X is less than 405. [3]

2 In a survey of 300 randomly chosen adults in Rickton, 134 said that they exercised regularly. This information was used to calculate an $\alpha\%$ confidence interval for the proportion of adults in Rickton who exercise regularly. The upper bound of the confidence interval was found to be 0.487, correct to 3 significant figures.

Find the value of α correct to the nearest integer. [4]

3 A website owner finds that, on average, his website receives 0.3 hits per minute. He believes that the number of hits per minute follows a Poisson distribution.

(a) Assume that the owner is correct.

(i) Find the probability that there will be at least 4 hits during a 10-minute period. [3]

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(ii) Use a suitable approximating distribution to find the probability that there will be fewer than 40 hits during a 3-hour period. [4]

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A friend agrees that the website receives, on average, 0.3 hits per minute. However, she notices that the number of hits during the day-time (9.00 am to 9.00 pm) is usually about twice the number of hits during the night-time (9.00 pm to 9.00 am).

(b) **(i)** Explain why this fact contradicts the owner's belief that the number of hits per minute follows a Poisson distribution. [1]

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(ii) Specify separate Poisson distributions that might be suitable models for the number of hits during the day-time and during the night-time. [2]

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4 The masses, in kilograms, of chemicals A and B produced per day by a factory are modelled by the independent random variables X and Y respectively, where $X \sim N(10.3, 5.76)$ and $Y \sim N(11.4, 9.61)$. The income generated by the chemicals is \$2.50 per kilogram for A and \$3.25 per kilogram for B.

(a) Find the mean and variance of the daily income generated by chemical A. [2]

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(b) Find the probability that, on a randomly chosen day, the income generated by chemical A is greater than the income generated by chemical B. [6]

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5 In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

6 A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $\mathrm{P}(a < X < b) = p$ and $\mathrm{P}(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

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(b) Find $\mathrm{P}(b < X < 6 - a)$ in terms of p. [2]

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It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

7 A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

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Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Additional Page

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