Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*2108969358*

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.


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**1** A curve is such that its gradient at a point $(x, y)$ is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve. [4]

**2** The circle with equation $(x - 3)^2 + (y - 5)^2 = 40$ intersects the $y$-axis at points $A$ and $B$.

**(a)** Find the $y$-coordinates of $A$ and $B$, expressing your answers in terms of surds. [2]

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**(b)** Find the equation of the circle which has $AB$ as its diameter. [2]

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**3** **(a)** Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where $a$, $b$ and $c$ are constants to be found. [3]

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**(b)** Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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**4** **(a)** Expand the following in ascending powers of $x$ up to and including the term in $x^2$.

**(i)** $(1 + 2x)^5$. [1]

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**(ii)** $(1 - ax)^6$, where $a$ is a constant. [2]

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In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of $x^2$ is $-5$.

**(b)** Find the possible values of $a$. [4]

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**5** The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

**(a)** Find the possible values of the constant $p$. [3]

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**(b)** One of the values of $p$ found in **(a)** is a negative fraction.

Use this value of $p$ to find the sum to infinity of this progression. [4]

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**6** A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where $c$ is a constant. The line is a tangent to the curve at point $P$.

Find the possible values of $c$ and the corresponding coordinates of $P$. [7]

**7** The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

**(a)** State the range of f. [1]

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**(b)** Obtain an expression for $f^{-1}(x)$ and state the domain of $f^{-1}$. [4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

**(c)** Obtain a simplified expression for $gf(x)$. [2]

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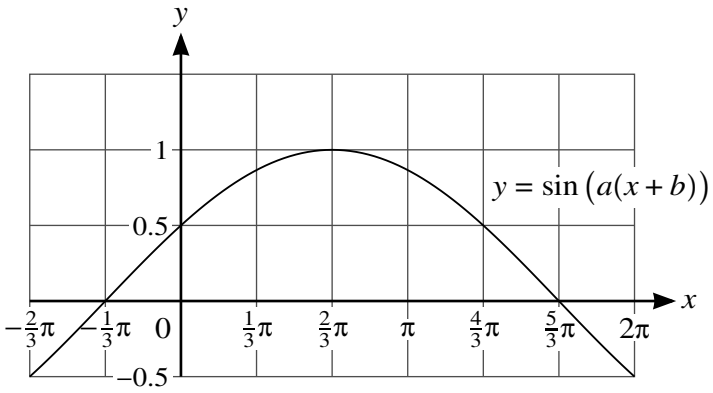
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**8**




The diagram shows part of the graph of $y = \sin\big(a(x + b)\big)$, where $a$ and $b$ are positive constants.

**(a)** State the value of $a$ and one possible value of $b$. [2]

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Another curve, with equation $y = \mathrm{f}(x)$, has a single stationary point at the point $(p, q)$, where $p$ and $q$ are constants. This curve is transformed to a curve with equation

$$y = -3\mathrm{f}\big(\tfrac{1}{4}(x + 8)\big).$$

**(b)** For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of $p$ and $q$. [3]

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**9** A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

**(a)** Find the equation of the normal to the curve at the point $A\,(4,\ 3)$, giving your answer in the form $y = mx + c$. [3]

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A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at $A$ the rate of increase of the $x$-coordinate is $3\,\text{cm}\,\text{s}^{-1}$.

**(b)** Find the rate of increase of the $y$-coordinate at $A$. [2]

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At $A$ the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the $y$-coordinate is constant at $5\,\text{cm}\,\text{s}^{-1}$.

**(c)** As the point moves down the normal, find the rate of change of its $x$-coordinate. [3]

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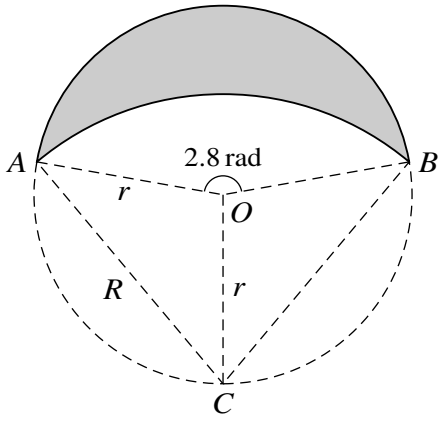
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**10**




The diagram shows points $A$, $B$ and $C$ lying on a circle with centre $O$ and radius $r$. Angle $AOB$ is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre $O$ and radius $r$. The lower arc is part of a circle with centre $C$ and radius $R$.

**(a)** State the size of angle $ACO$ in radians. [1]

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**(b)** Find $R$ in terms of $r$. [1]

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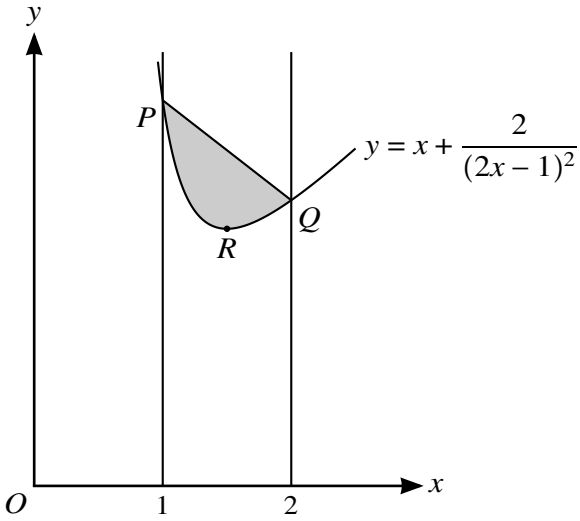
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**(c)** Find the area of the shaded region in terms of $r$. [7]

**11**




The diagram shows part of the curve with equation $y = x + \frac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at $P$ and $Q$ respectively and $R$ is the stationary point on the curve.

**(a)** Verify that the $x$-coordinate of $R$ is $\frac{3}{2}$ and find the $y$-coordinate of $R$. [4]

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**(b)** Find the exact value of the area of the shaded region. [6]

## Additional Page

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