Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*6217921484*

**MATHEMATICS** **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.


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**1** Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where $a$ and $b$ are integers. [4]

**2** The polynomial $2x^3 - x^2 + a$, where $a$ is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$.

**(a)** Find the value of $a$. [2]

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**(b)** When $a$ has this value, solve the inequality $\mathrm{p}(x) < 0$. [4]

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**3** The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the $x$-coordinate of this point, giving your answer correct to 3 significant figures. [6]

**4** **(a)** Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of $R$ and give $\alpha$ correct to 2 decimal places. [3]

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**(b)** Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

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**5** **(a)** Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where $x$ and $y$ are real and exact. [3]

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**(b)** On a sketch of an Argand diagram with origin $O$, show points $A$ and $B$ representing the roots of the equation in part **(a)**. [1]

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**(c)** Find the exact modulus and argument of each root. [3]

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**(d)** Hence show that the triangle $OAB$ is equilateral. [1]

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**6** Relative to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

**(a)** Using a scalar product, find the cosine of angle $BAC$. [4]

**(b)** Hence find the area of triangle $ABC$. Give your answer in a simplified exact form. [4]

**7** The variables $x$ and $\theta$ satisfy the differential equation

$$x \sin^2 \theta \frac{\mathrm{d}x}{\mathrm{d}\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

**(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to $\theta$ is $-\operatorname{cosec}^2 \theta$.) [1]

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**(b)** Solve the differential equation and find the value of $x$ when $\theta = \frac{1}{6}\pi$. [7]

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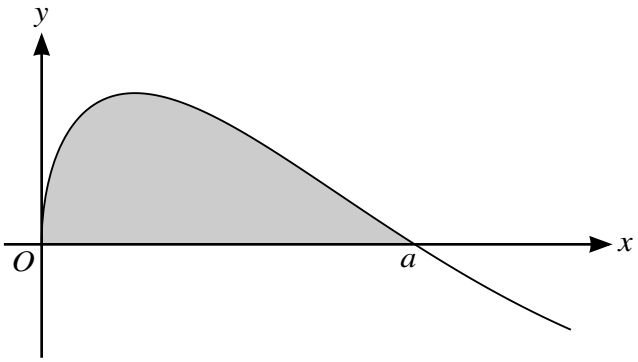
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**8**




The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the $x$-axis at the point where $x = a$.

**(a)** State the exact value of $a$. [1]

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**(b)** Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the $x$-axis. [7]

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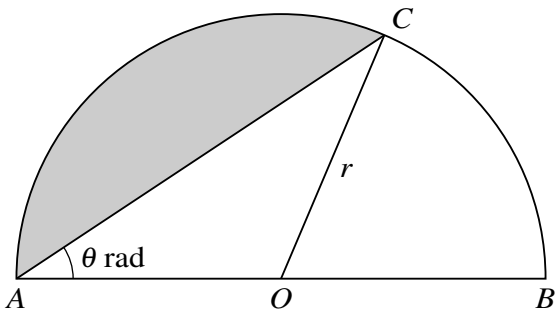
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**9**




The diagram shows a semicircle with diameter $AB$, centre $O$ and radius $r$. The shaded region is the minor segment on the chord $AC$ and its area is one third of the area of the semicircle. The angle $CAB$ is $\theta$ radians.

**(a)** Show that $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$. [4]

**(b)** Verify by calculation that $0.5 < \theta < 0.7$. [2]

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**(c)** Use an iterative formula based on the equation in part **(a)** to determine $\theta$ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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**10** Let $\mathrm{f}(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

**(a)** Express $\mathrm{f}(x)$ in partial fractions. [5]

**(b)** Find the exact value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$. Give your answer as a single logarithm. [5]

## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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