Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

8429791 1

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

© UCLES 2022

BLANK PAGE

1 Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the equation of the circle with centre A which passes through B. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

3 **(a)** Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

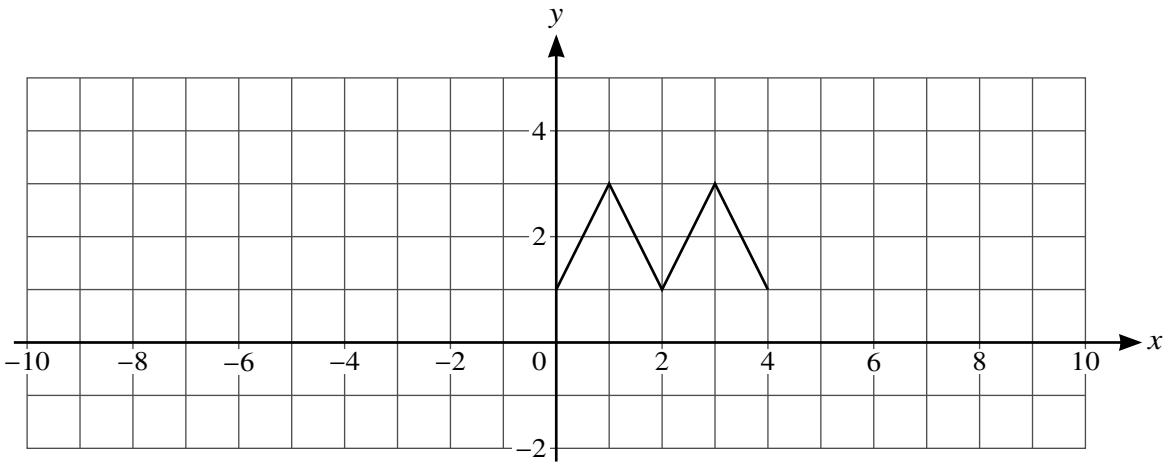
Find the 50th term. [4]

© UCLES 2022

5 The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

7 **(a)** Prove the identity $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$. [3]

(b) Hence find the exact solutions of the equation $\frac{\sin\theta}{\sin\theta+\cos\theta}+\frac{\cos\theta}{\sin\theta-\cos\theta}=2$ for $0\leqslant\theta\leqslant\pi$. [4]

8 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) State the set of values of x for which y increases as x increases. [1]

.....

.....

.....

.....

.....

9 Functions f and g are defined by

$$\mathrm{f}(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$\mathrm{g}(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $\mathrm{gf}(x)$. [1]

.....

.....

.....

.....

.....

(b) Given that $\mathrm{gf}(2) = 11$, find the value of a. [2]

.....

.....

.....

.....

.....

.....

(c) Given that the graph of $y = \mathrm{f}(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

.....

.....

.....

.....

.....

.....

.....

It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(e) Explain why the composite function fg cannot be formed. [1]

.....

.....

.....

.....

.....

.....

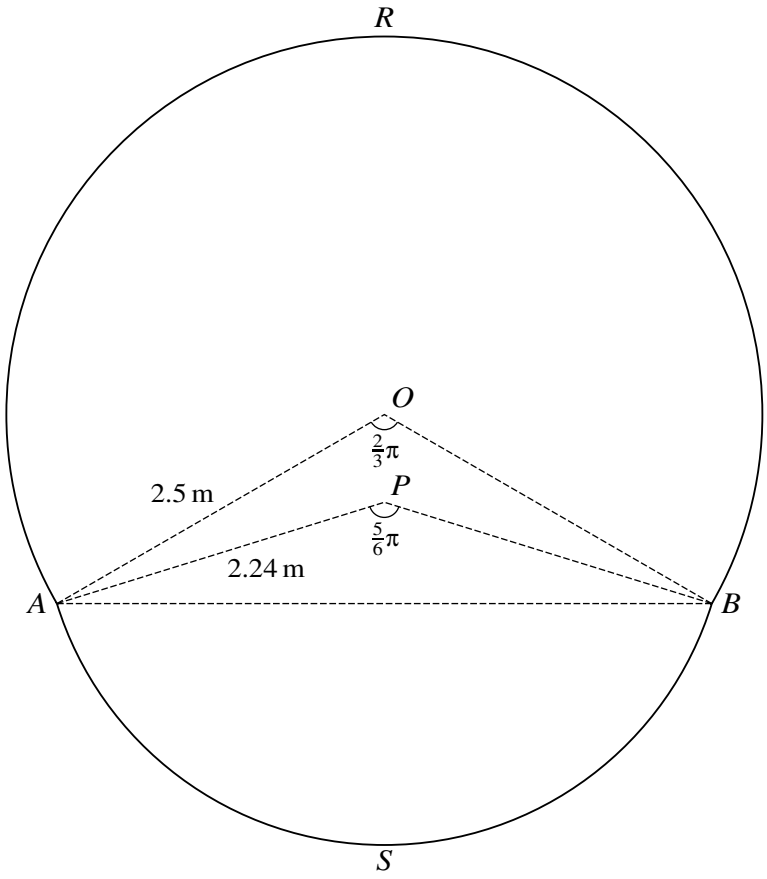
.....

.....

.....

.....

10



The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O, a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 **(a)** Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]

.....

.....

.....

.....

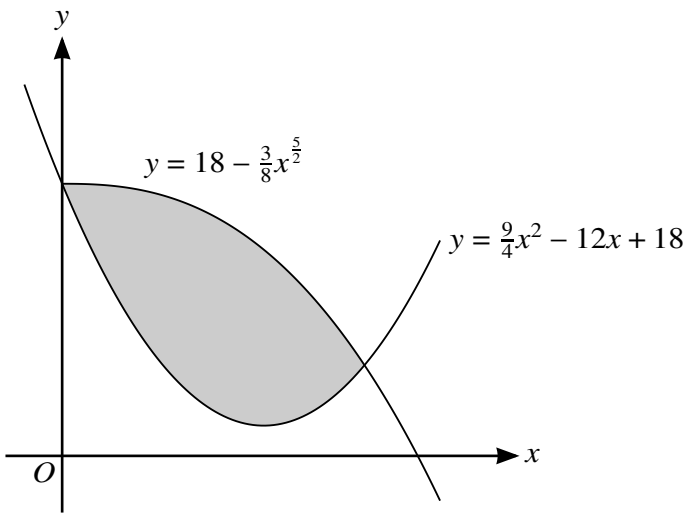
.....

.....

.....

.....

.....



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2022